V.I.ROMANOVSKIY INSTITUTE
OF MATHEMATICS

UNIVERSITY OF EXACT
AND SOCIAL SCIENCES

ABSTRACTS
OF THE INTERNATIONAL CONFERENCE
GIBBS MEASURES AND THE THEORY OF DYNAMICAL SYSTEMS

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V.I.Romanovskiy Institute of Mathematics

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ABSTRACTS

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May 20–21, 2024, Tashkent, Uzbekistan

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Gibbs measures and the theory of dynamical systems: Abstract of the international conference, May 20–21, 2024, Tashkent, Uzbekistan.

MAIN SECTIONS OF THE CONFERENCE:

- Algebra and analysis,
- Continuous time dynamical systems (Differential equations),
- Discrete-time dynamical systems,
- Stochastic processes.

The aim of the scientific conference is to discuss the current state and prospects for the development of the main directions of Gibbs measures and the theory of dynamical systems, as well as their applications in mathematical physics, mathematical biology and the theory of stochastic processes.

The conference languages: Uzbek, English and Russian.

This conference was organized on the basis of resolution №76 of the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan dated March 20, 2024.

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Contents

Abdullaev J.I., Khalkhuzhaev A.M., Makhmudov Kh.Sh. *Bound states of a system of two identical particles on a two-dimensional lattice* 11
Abdumalikov S.Sh. *Boundary value problem for subdiffusion equation on ladder graph with δ' type vertex conditions* 13
Abdurakhimova Sh.B. *Topological conjugacy of quadratic stochastic operators* ... 14
Abdurasulov K.K., Saydaliyev A. *Transposed Poisson structures on quasi-filiform Lie algebras* 16
Absalamov A.T. *Dynamical systems of gonosomal evolution operators with four parameters* 19
Aitzhanov Y.M., Kadyrov S. *Global stability analysis of a novel epidemic model with separate compartments for symptomatic and asymptomatic cases* 21
Akhmedov M.I. *On some classes of regular spaces and their images* 23
Alimov Z.S., Karimov E.T., Kerbal S. *Inverse-initial problem for time-fractional diffusion equation involving the Hadamard derivative* 24
Aliyev A.F. *Gibbs measures on the circle* 26
Arzikulov F.N., Khakimov U.I. *On description of finite dimensional inner Rickart and inner Baer Jordan algebras* 28
Arzikulov F.N., Qurbonov A.X. *On local automorphisms of finite-dimensional naturally graded associative algebras* 30
Ashurov R.R., Sobirov Z.A., Turemuratova A.A. *Inverse source problem for the space-time fractional parabolic equation on a metric star graph* 33
Asqarov J.N. *Translation-invariant Gibbs measures for the modified SOS model on the Cayley tree of order four* 35
Azizova M.A, Latipova D.A. *Spectral properties of the discrete Schrödinger operators* 37
Azizov M., Ismoilov S. *CPA-structure on small-dimensional nilpotent Lie algebras* 40
Bogachev L.V. *Large components in random decomposable structures, and the Bose–Einstein condensation* 42
Boltaev A.T., Turayev A.B. *On the spectrum of two particle discrete Schrödinger operators* 43
Boxonov Z.S. *Global dynamics of a two-dimensional mosquito population* 44
Dzhalilov A.A., Khomidov M.K. *A note on limiting Gibbs distributions for the infinite Toda chain* 46
Egamov D.O. *Periodic ground states corresponding to a non-normal subgroup of index four for the Ising model with competing interactions* 48
Eshimbetov M.R., Otaboyev Sh.I. *The Fokas' method for inverse control problem for heat equation on metric graph* 51
Eshmetova S.D., Khakimov O.N. *On dynamics of infinite dimensional Volterra operator* 52
Eshmurodov Sh.G., Sobirov Z.A. *Inverse source problems for the time-fractional pseudo-parabolic equation with the Riemann-Liouville derivative on a star graph* 54
Gafforov R.A., Kerbal S., Juraboev S.S. *Equivalence of elementary surfaces with respect to the movements in pseudo-Euclidean space* 55
Hakimova M.A. *The generalization of Preston's lemma* 56
Hamrayev A.Yu., To'rayev A.O'. *Dynamics of a quasi non-Volterra cubic stochastic operator* 58
Hasanov A., Rashidov S.G'. *Application of the Hankel transform to solving Cauchy problems for model partial differential equations* 60
Haydarov F.H. *Periodic Gibbs measures for continuous spin models on Cayley trees* 62
Haydarov F.H., Mavlonov I.M. *On positive fixed points of operator of Hammerstein type with degenerate kernel and Gibbs measures* 64
Hryniv O. *Invariance principle for a class of Gibbs random fields* 66
Isakov B.M. *The boundary condition for a mixed type model on the Cayley tree* 67

Jamilov U.U., Aralova K.A. *The dynamics of superposition of Volterra and permuted Volterra quadratic operators* 69
Jamilov U.U., Baratov B.S. *The asymptotical behavior of trajectories of a separable cubic operator* 71
Jamilov U.U., Ibragimov M.M. *On automorphisms of a three-dimensional non-Volterra genetic algebra* 73
Jamilov U.U., Khudoyberdiev Kh.O. *A regular non-Volterra quadratic operator* 75
Jamilov U.U., Mukhitdinov R.T. *On convex combination of non-Volterra quadratic operators* 78
Jamilov U.U., Rajabov S.M. *A regular non-Volterra quadratic stochastic operator* 81
Juraev I.T. *Fixed points for a type of Volterra quadratic operators* 84
Kadyrov S. *Mathematical modeling of prevalence of HIV using continuous dynamical systems* 86
Karshiboev O.Sh., Umaralieva G.Sh. *Periodic Gibbs measures for the Kittel's zipper model on the Cayley tree* 88
Kaygorodov I. *Transposed Poisson algebras* 90
Khakimov R.M., Bozorqulov A.A. *k_m-translation-invariant splitting Gibbs measures in the case $g = wand$* 91
Khakimov R.M., Jurayev Sh.M., Sattarov I.A. *On the translation-invariant p-adic Gibbs measure for theHC model with four states on the Cayley tree: in the "stick" case* 94
Khakimov R.M., Makhammadaliev M.T. *Non uniqueness conditions of translation-invariant Gibbs measures for the hard-core model with a countable set of spin values* 97
Khakimov R.M., Makhammadaliev M., Mutalliev N.N. *Three state hard-core models on the binary tree: translation invariant Gibbs measures* 99
Kholmatov F.M., Mukhamadiev F.G. *A note homotopy properties of the space of compact maximal linked systems* 101
Khomidov M.K. *Limit theorem of rescaled hitting time functions for quadratic irrational rotations* 103
Khudayarov S.S., Dilmurodov E.B. *About dynamic systems of a QnSO* 105
Kurbanbaev T.K. *Almost inner derivations of finite dimensional simple Leibniz algebras* 107
Kurbanov Sh.Kh., Kasimova M.U. *Threshold effect of the generalized Friedrichs model* 108
Lakaev S.N., Kurbanov Sh.Kh., Abduvayitov S.S. *The discrete spectrum of the generalized Friedrichs model with a rank-two perturbation* 110
Lakaev S.N., Turayev A.B. *The number of eigenvalues of the two particle discrete Schrödinger operators* 112
Lingmin L. *Expanding property and statistical laws for p-adic subhyperbolic maps* 114
Meyliev Sh.U., Mukhamadiev F.G. *Completeness type properties of modification of the Niemytzki plane* 115
Mirzayev A.N. *A non-local problem for a time fractional wave equation on metric graph* 116
Mukhamadiev F.G. *Some homotopy properties of the space of thin complete linked systems* 117
Narkuziev B.A., To'xtamurodov J.O. *Application of cubic matrices to cryptography* 119
Nosirova M.S., Mukhamadiev F.G. *On homotopy properties of the $n-$fold symmetric product of topological spaces* 121
Nuriddinov O.O. *Description of local derivations on low-dimensional Jordan algebras* 123
Oltiboyev D.N. *Relation between weighted Green functions and equilibrium states* 126
Orinbaev P.R., Klichbaeva M.A. *The modular and cyclic modular spectrum of partial integral operator in Kaplansy-Hilbert modules* 127

Qirg'izbayeva M.M., Khakimov O.N. *On dynamics of ratsional maps over* $\mathbb{Q}_2$ .. 129
Quttibayev A.U., Khakimov O.N. *On dynamics of Ising-Potts map over* $\mathbb{Q}_2$ 131
Rakhimov A.A., Tukhtasinov T.Sh. *Subadditive measure on projectors of a real W*-algebra* 133
Rakhimov K.U., Tursunov I.I. *Uniqueness of solution of the Cauchy problem for airy-type equations with various fractional derivatives given on the star graph* 135
Rakhimov K. *Dynamics of Henon-like maps* 137
Rahmatullaev M.M., Abdukaxorova Z.T. *Fixed points of a mapping related to the Ising model on* $\mathbb{Q}_p^2$ 138
Rahmatullaev M.M., Abraev B.U. *Ground states and Gibbs measures for the SOS model with an external field and countable set of spin values on a Cayley tree of order two* 140
Rahmatullaev M.M., Burxonova Z.A. *Constructive Gibbs measures for the Ising model on the Cayley tree* 142
Rahmatullaev M.M., Rasulova M.A. *Translation-invariant Gibbs measures for the Potts-SOS model with an external field* 144
Rahmatullayev M.M., Samijonova N.D. *p-Adic quasi Gibbs measures for the Potts model with an external field on a Cayley tree of order two* 146
Rasulova M.A. *H_a-periodic ground states for the Vannimenus model on the Cayley tree of order two* 148
Rasulova M.A., Hakimova M.A. *Periodic ground states for the Chui-weeks's model on the Cayley tree of order two* 150
Rozikov U.A., Axmanova M.M. *On Gibbs measures of p-SOS model on Cayley tree of order three* 152
Rozikov U.A., Djurayeva Ch.G. *On an Hammerstein's integral equation related to Gibbs measures* 154
Sadullaev A.K. *The path in the space* $SP_G^n X$ 156
Samatov B.T., Turgunboyeva M.A. *Differential game with the ℓ-capture problem for Pontryagin's control example* 158
Sattarov I.A. *Criterion for uniqueness and non-uniqueness of the p-adic Markov chain with a countable state* 160
Sattarov I.A., Abdurahmonova M.Sh. *p-Adic dynamical systems of the function* ax^{-k} 162
Sattorov D.X., Khakimov O.N. *Regularity of positive Riesz type stochastic operators* 163
Shermatova Z.Kh. *Transposed Poisson structures on infinite dimensional Lie algebras* 165
Shoyimardonov S.K., Zoyidjonova L.A. *Fixed points and stability of the phytoplankton-zooplankton system* 167
Sneha Latha.M., Parandhama Areti, Kulanov I.B. *MHD Stefan flow of Casson non-fluid through a porous medium in the presence of chemical reaction with the effect of Thompson and Troian slip over a plate in the company of radiation* 169
Sobirov Z.A., Eshimbetov M.R. *Fokas method for the heat equation on metric graphs* 172
Tashpulatov S.M. *Four electron systems in the impurity Hubbard model. First singlet state. One-dimensional case* 174
Tukhtabaev A.M., Samijonova N.D. *Weakly periodic p-adic quasi Gibbs measures for the Potts model on a Cayley tree* 177
Tursunova M.S. *Dynamical system of a quadratic function* 179
Umarov S.R. *On representation formulas of fractional order linear systems* 180
Urazmatov G.Kh. *On maximal solvable extension of given nilradical containing characteristically nilpotent ideals* 181
Xasanova K.E., Khakimov O.N. *On dynamics of linear operators over non-Archimedean sequence spaces* 183
Yusupov B.B. *Local automorphisms of p-filiform Leibniz algebras* 185
Zoyidov A.N. *On σ-compact of modification Niemytzki plane* 188
Zoyidov A.N. *A note modification of Niemytzki plane* 189

Mundarija

Abdulboqiyev X. *Kaputo kasr tartibli operatorni o‘z ichiga oluvchi xususiy hosilali differensial tenglama uchun chegaraviy masala* 190
Abduxakimov S.X., Bayzaqov M.B. *Panjaradagi ikki fermionli sistemaga mos Shrodinger operatorining xos qiymatlari soni va uning joylashgan o‘rni* 192
Ergashov O.H. *Bir nostoxastik kvadratik operator qo‘zg‘almas nuqtalari haqida* 195
Mo‘minov A.T. *To‘rt o‘lchamli filiform Leybnits algebralarining uzunliklari* 197
Nabijonova F.B. *Spin faktorlari bo‘yicha 2-lokal avtomorfizmlar tavsifi* 199
Nishonova Sh.T., Usmonov D.A. *Singulyar koeffitsientga ega bo‘lgan aralash turdagi tenglama uchun bir chegaraviy masala haqida* 200
Olimova D.B. *Ba’zi kichik o‘lchamli Li algebralarining tasvirlari* 201
Omonova A.N. *Involyutsiyali birinchi tartibli differensial tenglama uchun Bitsadze - Samariskiy masalasi* 203
Quljonov O‘.N. *Panjaradagi bir zarrachali Shryodinger operatorining spektral xossalari* 205
Qurbonov H., Absalamova D. *Navbat uzunligi chegaralangan imtiyozli xizmat ko‘rsatish tarmog‘ining bandlik davri* 207
Quziboyev S.H. *Abel C*-algebralarning tasnifi haqida Gelfand-Naymark teoremasi* . 209
Rafiqov A.N., Azimjonov A.P. *Temperlangan Hilfer operatori buziladigan oddiy differensial tenglama uchun Koshi masalasi* 210
Xamdamova D.R., Khakimov O.N. *p-Adik algebraik tenglama va uning tatbiqlari* 212
Yakubova D.U., Sheraliyeva S.A. *Kichik o‘lchamli Shrodinger algebrasining anti-differensiallashlari* 214
Yo‘ldosheva N.A. *Yadrosi ajraluvchi Gammershteyn tipidagi operatorlarning qo‘zg‘almas nuqtalari haqida* 215

Содержание

Апаков Ю.П., Нуритдинова З.Н. *О разрешимости краевой задачи для неоднородного уравнения четвертого порядка с кратными характеристиками* 217
Апаков Ю.П., Хамитов А.А. *Об одной краевой задаче для неоднородного уравнения третьего порядка с кратными характеристиками в трехмерном пространстве* 218
Ахмедов О.У. *О трансляционно-инвариантных p-адических квази-мерах Гиббса для модели SOS* 219
Бегжанова К.У. *Аффинные гомеоморфизмы пространства полуаддитивных функционалов* 221
Бердимуратова Ш.К. *Теорема Гельфанда-Наймарка* 222
Болтаев Х.Х. *Вещественные неизоморфные W*-подалгебры* 224
Водинчар Г.М., Фещенко Л.К. *Составление уравнений некоторых классов динамических систем с заданными инвариантами с помощью символьных вычислений* 226
Даужанов А.Ш. *Теорема типа Картана о дифференциальных свойствах логарифмического потенциала* 228
Дехконов Ж.Д., Нишонова М.А. *О конструктивном описании мер Гиббса для модели Поттса на дереве Кэли* 230
Джалилов Ш.А. *Инвариантная мера и символическая динамика для отображений окружности с изломами* 232
Жураев Ф.М. *Краевая задача на параллельных характеристиках для нагруженного уравнения параболо-гиперболического типа* 234
Иргашев Б.Ю. *Задача Дирихле для уравнения высокого порядка с производной Капуто* 236
Казаков Е.А., Водинчар Г.М. *О динамической системе, являющейся моделью двумодового гидромагнитного динамо с памятью* 238
Ким Д.И. *AW*-модуль над вещественной абелевой AW*-алгеброй* 240
Курбанбаев Т.К., Искендеров А.С. *Почти внутренние дифференцирования нильпотентных n-Лиевых алгебр* 241
Курбанов Х., Базарова У. *Взаимоотношение между распределениями стационарных длин очередей систем M|G|1 и M|G|1|N* 243
Мадгозиев Г.Т. *Неединственности меры Гиббса для одной модели на дереве Кэли* 245
Мадгозиев Г.Т., Юлдашев С.А. *Периодические меры Гиббса для HC-модели на дереве Кэли* 247
Мухтаров Я., Розет И.Г. *Качественное исследование режимов стохастики в одной динамической системе* 249
Отакулов С., Рахимов Б.Ш. *О свойстве локально-относительной управляемости дифференциального включения* 251
Отакулов С., Хайдаров Т.Т. *Минимаксная задача управления для системы с параметрами в условиях неполноты информации* 253
Паровик Р.И. *Исследование хаотических и регулярных режимов динамических систем с помощью алгоритма тест 0-1* 255
Расулов Х.Р. *Об одной динамической системе двуполой популяции* 257
Садуллаев А. *Новый подход к изучению m−выпуклых функций* 259
Тугенов З.Т. *Необходимое условие существования периодических вещественных мер* 262
Тухтамуродова Т.М. *О дискретном спектре одного частично интегрального оператора с вырожденным ядром* 264
Хакимов Р.М., Тожибоев Б.З. *О периодических мерах Гиббса для HC-модели с четырьмя состояниями в случае графа типа "палка"* 266

Хакимов Р.М., Умирзакова К.О. *О мерах Гиббса для одной плодородных НС моделей с жесткими взаимодействиями* 269
Халхужаев А.М., Боймуродов Ж.Х., Буронов Ф.Х. *О связанных состояниях системы трех частиц на решѣтке* 271
Халхужаев А.М., Хужамиеров И.А. *О числе собственных значений оператора Шредингера, соотвстствующего системе трех частиц на решетке* 273
Хатамов Н.М., Маликов Н.Н., Мамаджанова М.М. *Новые неупорядоченные трансляционно-инвариантные меры Гиббса в молекулах ДНК для модели Поттса* 275
Хатамов Н.М., Мамаджанова М.М. *Система функциональных уравнений ренатурации ДНК-РНК для модели Блюма-Капеля с внешним полем* 277
Хатамов Н.М., Муйдинов Д.Н. *2-Периодические градиентные меры Гиббса для модели Sos с альтернативным магнетизмом* 279
Ходжиев Б.Г. *О трансляционно-инвариантных основных состояниях модели SOS с конкурирующими взаимодействиями Блюма-Капеля на дереве Кэли* 282

BOUND STATES OF A SYSTEM OF TWO IDENTICAL PARTICLES ON A TWO-DIMENSIONAL LATTICE

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In this work, we consider the Hamiltonian H of a system of two particles (bosons) on a two-dimensional lattice $\mathbb{Z}^2$ with a potential function $\overline{v}$, whose support is the set $D_2 = \{(n_1, n_2) \in \mathbb{Z}^2 : \ |n_1| \leq 2\}$ which generalize results of [1]. We study the discrete spectrum of the family of Schrödinger operators $H(\mathbf{k})$, $\mathbf{k} = (k_1, k_2) \in \mathbb{T}^2$, (see (1)) corresponding to the Hamiltonian H. Operator $H(\mathbf{k}), \mathbf{k} \in \mathbb{T}^2$ acts on the Hilbert space $L_2^e(\mathbb{T}^2) := \{f \in L_2(\mathbb{T}^2) : f(-\mathbf{q}) = f(\mathbf{q})\}$ by the form

$$H(\mathbf{k}) = H_0(\mathbf{k}) - V. \tag{1}$$

The unperturbed operator $H_0(\mathbf{k})$ is an operator of multiplication by the function

$$\varepsilon_{\mathbf{k}}(\mathbf{q}) = \varepsilon\left(\frac{\mathbf{k}}{2} + \mathbf{q}\right) + \varepsilon\left(\frac{\mathbf{k}}{2} - \mathbf{q}\right) = 4 - 2\cos\frac{k_1}{2}\cos q_1 - 2\cos\frac{k_2}{2}\cos q_2. \tag{2}$$

The perturbation operator V is the integral operator on $L_2^e(\mathbb{T}^2)$ with the kernel $v(\mathbf{q} - \mathbf{s})$,

$$\frac{1}{2\pi}v(\mathbf{q} - \mathbf{s}) = \frac{1}{2\pi}(F\hat{v})(\mathbf{q} - \mathbf{s}).$$

We assume that

$$\hat{v}(\mathbf{n}) = \hat{v}(n_1, n_2) = \begin{cases} \bar{v}(|\mathbf{n}|), & \text{if} \quad |n_1| \leq 2 \\ 0, & \text{if} \quad |n_1| \geq 3, \end{cases} \tag{3}$$

where $\bar{v} : \mathbb{Z}_+ \to \mathbb{R}$ is a decreasing function on $\mathbb{Z}_+ = \mathbb{N}\bigcup\{0\}$ and $\bar{v} \in \ell_2(\mathbb{Z}_+)$, $|\mathbf{n}| = |n_1| + |n_2|$. Additionally, we assume that $\bar{v}(0) > \bar{v}(1) > \bar{v}(2) > \ldots > 0$. Under condition (3) the Hamiltonian $\hat{H}$ is bounded and self-adjoined in the space $\ell_2^{sym}((\mathbb{Z}^2)^2)$ and operator V belongs to the class of Hilbert-Schmidt operators.

It is known that the space $L_2^e(\mathbb{T}^2)$ can be decomposed into the direct sum $L_2^e(\mathbb{T}^2) = L_2^{--}(\mathbb{T}^2) \oplus L_2^{++}(\mathbb{T}^2)$ (see [2]), subspaces $L_2^{--}(\mathbb{T}^2)$ and $L_2^{++}(\mathbb{T}^2)$ are represented as the tensor product $L_2^{--}(\mathbb{T}^2) = L_2^-(\mathbb{T}) \otimes L_2^-(\mathbb{T})$, and $L_2^{++}(\mathbb{T}^2) = L_2^+(\mathbb{T}) \otimes L_2^+(\mathbb{T})$, where $L_2^-(\mathbb{T}) = \{f \in L_2(\mathbb{T}) : f(-q) = -f(q)\}$ and $L_2^+(\mathbb{T}) = \{f \in L_2(\mathbb{T}) : f(-q) = f(q)\}$.

For any $n \in \mathbb{N}$ we introduce the following notations

$$C^+(n) = \bar{v}(n+1) + 2\bar{v}(n+2) + \sqrt{\bar{v}^2(n+1) + 4\bar{v}^2(n+2)},$$

$$C^-(n) = \bar{v}(n+1) + 2\bar{v}(n+2) - \sqrt{\bar{v}^2(n+1) + 4\bar{v}^2(n+2)}$$

and

$$card\left\{n\in\mathbb{N}: C^{+}(n)>2\cos\frac{k_1}{2}\right\}=n^{+}(k_1),\quad card\left\{n\in\mathbb{N}: C^{-}(n)>2\cos\frac{k_1}{2}\right\}=n^{-}(k_1).$$

Note that subspaces $L_2^{--}(\mathbb{T}^2)$ and $L_2^{++}(\mathbb{T}^2)$ are invariant under the operator $H(\mathbf{k})$.

Since the space $L_2^e(\mathbb{T}^2)$ is expressed as the direct sum of the subspaces $L_2^{--}(\mathbb{T}^2)$ and $L_2^{++}(\mathbb{T}^2)$, the operator $H(\mathbf{k})$ is also expressed as the direct sum of the operators $H^{--}(\mathbf{k})$ and $H^{++}(\mathbf{k})$, i.e. $H(\mathbf{k})=H^{--}(\mathbf{k})\oplus H^{++}(\mathbf{k})$.

Theorem 1. The operator $H^{--}((k_1,\pi))$ has exactly $n^{+}(k_1)+n^{-}(k_1)$ eigenvalues (with the multiplicity) lying below the essential spectrum.

Theorem 2. The operator $H^{++}((k_1,\pi))$ has an infinite number eigenvalues lying below the essential spectrum.

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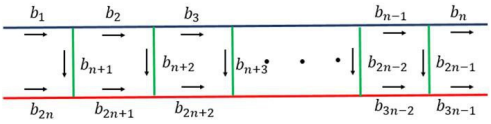
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BOUNDARY VALUE PROBLEM FOR SUBDIFFUSION EQUATION ON LADDER GRAPH WITH δ' TYPE VERTEX CONDITIONS

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We consider the ladder type metric graph Γ with equal bonds given in the figure below.



We denote edges of the graph as $e_k,\ k = \overline{0, 3n-1}$. We define the directions on the graph as it shown in the figure and define coordinates in each edge from 0 to L.

On the each edge of the above defined graph Γ we consider fractional differential equations:

$$\partial_{0t}^{\alpha} u^{(k)}(x,t) = u_{xx}^{(k)}(x,t) + f^{(k)}(x,t),\ x \in e_k, 0 < t \le T, k = \overline{1, 3n-1}, \tag{1}$$

where $\partial_{0t}^{\alpha}(\cdot)$ is the Caputo fractional derivative with $0 < \alpha < 1$ and $f^{(k)}(x,t)$ are given functions.

Subject to the vertex conditions

$$u_k|_{x=L} = u_{k+1}|_{x=0} = u_{k+n}|_{x=0},$$

$$u_{kx}|_{x=L} = u_{(k+1)x}|_{x=0} + u_{(k+n)x}|_{x=0}, k = 1, 2, ..., n-1 \tag{2}$$

$$u_{k-(n-1)}|_{x=L} = u_k|_{x=L} = u_{k+1}|_{x=0},$$

$$u_{(k-n+1)x}|_{x=L} + u_{kx}|_{x=L} = u_{(k+1)x}|_{x=0}, k = 2n, 2n+1, ..., 3n-2, 0 \le t \le T, \tag{3}$$

boundary conditions

$$u_j|_{x=0} = 0, j \in \{1, 2n\}, u_l|_{x=L} = 0, l \in \{n, 3n-1\}, t \ge 0, \tag{4}$$

and initial condition

$$u_k|_{t=0} = \varphi_k(x),\ x \in \bar{e_k},\ k = 1, 2, ..., 3n-1, \tag{2}$$

where $\varphi^{(k)}(x)$ are sufficiently smooth given functions.

We construct the solution of the problem using the method of separating of variables. Uniqueness of the solution is shown using the method of energy integrals.

TOPOLOGICAL CONJUGACY OF QUADRATIC STOCHASTIC OPERATORS

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Let S^{m-1} be the simplex:

$$S^{m-1} = \{x = (x_1, ..., x_m) \in R^m : x_i \geq 0, \sum_{i=1}^{m} x_i = 1\}. \quad (1)$$

In this work we study the topological conjugacy between some eigenstates of a quadratic stochastic operator with variable coefficients, that is mapping S^1 into itself.

For a variable coefficient $p(x)$ define an evolution operator $V_{a,b,c} : z = (x, y) \in S^1 \to z' = (x', y') \in S^1$ as the following:

$$V \equiv V_{a,b,c} : \begin{cases} x' = x^2 + 2p(x)xy, \\ y' = 2(1 - p(x))xy + y^2 \end{cases} \quad (2)$$

where

$$p(x) = \begin{cases} a, & x \leq \frac{1}{3}, \\ b, & \frac{1}{3} < x < \frac{2}{3}, \\ c, & x \geq \frac{2}{3} \end{cases}$$

with $a, b, c \in [0, 1]$. One has to study the dynamical system generated by the evolution operator $V_{a,b,c}$. But this problem can be reduced to the operators which are not topological conjucate. The notion of topological conjugacy is important in the studying of iterated functions and more generally dynamical systems ,if the dynamics of one iterated function can be solved, then those for any topologically conjugate function follow trivially ([1]-[5]). We give its definition.

Definition. Let $f : A \to A$ and $g : B \to B$ be two maps. f and g are said to be topologically conjugate if there exists a homeomorphism $h : A \to B$ such that, $h \circ f = g \circ h$. The homeomorphism h is called a topologically conjugacy.

Using $x + y = 1$ the operator (2) can be reduced to the mapping $f_{a,b,c} : [0, 1] \to [0, 1]$ defined by

$$f_{a,b,c}(x) = \begin{cases} (1 - 2a)x^2 + 2ax, & x \leq \frac{1}{3}, \\ (1 - 2b)x^2 + 2bx, & \frac{1}{3} < x < \frac{2}{3}, \\ (1 - 2c)x^2 + 2cx, & x \geq \frac{2}{3} \end{cases} \quad (3)$$

here $a, b, c \in [0, 1]$.

The cases: $a \neq \frac{1}{2}$, $b = \frac{1}{2}$, $c = \frac{1}{2}$ and $a = \frac{1}{2}$, $b = \frac{1}{2}$, $c \neq \frac{1}{2}$.

In this cases the functions are

$$f_a(x) = \begin{cases} (1 - 2a)x^2 + 2ax, & x \in [0, \frac{1}{3}], \\ x, & x \in (\frac{1}{3}, 1] \end{cases} \quad (4)$$

and

$$g_c(x) = \begin{cases} x, & x \in [0, \frac{2}{3}), \\ (1-2c)x^2 + 2cx, & x \in [\frac{2}{3}, 1] \end{cases} \tag{5}$$

The sets of fixed points are $Fix(f_a) = \{0\} \cup (\frac{1}{3}, 1]$ and $Fix(g_c) = [0, \frac{2}{3}) \cup \{1\}$.

Theorem. The functions $f_a(x)$ and $g_c(x)$ are topologically conjugate if $a + c = 1$.

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TRANSPOSED POISSON STRUCTURES ON QUASI-FILIFORM LIE ALGEBRAS

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Bai et.al. [1] introduced a dual notion of the Poisson algebra, which is called a *transposed Poisson algebra*, by changing the roles of the two multiplications in the Leibniz rule. Poisson algebra is introduced to commutative associative algebras by their derivation. Similarly, on a Lie algebra, the concept of a transposed Poisson algebra is defined by the $\frac{1}{2}$-derivation.

Definition 1. Let $\mathfrak{L}$ be a vector space equipped with two bilinear operations

$$\cdot,\ [-,-] : \mathfrak{L} \otimes \mathfrak{L} \to \mathfrak{L}.$$

The triple $(\mathfrak{L}, \cdot, [-,-])$ is called a **Poisson algebra** if $(\mathfrak{L}, \cdot)$ is a commutative associative algebra and $(\mathfrak{L}, [-,-])$ is a Lie algebra which satisfies the compatibility condition

$$[x, y \cdot z] = [x, y] \cdot z + y \cdot [x, z]. \tag{1}$$

Eq. (1) is called **Leibniz rule** since the adjoint operators of Lie algebra are derivations of the commutative associative algebra.

Definition 2. Let $\mathfrak{L}$ be a vector space equipped with two bilinear operations

$$\cdot,\ [-,-] : \mathfrak{L} \otimes \mathfrak{L} \to \mathfrak{L}.$$

The triple $(\mathfrak{L}, \cdot, [-,-])$ is called a **transposed Poisson algebra** if $(\mathfrak{L}, \cdot)$ is a commutative associative algebra and $(\mathfrak{L}, [-,-])$ is a Lie algebra which satisfies the following compatibility condition

$$2z \cdot [x, y] = [z \cdot x, y] + [x, z \cdot y]. \tag{2}$$

Eq. (2) is called **transposed Leibniz rule** because the roles of two binary operations in the Leibniz rule in a Poisson algebra are switched. Further, the resulting operation is rescaled by introducing a factor 2 on the left-hand side.

Definition 3. Let $(\mathfrak{L}, [-,-])$ be an algebra with a multiplication $[-,-]$, and φ be a bilinear map. Then φ is a $\frac{1}{2}$-**derivation** if it satisfies:

$$\varphi([x, y]) = \frac{1}{2}([\varphi(x), y] + [x, \varphi(y)]). \tag{3}$$

Note that $\frac{1}{2}$-derivations are a particular case of δ-derivations introduced by Filippov in [2]. It is easy to see from Definition 3 that $[\mathfrak{L}, \mathfrak{L}]$ and $Ann(\mathfrak{L})$ are invariant under any $\frac{1}{2}$-derivation of $\mathfrak{L}$. Definition 2 and Definition 3 immediately imply the following lemma.

Lemma 1. Let $(\mathfrak{L}, [-,-])$ be a Lie algebra and $\cdot$ a new binary (bilinear) operation on $\mathfrak{L}$. Then $(\mathfrak{L}, \cdot, [-,-])$ is a transposed Poisson algebra if and only if $\cdot$ is commutative and associative and for every $z \in \mathfrak{L}$ the multiplication by z in $(\mathfrak{L}, \cdot)$ is a $\frac{1}{2}$-derivation of $(\mathfrak{L}, [-,-])$.

Let L be an n-dimensional quasi-filiform Lie algebra:

$$g^1_{(n,1)}: \begin{cases} [e_1,e_i]=e_{i+1}, & 2\le i\le n-2,\\ [e_i,e_{n-i}]=(-1)^i e_n, & 2\ \le i\le \frac{n-1}{2},\ n\ge 5\ \ and\ \ n\ is\ odd \end{cases}$$

where $\{e_1,e_2,\dots,e_n\}$ is a basis of the algebra.

We will derive and calculate $\frac{1}{2}$-derivation of quasi-filiform Lie algebra.

Theorem 1. Any $\frac{1}{2}$-derivations of the algebra $g^1_{(n,1)}$ has the form

- for $n=5$:

$$\begin{cases} \varphi(e_1)=\alpha_1e_1+\alpha_2e_2+\alpha_3e_3+\alpha_4e_4+\alpha_5e_5,\ \varphi(e_2)=\beta_1e_1+\beta_2e_2+\beta_3e_3+\beta_4e_4+\beta_5e_5,\\ \varphi(e_3)=\frac{1}{2}(\alpha_1+\beta_2)e_3+\frac{1}{2}\beta_3e_4-\frac{1}{2}\alpha_3e_5,\\ \varphi(e_4)=\frac{1}{4}(3\alpha_1+\beta_2)e_4+\frac{1}{2}\alpha_2e_5,\ \varphi(e_5)=\frac{1}{2}\beta_1e_4+\frac{1}{4}(\alpha_1+3\beta_2)e_5, \end{cases}$$

- for $n\ge 7$:

$$\begin{cases} \varphi(e_1)=\sum\limits_{i=1}^{n}\alpha_ie_i,\ \varphi(e_2)=\alpha_1e_2+\beta_{n-2}e_{n-2}+\beta_{n-1}e_{n-1}+\beta_ne_n,\\ \varphi(e_3)=\alpha_1e_3+\frac{1}{2}\beta_{n-2}e_{n-1}-\frac{1}{2}\alpha_{n-2}e_n,\\ \varphi(e_i)=\alpha_1e_i+\frac{(-1)^i}{2}\alpha_{n-i+1}e_n,\ 4\le i\le n-1,\ \varphi(e_n)=\alpha_1e_n. \end{cases}$$

Now, we will describe all transposed Poisson algebra structures on quasi-filiform Lie algebra.

Theorem 2. Let $(g^1_{(5,1)},\cdot,[-,-])$ be a transposed Poisson algebra structure defined on the Lie algebra $g^1_{(5,1)}$. Then the multiplication of $(g^1_{(5,1)},\cdot)$ has the following form:

$\mathbf{TP}_1(g^1_{(5,1)})$: $e_1\cdot e_1=\alpha_4e_4+\alpha_5e_5,\ e_1\cdot e_2=\beta_4e_4+\beta_5e_5,\ e_2\cdot e_2=\beta_9e_4+\beta_{10}e_5$;

$\mathbf{TP}_2(g^1_{(5,1)})$: $\begin{cases} e_1\cdot e_1=e_3+\alpha_4e_4+\alpha_5e_5,\ e_1\cdot e_2=\beta_3e_3+\beta_4e_4+\beta_5e_5,\ e_1\cdot e_3=\frac{1}{2}\beta_3e_4-\frac{1}{2}e_5,\\ e_2\cdot e_2=\beta_3^2e_3+\beta_9e_4+\beta_{10}e_5,\ e_2\cdot e_3=\frac{1}{2}\beta_3^2e_4-\frac{1}{2}\beta_3e_5; \end{cases}$

$\mathbf{TP}_3(g^1_{(5,1)})$: $\begin{cases} e_1\cdot e_1=\alpha_1e_1+e_2+\alpha_3e_3+\alpha_4e_4+\alpha_5e_5,\\ e_1\cdot e_2=-\frac{1}{4}(\alpha_1-\beta_2)^2e_1+\beta_2e_2-\frac{1}{2}\alpha_3(\alpha_1-\beta_2)e_3+\beta_4e_4+\beta_5e_5,\\ e_1\cdot e_3=\frac{1}{2}(\alpha_1+\beta_2)e_3-\frac{1}{4}\alpha_3(\alpha_1-\beta_2)e_4-\frac{1}{2}\alpha_3e_5,\\ e_1\cdot e_4=\frac{1}{4}(3\alpha_1+\beta_2)e_4+\frac{1}{2}e_5,\\ e_1\cdot e_5=-\frac{1}{8}(\alpha_1-\beta_2)^2e_4+\frac{1}{4}(\alpha_1+3\beta_2)e_5,\\ e_2\cdot e_2=-\frac{1}{4}(\alpha_1-\beta_2)^2\beta_2e_1+\frac{1}{4}\left(-\alpha_1^2-2\alpha_1\beta_2+3\beta_2^2\right)e_2+\frac{1}{4}\alpha_3(\alpha_1-\beta_2)^2e_3+\\ \qquad+\frac{1}{8}\Big(\alpha_1^2(\alpha_5\beta_2-\beta_5)-\beta_2(2\alpha_4\beta_2-\alpha_5\beta_2^2-10\beta_4+\beta_2\beta_5)+\\ \qquad+2\alpha_1\left(\alpha_4\beta_2-\alpha_5\beta_2^2-\beta_4+\beta_2\beta_5\right)\Big)e_4+\\ \qquad+\frac{1}{4}\left(2\beta_4-2\alpha_4\beta_2-3\alpha_5\beta_2^2+3\alpha_1(\alpha_5\beta_2-\beta_5)+7\beta_2\beta_5\right)e_5,\\ e_2\cdot e_3=\frac{1}{4}(\beta_2^2-\alpha_1^2)e_3+\frac{1}{8}\alpha_3(\alpha_1-\beta_2)^2e_4+\frac{1}{4}\alpha_3(\alpha_1-\beta_2)e_5,\\ e_2\cdot e_4=\frac{1}{4}\alpha_1(\beta_2-\alpha_1)e_4+\frac{1}{2}\beta_2e_5,\\ e_2\cdot e_5=-\frac{1}{8}(\alpha_1-\beta_2)^2\beta_2e_4-\frac{1}{4}\left(\alpha_1^2+\alpha_1\beta_2-2\beta_2^2\right)e_5,\\ e_3\cdot e_3=\frac{1}{8}(\beta_2^2-\alpha_1^2)e_4-\frac{1}{4}(\alpha_1+\beta_2)e_5; \end{cases}$

where the transposed Poisson algebra has its products with respect to the bracket $[-,-]$, and the remaining products are equal to zero.

Theorem 3. Let $(g^1_{(n,1)}, \cdot, [-,-])$ be a transposed Poisson algebra structure defined on the Lie algebra $g^1_{(n,1)}$ and $n \geq 7$. Then, the multiplication of $(g^1_{(n,1)}, \cdot)$ has the following form:

$$\mathbf{TP}_1(g^1_{(n,1)}): \begin{cases} e_1 \cdot e_1 = \sum\limits_{j=4}^{n} \alpha_j e_j,\ e_1 \cdot e_2 = \beta_1 e_{n-2} + \beta_2 e_{n-1} + \beta_3 e_n, \\ e_1 \cdot e_3 = \frac{1}{2}\beta_1 e_{n-1} - \frac{1}{2}\alpha_{n-2} e_n,\ e_1 \cdot e_j = \frac{(-1)^j}{2}\alpha_{n-j+1} e_n,\ 4 \leq j \leq n-3, \\ e_2 \cdot e_2 = \beta_4 e_{n-2} + \beta_5 e_{n-1} + \beta_6 e_n,\ e_2 \cdot e_3 = \frac{1}{2}\beta_4 e_{n-1} - \frac{1}{2}\beta_1 e_n; \end{cases}$$

$$\mathbf{TP}_2(g^1_{(n,1)}): \begin{cases} e_1 \cdot e_1 = e_3 + \sum\limits_{j=4}^{n} \alpha_j e_j,\ e_1 \cdot e_2 = \beta_1 e_{n-2} + \beta_2 e_{n-1} + \beta_3 e_n, \\ e_1 \cdot e_3 = \frac{1}{2}\beta_1 e_{n-1} - \frac{1}{2}\alpha_{n-2} e_n,\ e_1 \cdot e_j = \frac{(-1)^j}{2}\alpha_{n-j+1} e_n,\ 4 \leq j \leq n-3, \\ e_1 \cdot e_{n-2} = -\frac{1}{2} e_n,\ e_2 \cdot e_2 = \beta_4 e_{n-1} + \beta_5 e_n,\ e_2 \cdot e_3 = -\frac{1}{2}\beta_1 e_n; \end{cases}$$

$$\mathbf{TP}_3(g^1_{(n,1)}): \begin{cases} e_1 \cdot e_1 = e_2 + \sum\limits_{j=3}^{n} \alpha_j e_j,\ e_1 \cdot e_2 = 2\beta_1 e_{n-1} + \beta_2 e_n, \\ e_1 \cdot e_3 = -\frac{1}{2}\alpha_{n-2} e_n,\ e_1 \cdot e_j = \frac{(-1)^j}{2}\alpha_{n-j+1} e_n,\ 4 \leq j \leq n-2, \\ e_1 \cdot e_{n-1} = \frac{1}{2} e_n,\ e_2 \cdot e_2 = \beta_1 e_n; \end{cases}$$

where the transposed Poisson algebra has its products with respect to the bracket $[-,-]$, and the remaining products are equal to zero.

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DYNAMICAL SYSTEMS OF GONOSOMAL EVOLUTION OPERATORS WITH FOUR PARAMETERS

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In this piece of work we study dynamical system generated by a concrete non-linear multidimensional operator describing a gonosomal evolution. Our model is related to a bisexual population. We note that investigation of dynamical systems generated by evolution operators of free and bisexual population can be reduced to the study of nonlinear dynamical systems (see [1], [2], [3], for more details). Since there is no any general theory of non-linear operators, for each concrete such operator one has to use a specific method of investigation.

We discuss one example of sex-linked inheritance. Hemophilia is a lethal recessive X-linked disorder: a female carrying two alleles for hemophilia die. Therefore, if we denote by X^h the gonosome X carrying the hemophilia, there are only two female genotypes: XX and XX^h (X^hX^h is lethal) and two male genotypes: XY and X^hY. We have four types of crosses defined as

$$\begin{aligned} XX \times XY &\to aXX,\ (1-a)XY,\\ XX \times X^hY &\to bXX^h,\ (1-b)XY,\\ XX^h \times XY &\to bXX,\ (a-b)XX^h,\ cXY, (1-a-c)X^hY,\\ XX^h \times X^hY &\to bXX^h,\ dXY,\ (1-b-d)X^hY. \end{aligned}$$

Let $F = \{XX, XX^h\}$ and $M = \{XY, X^hY\}$ be the sets of female and male genotypes respectively. Assume that state of the set F is given by a real vector (x, y) and state of M by a real vector (u, v). Then a state of the set $F \cup M$ is given by the vector $t = (x, y, u, v) \in \mathbb{R}^4$.

We consider the following subsets of $\mathbb{R}^4$:

$$S^3 = \Big\{ s = (x, y, u, v) \in \mathbb{R}^4 :\ x \geq 0,\ y \geq 0,\ u \geq 0,\ v \geq 0,\ x + y + u + v = 1 \Big\}$$

which is the three-dimensional simplex and

$$\Theta = \{ s = (x, y, u, v) \in S^3 : (x, y) = (0, 0)\ or\ (u, v) = (0, 0) \};$$
$$S^{2,2} = S^3 \setminus \Theta.$$

If $t' = (x', y', u', v')$ is a state of the system $F \cup M$ in the next generation, then by the above rule we get the evolution operator $W : S^{2,2} \to S^{2,2}$ defined by

$$W : \begin{cases} x' &= \dfrac{axu + byu}{(x+y)(u+v)},\\ y' &= \dfrac{bxv + (a-b)yu + byv}{(x+y)(u+v)},\\ u' &= \dfrac{(1-a)xu + (1-b)xv + cyu + dyv}{(x+y)(u+v)},\\ v' &= \dfrac{(1-a-c)yu + (1-b-d)yv}{(x+y)(u+v)}. \end{cases} \tag{1}$$

where a, b, c, $d \in (0;1)$ and $a+c<1, \quad b+d<1$.

The main problem for a given operator W and arbitrarily initial point $s^{(0)} \in S^{2,2}$, is to describe the limit points of the trajectory $\{s^{(m)}\}_{m=0}^{\infty}$, where $s^{(m)} = W^m(s^{(0)}) = \underbrace{W(W(...W}_{m}(s^{(0)}))...)$.

For the fixed points and for the trajectories of the operator W we have the following results:

Theorem. The operator $W : S^{2,2} \to S^{2,2}$ given by (1) has unique fixed point $s_0 = (a, 0, 1-a, 0)$ and for any initial point $s \in S^{2,2}$ we have

$$\lim_{m\to\infty} W^m(s) = s_0 = (a, 0, 1-a, 0). \tag{2}$$

Let $s^{(0)} = (x^{(0)}, y^{(0)}, u^{(0)}, v^{(0)}) \in S^{2,2}$ be an initial state (the probability distribution on the set $\{XX, XX^h; XY, X^hY\}$ of genotypes). This Theorem has the following biological interpretations: when time goes to infinity, the population tends to the equilibrium state $s_0 = (a, 0, 1-a, 0)$, meaning that the future of the population is stable: genotypes XX and XY are survived always, but the genotypes XX^h and X^hY will asymptotically disappear. Consequently, only healthy chromosomes will survive.

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GLOBAL STABILITY ANALYSIS OF A NOVEL EPIDEMIC MODEL WITH SEPARATE COMPARTMENTS FOR SYMPTOMATIC AND ASYMPTOMATIC CASES

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In this study, we explore the global stability of a novel epidemic model that integrates reported and unreported cases (Eq.(1)), distinguishing between symptomatic and asymptomatic individuals. Using a Lyapunov function, we demonstrate the model's stability, highlighting the crucial role of asymptomatic cases in shaping disease dynamics and control effectiveness. Our findings contribute to biomathematics, emphasizing the need for ongoing research to refine our understanding and control strategies. We suggest future research directions, emphasizing the need for ongoing studies to refine our understanding and control strategies.

SEI_rI_uR model:

$$\begin{cases} \frac{dS}{dt} = \Lambda - \frac{\beta S(I_r+I_u)}{N} - \mu S + \alpha R \\ \frac{dE}{dt} = \frac{\beta S(I_r+I_u)}{N} - \lambda_r E - \lambda_u E - \mu E \\ \frac{dI_r}{dt} = \lambda_r E - \gamma I_r - \delta I_r - \mu I_r \\ \frac{dI_u}{dt} = \lambda_u E - \gamma I_u - \mu I_u \\ \frac{dR}{dt} = \gamma I_r + \gamma I_u - \mu R - \alpha R \end{cases} \tag{1}$$

Using Next generation matrix method [2] we find basic reproduction number of Eq.(1):

$$\mathcal{R}_0 = \frac{\beta\Lambda}{N\mu(\lambda_r+\lambda_u+\mu)}\left(\frac{\lambda_r}{\mu+\gamma+\delta}+\frac{\lambda_u}{\mu+\gamma}\right)$$

Our main theorem is

Theorem 1. For $\mathcal{R}_0 < 1$, the disease free equilibrium ε_0 of the system **??** is globally asymptotically stable (GAS) on $\mathcal{G} = \{(S, E, I_r, I_u, R) \in \mathbb{R}^5_+ \text{ such that } S+E+I_r+I_u+R \leq \frac{\Lambda}{\mu}\}$.

Also we investigate elasticity of $\mathcal{R}_0$ and make fitting for UK data. We make simulations using real-life initial conditions.

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ON SOME CLASSES OF REGULAR SPACES AND THEIR IMAGES

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A space X is *developable* [2, 3] if there exists a sequence $\{\gamma_m : m \in \mathbb{N}\}$ of open covers of X such that, for each $x \in X$, $\{st(x, \gamma_m) : m \in \mathbb{N}\}$ is a local base at x.

Such a sequence of covers is called a *development* for X. It is well known that every metrizable space is developable, and every developable space is clearly first countable.

A regular developable space is a *Moore space* [2, 3].

A family $\mathcal{U}$ is called *σ-closure preserving* [1] if it is represented as a union of countably many closure preserving subfamilies.

An *M_1-space* [2, 3] is a regular space having a σ-closure preserving base.

A collection $\mathcal{B}$ (not necessarily open) of subsets of a regular space X is a *quasi-base* [1] if whenever $x \in X$ and U is a neighbourhood of x, there exists a $B \in \mathcal{B}$ such that $x \in IntB \subset B \subset U$.

An *M_2-space* [2, 3] is a regular space having a σ-closure preserving quasi-base.

We received the following results.

Theorem 1. Let a mapping $f : X \to Y$ is an injective, continuous open mapping. If a space X is developable space, then the space Y is also developable space.

Theorem 2. Let a mapping $f : X \to Y$ is an injective, continuous open mapping. If a space X is Moore space, then the space Y is also Moore space.

Theorem 3. Let a mapping $f : X \to Y$ is an injective, continuous open mapping. If a space X is M_1-space, then the space Y is also M_1-space.

Theorem 4. Let a mapping $f : X \to Y$ is an injective, continuous open mapping. If a space X is M_2-space, then the space Y is also M_2-space.

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INVERSE-INITIAL PROBLEM FOR TIME-FRACTIONAL DIFFUSION EQUATION INVOLVING THE HADAMARD DERIVATIVE

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It is well-known that many wave processes were modeled using time-fractional wave equations [1]. There are many works devoted to the studying direct and inverse problems for time-fractional wave equations with local and non-local conditions (see for instance, [2-4]).

In the present work, we have considered an inverse-initial problem for the time-fractional wave equation with the Hadamard fractional derivative in a rectangular domain. Let us consider an equation

$$ {}_HD^{\alpha}_{1t}u(t,x) - u_{xx}(t,x) = f(t,x) \tag{1}$$

in a rectangular domain $\Omega = \{(t,x) : 0 < x < 1,\ 1 < t < T\}$. Here $f(t,x)$ is a given function, T is a positive real number and

$$ {}_HD^{\alpha}_{1t}g(t) = \left(t\frac{d}{dt}\right)^n \frac{1}{\Gamma(n-\alpha)} \int\limits_1^t \left(\log\frac{t}{s}\right)^{n-\alpha+1} \frac{g(s)}{s}ds \ \ t > 1$$

represents the Hadamard fractional derivative of order $\alpha (1 < \alpha \le 2$, $\log(..) = \ln(..)$ [5].

Problem. To find a pair of functions $\{u(t,x); \psi(x)\}$ satisfying

- the equation (1) in Ω;
- regularity conditions $u(\cdot,x) \in C^{\alpha}_{\gamma,\log}[1,T]$, $u(t,\cdot) \in C^1[0,1] \cap C^2(0,1)$;
- boundary conditions $u(t,0) = u(t,1) = 0$, $1 \le t \le T$;
- initial conditions

$$ {}_HI^{2-\alpha}_{1t}u(t,x)\big|_{t=1+} = \varphi(x),\ 0 \le x \le 1,\ {}_HD^{\alpha-1}_{1t}u(t,x)\big|_{t=1+} = \psi(x),\ 0 < x < 1;$$

- over-determination condition $u(\xi,x) = \zeta(x)$, $0 \le x \le 1$ for fixed $\xi \in (1,T]$.

Here $\varphi(x)$ and $\zeta(x)$ are given functions, ${}_HI^{\beta}_{1t}$ represents the Hadamard fractional integral of order $\beta > 0$ given

$$ {}_HI^{\beta}_{1t}g(t) = \frac{1}{\Gamma(\beta)} \int\limits_1^t \left(\log\frac{t}{s}\right)^{\beta-1} \frac{g(s)}{s}ds,\ t > 1,$$

class of functions $C^{\alpha}_{\delta,\gamma}(..)$ with $0<\gamma\leq 1$ is given by [5]

$$C^{n}_{\delta,\gamma}[a,b]=\left\{g:\ \|g\|_{C^{n}_{\delta,\gamma}}=\sum_{k=0}^{n-1}\|\delta^{k}g\|_{C}+\|\delta^{n}g\|_{C_{\gamma,\log}}\right\},\ C^{0}_{\delta,\gamma}[a,b]=C_{\gamma,\log}[a,b],\ \delta=t\frac{d}{dt}.$$

We will use the method of separation of variables and will obtain series form solution as follows:

$$u(t,x)=\sum_{k=1}^{\infty}\left[\varphi_k(t-1)^{\alpha-1}E_{\alpha,\alpha}[-(k\pi)^2(\log t)^{\alpha}]+\psi_k(t-1)^{\alpha-2}E_{\alpha,\alpha-1}\left[-(k\pi)^2(\log t)^{\alpha}\right]+\right.$$

$$\left.+\int_1^t\left(\log\frac{t}{s}\right)^{\alpha-1}E_{\alpha,\alpha}\left[-(k\pi)^2\left(\log\frac{t}{s}\right)^{\alpha}\right]f_k(s)\frac{ds}{s}\right]\sin k\pi x,$$

$$\psi(x)=\sum_{k=1}^{\infty}\frac{1}{(\xi-1)^{\alpha-2}E_{\alpha,\alpha-1}\left[-(k\pi)^2(\log\xi)^{\alpha}\right]}\left[\zeta_k-\varphi_k(\xi-1)^{\alpha-1}E_{\alpha,\alpha}[-(k\pi)^2(\log\xi)^{\alpha}]-\right.$$

$$\left.-\int_1^{\xi}\left(\log\frac{\xi}{s}\right)^{\alpha-1}E_{\alpha,\alpha}\left[-(k\pi)^2\left(\log\frac{\xi}{s}\right)^{\alpha}\right]f_k(s)\frac{ds}{s}\right]\sin k\pi x;$$

Here φ_k, ψ_k, ζ_k and $f_k(t)$ are the Fourier coefficients of functions $\varphi(x),\psi(x),\zeta(x)$ and $f(t,x)$, correspondingly.

We impose certain conditions on the given functions to provide uniform convergence of corresponding infinite series.

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GIBBS MEASURES ON THE CIRCLE

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Thermodynamic formalism is one of the best tools to study circle homeomorphism with singularities. Ya. G. Sinai constructed in [1] the first example of a thermodynamic formalism for Anosov's flows, which was generalized later in the works of D. Ruelle [2], R. Bowen [3] and others for Smale's Axiom A systems. E B. Vul, Ya. G. Sinai and K. M. Khanin finally succeeded in establishing in [4] a thermodynamic formalism approach to Feigenbaum universality in families of critical interval maps, the first example of such an approach to nonhyperbolic systems, whereas the standard approach by Feigenbaum and others has been the renormalization group well known from statistical mechanics.

A.Dzhalilov constructed in [5] the first example of a thermodynamic formalism for circle homeomorphisms with rotation number $\rho = \frac{\sqrt{5}-1}{2}$ and critical singularity point. In [5], he constructed a Gibbs measure which is an invariant measure of shift map on the symbolic space. This provided calculating the Hausdorff dimension of the invariant measure for critical circle maps. After that, this approach can be seen in the work of many authors (see [6], [7], [8]).

For any circle homeomorphisms with same rotation number, their symbolic spaces are same. So we consider a symbolic space for only the simplest circle homeomorphism $T\rho(x) = x + \rho \mod 1$. In this work, we show that what kind of circle map correspond to shift map of that symbolic space. Naturally, invariant measures of that circle map is analogue of Gibbs measure of symbolic dynamics.

Let $T_\rho(x) = x+\rho \mod 1$ be circle homeomorphism with golden mean rotation number $\rho = \frac{\sqrt{5}-1}{2}$. Denote by q_n Fibonacci sequence, i. e., $q_0 = 1$, $q_1 = 1$, $q_{n+1} = q_n + q_{n-1}$, $n \geq 1$. Fix any point $x_0 \in S^1$. Its forward orbit $O^+_{T_\rho}(x_0) = \{x_i = T^i(x_0),\ i = 0, 1, 2, ...\}$ defines a sequence of natural partitions of the circle. Denote by $I^{(n)}_0(x_0)$ the closed interval in S^1 with endpoints x_0 and x_{q_n}. And also we define $I^{(n)}_i(x_0) = T^i(I^{(n)}_0(x_0)), i \geq 1$. Now, we can denote $P_n(x_0)$ n th dynamical partition with starting point x_0 as

$$P_n(x_0) = \{I^{(n)}_i(x_0),\ 0 \leq i < q_{n+1}\} \cup \{I^{(n+1)}_j(x_0),\ 0 \leq j < q_n\}.$$

Now we construct symbolic space using the dynamical partition $P_n(x_0)$. For this we take an arbitrary point $x \in S^1 \setminus O^+_{T_\rho}(x_0)$. If $x \in I^{(n+1)}_j(x_0),\ 0 \leq j < q_n\}$, for $n \geq 0$ $a_{n+1} := a_{n+1}(x) = a$. If $x \in I^{(n)}_i(x_0),\ 0 \leq i < q_{n+1}\}$, we know from the construction of the partition P_{n+1} from P_n that either $x \in I^{(n+2)}_i(x_0)$ or $x \in I^{(n+1)}_{i+q_n}(x_0)$. In the first case we put $a_{n+1} = 0$ and in the second $a_{n+1} = 1$. In this way we get a one-to-one correspondence

$$\varphi : S^1 \setminus O^+_{T_\rho}(x_0) \leftrightarrow \Theta := \{(a_1, ...a_n, ...),\ a_n \in \{a, 0, 1\};\ a_{n+1} = a \Leftrightarrow a_n = 0,\ n \geq 1\}.$$

Now we denote shift map as $\sigma((a_1, a_2, a_3, ...)) = (a_2, a_3, ...)$.

Theorem. The circle map which conjugate with σ shift map by φ is

$$T(x) = \begin{cases} (-x + \rho)\rho^{-1}, & \text{if } x \in [0, \rho); \\ (-x + 1)\rho^{-1}, & \text{if } x \in [\rho, 1). \end{cases}$$

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ON DESCRIPTION OF FINITE DIMENSIONAL INNER RICKART AND INNER BAER JORDAN ALGEBRAS

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The present paper is devoted to the classification problem of inner RJ-algebras. Inner RJ-algebras were introduced and studied in the papers [2], [1] and [3]. An inner RJ-algebra (inner BJ-algebra) is defined as follows: a Jordan algebra $\mathcal{J}$ is called an inner RJ-algebra (respectively an inner BJ-algebra) if, for each element $a \in \mathcal{J}^2$ (respectively subset $\mathcal{S} \subseteq \mathcal{J}^2$), there exists an idempotent $e \in \mathcal{J}$ such that ${}^{\perp_q}\{x\} \cap \mathcal{J}^2 = U_e(\mathcal{J}) \cap \mathcal{J}^2$ (respectively ${}^{\perp_q}\mathcal{S} \cap \mathcal{J}^2 = U_e(\mathcal{J}) \cap \mathcal{J}^2$), where $U_e(x) = 2(e \circ x) \circ e - e^2 \circ x$, $\mathcal{J}^2 := \{a^2 : a \in \mathcal{J}\}$, for each nonempty subset $\mathcal{S}$ of a Jordan algebra $\mathcal{J}$, the (inner) quadratic annihilator of $\mathcal{S}$ is defined by

$$ {}^{\perp_q}\mathcal{S} := \{a \in \mathcal{J} : U_s a = 0 \text{ for all } s \in \mathcal{S}\}.$$

Inner RJ- and BJ-algebras are also mentioned in [5].

$\mathcal{J}$ is an inner BJ-algebra if, and only if, it is an inner RJ-algebra and the set of all idempotents of $\mathcal{J}$ is a complete lattice (cf. [2, Theorem 3.5]). Let $\mathcal{J}$ be a Jordan algebra, S be an arbitrary subset of $\mathcal{J}$ and, let

$$\mathcal{J}^2 = \{a^2 : a \in \mathcal{J}\},\ {}^{\perp_q}S := \{x \in \mathcal{J} : U_a x = 0, (\forall a \in S)\}.$$

Then, for a Jordan algebra $\mathcal{J}$, we consider the following condition:

(A) for every element $x \in \mathcal{J}$, there exists an idempotent $e \in \mathcal{J}$ such that

$$ {}^{\perp_q}\{x\} \cap \mathcal{J}^2 = U_e(\mathcal{J}) \cap \mathcal{J}^2.$$

Definition. A Jordan algebra satisfying condition (A) is called an inner RJ-algebra, and condition (A) itself is called inner Rickart condition.

Theorem 1. Let $\mathcal{J}$ be a nilpotent Jordan algebra. Then $\mathcal{J}$ is an inner RJ-algebra if and only if, for any element a in $\mathcal{J}$, $a^2 = 0$.

Theorem 2. Let $\mathcal{J}$ be an inner RJ-algebra with an idempotent e and a finite-dimensional associative nilradical $\mathcal{N}$ such that $\mathcal{J} = \mathbb{F}e \dot{+} \mathcal{N}$. Then, for any basis $\{e_1, e_2, \ldots, e_n\}$ of $\mathcal{N}$, the following conditions are valid

$$e_i e_j = 0,\ i, j = 1, 2, \ldots, n, \tag{1.1}$$

$$e(ee_i) = ee_i,\ i = 1, 2, \ldots, n. \tag{1.2}$$

$$ee_i \in \mathcal{N}, e_i e \in \mathcal{N}, \tag{1.3}$$

Conversely, any Jordan algebra with a basis $\{e, e_1, e_2, \ldots, e_n\}$, where e is an idempotent element, satisfying the properties (1.1)–(1.3), is an inner RJ-algebra.

Theorem 3. Let $\mathcal{J}$ be a finite-dimensional inner RJ-algebra over an algebraically closed field $\mathbb{F}$ of characteristic $\neq 2$ with a semisimple subalgebra $\mathcal{S}$ without nilpotent elements and an associative nilpotent radical $\mathcal{N}$ such that $\mathcal{J} = \mathcal{S} \dot{+} \mathcal{N}$. Then, if, for any idempotent $e \in \mathcal{S}$ and element $a \in \mathcal{D}$, $a^2 = 0$ and $2ea \neq a$, then $\mathcal{J}$ is an inner BJ-algebra.

In the following table indecomposable Jordan algebras over an algebraically closed field $\mathbb{F}$ of characteristic $\neq 2$ of low dimension is listed. The third column of the tables indicates whether the corresponding Jordan algebra is an inner RJ-algebra or not, i.e., if yes, then sign + is put in the appropriate place of the column, if not, then sign - is put. All notations of table 1 are taken from [4].

Table1

Two-dimensional Jordan algebras.

J	Multiplication table	Is this algebra an inner RJ-algebra (BJ-algebra)?	According to which theorem?
$\mathcal{B}_1$	$e_1^2 = e_1$, $e_1 n_1 = n_1$,	+	Theorem 2
$\mathcal{B}_2$	$e_1^2 = e_1$, $e_1 n_1 = \frac{1}{2} n_1$,	-	Theorem 2
$\mathcal{B}_3$	$n_1^2 = n_2$	-	Theorem 1

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ON LOCAL AUTOMORPHISMS OF FINITE-DIMENSIONAL NATURALLY GRADED ASSOCIATIVE ALGEBRAS

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The concept of local derivations was introduced by R. Kadison in his paper [4]. In this paper R. Kadison proved that each continuous local derivation from a von Neumann algebra into its dual Banach bemodule is a derivation. In [3] B.Jonson extended the above result by proving that every local derivation from a C*-algebra into its Banach bimodule is a derivation. Based on these results, many authors have studied local derivations on operator algebras.

Present paper is devoted to the study of local automorphisms on associative algebras. Local automorphisms of certain finite-dimensional simple Lie and Leibniz algebras are investigated in [1]. T.Becker, J.Escobar, C.Salas, and R.Turdibaev proved that the set of local automorphisms $LAut(sl_2)$ coincides with the group $Aut^{\mp}(sl_2)$ of all automorphisms and anti-automorphisms. Later M.Costantini proved that a linear map on a simple Lie algebra is a local automorphism if and only if it is either an automorphism or an anti-automorphism. F.N.Arzikulov, I.A.Karimjanov, and S.M.Umrzaqov established that every local automorphism on the solvable Leibniz algebras with null-filiform and naturally graded non-Lie filiform nilradicals, whose dimension of complementary space is maximal, is an automorphism. Recently, local automorphisms of Cayley algebras were considered in [2]. In the present paper automorphisms and local automorphisms of five-dimensional naturally graded associative algebra with characteristic sequence $C(A) = (3, 2)$ are studied. Namely, a general form of the matrices of automorphisms and local automorphisms of these algebras is clarified. It turns out that the general form of the matrix of an automorphism on these algebras does not coincide with the local automorphism's matrix's general form on these algebras. Therefore, these associative algebras have local automor-phisms that are not automorphisms. Let A be a naturally graded n-dimensional quasi-filiform associative algebra. Then, there are two possibilities for the characteristic sequence, either $C(A) = (n - 2, 1, 1)$ or $C(A) = (n - 2, 2)$.

The associative algebras

$$\pi_2 : \begin{cases} e_1e_1 = e_2, \\ e_1e_2 = e_2e_1 = e_3, \\ e_1e_4 = e_4e_1 = e_5, \\ e_4e_4 = e_5 \end{cases} \qquad \pi_3 : \begin{cases} e_1e_1 = e_2, \\ e_1e_2 = e_2e_1 = e_3, \\ e_1e_4 = e_5, \\ e_4e_4 = e_5 \end{cases}$$

are five-dimensional naturally graded associative algebras with characteristic sequence $C(A) = (3, 2)$ (Karimjonov-Ladra). Now we give a description of automorphisms of the associative algebras π_2 and π_3.

Definition. Let A be an algebra. A bijective linear map $\Phi{:}A \to A$ is called an automorphism, if, for any elements $x,\ y \in A$, $\Phi(xy) = \Phi(x)\Phi(y)$.

Theorem 1. A linear operator on the associative algebra π_2 is an automorphism if

and only if the matrix of this linear operator has the following matrix form

$$\begin{pmatrix} \alpha_{11} & 0 & 0 & 0 & 0 \\ \alpha_{21} & \alpha_{11}^2 & 0 & 0 & 0 \\ \alpha_{31} & 2\alpha_{11}\alpha_{21} & \alpha_{11}^3 & \alpha_{34} & 0 \\ \alpha_{41} & 0 & 0 & \alpha_{11}+\alpha_{41} & 0 \\ \alpha_{51} & 2\alpha_{11}\alpha_{41}+\alpha_{41}^2 & 0 & \alpha_{54} & (\alpha_{11}+\alpha_{41})^2 \end{pmatrix}$$

Theorem 2. A linear operator on the associative algebra π_3 is an automorphism if and only if the matrix of this linear operator has the following matrix form

$$\begin{pmatrix} \alpha_{11} & 0 & 0 & 0 & 0 \\ \alpha_{21} & \alpha_{11}^2 & 0 & 0 & 0 \\ \alpha_{31} & 2\alpha_{11}\alpha_{21} & \alpha_{11}^3 & \alpha_{34} & 0 \\ 0 & 0 & 0 & \alpha_{11} & 0 \\ \alpha_{51} & 0 & 0 & \alpha_{54} & \alpha_{11}^2 \end{pmatrix}$$

Now we give a description of local automorphisms of the associative algebras π_2 and π_3.

Definition. Let A be an algebra. A linear map $\Phi{:}A \to A$ is called a local automorphism, if for any element $v \in A$ there exists an automorphism $\varphi_v{:}A \to A$ depending on v such that $\Phi(v) = \varphi_v(v)$.

Theorem 3. A linear operator on the associative algebra π_2 is a local automorphism if and only if the matrix of this linear operator has the following matrix form

$$\begin{pmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{22}^2 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33}^3 & b_{34} & 0 \\ b_{41} & 0 & 0 & b_{44} & 0 \\ b_{51} & b_{52} & 0 & b_{54} & b_{55}^2 \end{pmatrix}$$

Theorem 4. A linear operator on the associative algebra π_3 is a local automorphism if and only if the matrix of this linear operator has the following matrix form

$$\begin{pmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{11}^2 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33}^3 & b_{34} & 0 \\ 0 & 0 & 0 & b_{11} & 0 \\ b_{51} & 0 & 0 & b_{54} & b_{33}^2 \end{pmatrix}$$

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INVERSE SOURCE PROBLEM FOR THE SPACE-TIME FRACTIONAL PARABOLIC EQUATION ON A METRIC STAR GRAPH

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We consider the star metric graph Γ with n bonds consists of a finite set of vertices $V = \{\nu_k\}_0^n$ and a finite set of edges $E = \{e_k\}_1^n$, where e_k connect the vertices ν_0 and ν_k, $k = \overline{1,n}$. By assigning the interval $(0, l_k)$, to the bond e_k, $k = \overline{1,n}$ of the graph we define the coordinates x_k on each of the bonds. Where the vertex ν_0 of the graph has a coordinate 0 in each bond. Further, without loss of generality, we will use x instead of x_k.

On each edge of the graph we consider the following space-time fractional parabolic equations

$$\partial_{0,t}^{\alpha} u_k(x,t) + \partial_{x,l_k}^{\beta}(\gamma_k(x) \mathrm{D}_{0,x}^{\beta} u_k(x,t)) = f(t) g_k(x,t) + h_k(x,t), \;\; x \in e_k, \;\; t \in (0,T], \;\; k = \overline{1,n}, \tag{1}$$

with the initial conditions

$$u_k(x,0) = 0, \quad x \in \bar{e}_k, \quad k = \overline{1,n},$$

the vertex conditions

$$I_{0,x}^{1-\beta} u_1(0,t) = I_{0,x}^{1-\beta} u_2(0,t) = ... = I_{0,x}^{1-\beta} u_n(0,t),$$

$$\sum_{k=1}^{n} (\gamma_k(0) \mathrm{D}_{0,x}^{\beta} u_k(0,t)) = 0, \;\; t \in (0,T],$$

the boundary conditions

$$I_{0,x}^{1-\beta} u_k(l_k,t) = 0, \;\; t \in (0,T], \;\; k = \overline{1,n},$$

and the overdetermination condition

$$\sum_{k=1}^{n} \int_0^{l_k} \eta_k(x) u_k(x,t)\, dx = \psi(t), \quad t \in (0,T],$$

where $\partial_{0,t}^{\alpha}$ denotes the generalized left Caputo fractional derivative of order $\alpha \in (0,1)$ with respect to the time variable t, $\mathrm{D}_{0,x}^{\beta}$ and ∂_{x,l_k}^{β}, $k = \overline{1,n}$ stand for the left Riemann Liouville and the right Caputo generalized fractional derivatives of order $\beta \in (\frac{1}{2}, 1)$ with respect to the spatial variable x, $I_{0,x}^{1-\beta}$ is the left Riemann-Liouville fractional integral of order $1-\beta$. The real-valued functions γ_k are absolutely continuous on $[0, l_k]$ and $0 < p_1 \leq |\gamma_k| \leq p_2$. $g_k(x,t)$, $h_k(x,t)$, $k = \overline{1,n}$, $\psi(t)$ are given functions, $f(t)$ is an unknown function.

We consider the inverse problem of determining the pair of functions $\{u(x,t), f(t)\}$ in the equation (1) in a bounded domain $G_T = \Gamma \times (0,T]$ in Sobolev spaces [1]. G.Leugering et al. [2], investigated the optimal control problems for parabolic fractional Sturm-Liouville equations on a star graph. They solved the problem using the method of Galerkin and obtained a weak solution. In our case, we use the functional method based on the priori estimates to show the uniqueness and existence of a strong solution.

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TRANSLATION-INVARIANT GIBBS MEASURES FOR THE MODIFIED SOS MODEL ON THE CAYLEY TREE OF ORDER FOUR

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The Cayley tree Γ^k (see, e.g., [1-4]) of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, from each vertex of which exactly $k+1$ edges issue. Let $\Gamma^k = (V, L, i)$, where V is the set of vertices of Γ^k, L is the set of edges of Γ^k and i is the incidence function associating each edge $l \in L$ with its endpoints $x, y \in V$. It is known (see [1]) that there exists a one-to-one correspondence between the set V of vertices of the Cayley tree of order $k \geq 1$ and the group G_k of the free products of $k+1$ cyclic groups $\{e, a_i\}$, $i = 1, ..., k+1$ of the second order (i.e. $a_i^2 = e$, $a_i^{-1} = a_i$) with generators $a_1, a_2, ..., a_{k+1}$.

We consider model, where the spin takes values in the set $\Phi = \{-1, 0, 1\}$.

The Hamiltonian of the modified SOS model has the form

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L} |\sigma(x) - \sigma(y)| \cos[\pi(\sigma(x) - \sigma(y))], \tag{1}$$

where $J \in \mathbb{R}$ is a nonzero coupling constant, $\sigma \in \Phi^V$.

Let $h : x \mapsto h_x = (h_{-1,x}, h_{0,x}, h_{1,x}) \in \mathbb{R}^3$ be a real vector-valued function of $x \in V \setminus \{x^0\}$. We consider the probability distributions $\mu^{(n)}$ on Φ^{V_n} for given $n = 1, 2, ...$ defined by

$$\mu^{(n)}(\sigma_n) = Z_n^{-1} \exp\big(-\beta H(\sigma_n) + \sum_{x \in W_n} h_{\sigma(x),x}\big), \tag{2}$$

where $\sigma_n \in \Phi^{V_n}$ and Z_n is the partition function. We say that the probability distributions $\mu^{(n)}$ are compatible if $\forall n \geq 2$ and $\sigma_{n-1} \in \Phi^{V_{n-1}}$:

$$\sum_{\omega_n \in \Phi^{W_n}} \mu^{(n)}\big(\sigma_{n-1} \vee \omega_n\big) = \mu^{(n-1)}\big(\sigma_{n-1}\big), \tag{3}$$

here $\big(\sigma_{n-1} \vee \omega_n\big) \in \Phi^{V_n}$ is the concatenation of σ_{n-1} and ω_n.

If for probability distributions $\mu^{(n)}$ on Φ^{V_n} the equality (3) holds, then by the Kolmogorov extension theorem there is a unique measure μ on Φ^V such that, $\forall n$ and $\sigma_n \in \Phi^{V_n}$,

$$\mu\big(\{\sigma \mid \sigma_{V_n} = \sigma_n\}\big) = \mu^{(n)}(\sigma_n).$$

The measure μ is called splitting Gibbs measure (SGM) corresponding to Hamiltonian H and function $h : x \mapsto h_x, x \neq x^0$.

The next theorem expresses the requirement on function h that the distributions $\mu^{(n)}(\sigma_n)$ satisfy the compatibility conditions.

Theorem 1. Defined as in (2) the probability distributions $\mu^{(n)}(\sigma_n), n = 1, 2, ...,$ satisfy the compatibility condition if and only if

$$h_x^* = \sum_{y \in S(x)} F\big(h_y^*, \theta\big) \tag{4}$$

is satisfied for any $x \in V \setminus \{x^0\}$, where $\theta = \exp(J\beta)$ and also $\beta = 1/T$ is the inverse temperature. Here h_x^* represents the vector $(h_{-1,x} - h_{1,x}, h_{0,x} - h_{1,x})$ and the vector function $F(\cdot, \theta) : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as follows $F(h, \theta) = \big(F_{-1}(h, \theta), F_0(h, \theta)\big)$ where

$$F_i(h, \theta) = \ln \frac{\theta \exp(h_i - h_1) + \exp(h_i - h_1) + \theta}{\frac{1}{\theta^2} \exp(h_i - h_1) + \theta \exp(h_i - h_1) + 1},$$

$i = -1, 0$.

Namely, for any collection of functions satisfying the functional equation (4) there exists a unique splitting Gibbs measure, the correspondence being one-to-one.

Theorem 2. Let N_{TISGM} be the number of translation-invariant SGMs for the modified SOS model on the Cayley tree with order four. The following statements are true:

$$N_{TISGM} = \begin{cases} 3, & \text{if } \ \theta \in M_{\eta_1,\eta_2} \cap L, \\ 2, & \text{if } \ \eta_1(\theta) = 1 \text{ or } \eta_2(\theta) = 1, \\ 1, & \text{if } \ \text{otherwise}, \end{cases}$$

where $L = (0; \sqrt{\frac{3+\sqrt{129}}{20}})$ and $M_{\eta_1,\eta_2} = \{\theta : \eta_1(\theta) < 1 < \eta_2(\theta)\}$.

In fact

$$\eta_i(\theta) = \frac{1}{y_i^*} \frac{2\theta + y_i^{*4}}{\frac{1}{\theta^2} + 1 + \theta y_i^{*4}},$$

where

$$y_1^* = \sqrt[4]{\frac{-10\theta^4 + 3\theta^2 + 3 + \sqrt{(10\theta^4 - 3\theta^2 - 3)^2 - 4\theta^3(2\theta^3 + 2\theta)}}{2\theta^3}}$$

and

$$y_2^* = \sqrt[4]{\frac{-10\theta^4 + 3\theta^2 + 3 - \sqrt{(10\theta^4 - 3\theta^2 - 3)^2 - 4\theta^3(2\theta^3 + 2\theta)}}{2\theta^3}}.$$

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SPECTRAL PROPERTIES OF THE DISCRETE SCHRÖDINGER OPERATORS

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We study the family $H_{\gamma\lambda\mu\eta}(K)$, $K \in \mathbb{T}^2$, of discrete Schrödinger operators, associated to the Hamiltonian of a system of two identical bosons with interaction magnitudes γ, λ, μ and η. Under certain conditions on the potential, we prove there existence of eigenvalues of the discrete Schrödinger operator $H_{\gamma\lambda\mu\eta}(0)$. Applying this result for any $K \in \mathbb{T}^2$ we give a lower bound for the number of the discrete eigenvalues of the operator $H_{\gamma\lambda\mu\eta}(K)$.

The fiber operator $H_{\gamma\lambda\mu\eta}(K)$, $K \in \mathbb{T}^2$, is a self-adjoint operator defined in $L^{2,e}(\mathbb{T}^2)$ as

$$H_{\gamma\lambda\mu\eta}(K) := H_0(K) + V_{\gamma\lambda\mu\eta},$$

where the unperturbed operator $H_0(K)$ is the multiplication operator by the function

$$\mathcal{E}_K(p) := 2\sum_{i=1}^{2}\Big(1 - \cos\tfrac{K_i}{2}\,\cos p_i\Big),$$

and the perturbation $V_{\gamma\lambda\mu\eta}$ is defined as

$$\begin{aligned} V_{\gamma\lambda\mu\eta}f(p) \;&= \tfrac{\gamma}{4\pi^2}\int_{\mathbb{T}^2} f(q)\,\dot{\mathrm{q}} + \tfrac{\lambda}{4\pi^2}\int_{\mathbb{T}^2}(\cos p_1\cos q_1 + \cos p_2\cos q_2)f(q)\,\dot{\mathrm{q}} \\ &+\tfrac{\mu}{4\pi^2}\int_{\mathbb{T}^2}(\cos 2p_1\cos 2q_1 + \cos 2p_2\cos 2q_2)f(q)\,\dot{\mathrm{q}} \\ &+\tfrac{\eta}{4\pi^2}\int_{\mathbb{T}^2}(\cos 3p_1\cos 3q_1 + \cos 3p_2\cos 3q_2)f(q)\,\dot{\mathrm{q}} \\ &+\tfrac{\mu}{4\pi^2}\cos p_1\cos p_2\int_{\mathbb{T}^2}\cos q_1\cos q_2 f(q)\,\dot{\mathrm{q}} + \tfrac{\mu}{4\pi^2}\sin p_1\sin p_2\int_{\mathbb{T}^2}\sin q_1\sin q_2 f(q)\,\dot{\mathrm{q}} \\ &+\tfrac{\eta}{4\pi^2}\cos 2p_1\cos p_2\int_{\mathbb{T}^2}\cos 2q_1\cos q_2 f(q)\,\dot{\mathrm{q}} + \tfrac{\eta}{4\pi^2}\sin 2p_1\sin p_2\int_{\mathbb{T}^2}\sin 2q_1\sin q_2 f(q)\,\dot{\mathrm{q}} \\ &+\tfrac{\eta}{4\pi^2}\cos p_1\cos 2p_2\int_{\mathbb{T}^2}\cos q_1\cos 2q_2 f(q)\,\dot{\mathrm{q}} + \tfrac{\eta}{4\pi^2}\sin p_1\sin 2p_2\int_{\mathbb{T}^2}\sin q_1\sin 2q_2 f(q)\,\dot{\mathrm{q}}. \end{aligned} \tag{1}$$

In the literature the parameter $K \in \mathbb{T}^2$ is called the *two-particle quasi-momentum* and the fiber $H_{\gamma\lambda\mu\eta}(K)$ is called the *discrete Schrödinger operator* associated to the two-particle Hamiltonian $\hat{H}_{\gamma\lambda\mu\eta}$.

Depending on the parameters $\gamma, \lambda, \mu, \eta \in \mathbb{R}$ the rank of $V_{\gamma\lambda\mu\eta}$ varies, but never exceeds 13. Hence, by the classical Weyl theorem for any $K \in \mathbb{T}^2$ the essential spectrum $\sigma_{\mathrm{ess}}(H_{\gamma\lambda\mu\eta}(K))$ of $H_{\gamma\lambda\mu\eta}(K)$ coincides with the spectrum of $H_0(K)$:

$$\sigma_{\mathrm{ess}}(H_{\gamma\lambda\mu\eta}(K)) = \sigma(H_0(K)) = [\mathcal{E}_{\min}(K), \mathcal{E}_{\max}(K)],$$

where

$$\begin{aligned} \mathcal{E}_{\min}(K) :=&\quad \min_{p\in\mathbb{T}^2}\,\mathcal{E}_K(p) = 2\sum_{i=1}^{2}\Big(1 - \cos\tfrac{K_i}{2}\Big) \geq \mathcal{E}_{\min}(0) = 0, \\ \mathcal{E}_{\max}(K) :=&\quad \max_{p\in\mathbb{T}^2}\,\mathcal{E}_K(p) = 2\sum_{i=1}^{2}\Big(1 + \cos\tfrac{K_i}{2}\Big) \leq \mathcal{E}_{\max}(0) = 8. \end{aligned}$$

Let we consider the following closed subspaces of $L^{2,\mathrm{e}}(\mathbb{T}^2)$.

$$L^{2,\mathrm{ees}}(\mathbb{T}^2) = \{f \in L^{2,\mathrm{e}}(\mathbb{T}^2) : f(p_1,p_2) = f(p_2,p_1) = f(-p_1,p_2),\ \forall p_1,p_2 \in \mathbb{T}\},$$
$$L^{2,\mathrm{oos}}(\mathbb{T}^2) = \{f \in L^{2,\mathrm{e}}(\mathbb{T}^2) : f(p_1,p_2) = f(p_2,p_1) = -f(-p_1,p_2),\ \forall p_1,p_2 \in \mathbb{T}\},$$
$$L^{2,\mathrm{eea}}(\mathbb{T}^2) = \{f \in L^{2,e}(\mathbb{T}^2) : f(p_1,p_2) = -f(p_2,p_1) = f(p_1,-p_2),\ \forall p_1,p_2 \in \mathbb{T}\},$$
$$L^{2,\mathrm{ooa}}(\mathbb{T}^2) = \{f \in L^{2,e}(\mathbb{T}^2) : f(p_1,p_2) = -f(p_2,p_1) = -f(p_1,-p_2),,\ \forall p_1,p_2 \in \mathbb{T}\}.$$

We note that

$$L^{2,e}(\mathbb{T}^2) = L^{2,\mathrm{ees}}(\mathbb{T}^2) \oplus L^{2,\mathrm{oos}}(\mathbb{T}^2) \oplus L^{2,eea}(\mathbb{T}^2) \oplus L^{2,ooa}(\mathbb{T}^2).$$

Moreover, the spaces $L^{2,\mathrm{oos}}(\mathbb{T}^2), L^{2,\mathrm{ees}}(\mathbb{T}^2), L^{2,\mathrm{eea}}(\mathbb{T}^2)$ *and* $L^{2,ooa}(\mathbb{T}^2)$ are reducing for $H_{\gamma\lambda\mu\eta}$ and therefore

$$\sigma\Big(H_{\gamma\lambda\mu\eta}(0)\Big) = \bigcup_{\theta\in\{\mathrm{ees,oos,eea,ooa}\}} \sigma\Big(H_{\gamma\lambda\mu\eta}(0)\big|_{L^{2,\theta}(\mathbb{T}^2)}\Big),$$

where $A\big|_{\mathcal{H}}$ is the restriction of a self-adjoint operator on the subspace $\mathcal{H}$.

Let we denote by $H^{\theta}_{\gamma\lambda\mu\eta}$ resp. $V^{\theta}_{\gamma\lambda\mu\eta}$ the respective part of the operator $H_{\gamma\lambda\mu\eta}$ resp. $V_{\gamma\lambda\mu\eta}$ in the reducing subspace $L^{2,\theta}(\mathbb{T}^2), \theta \in \{\mathrm{ees}, \mathrm{oos}, \mathrm{eea}, \mathrm{ooa}\}$. The equality

$$2\cos x\cos y + 2\cos z\cos t = (\cos x + \cos z)(\cos y + \cos t) + (\cos x - \cos z)(\cos y - \cos t)$$

and the representation of $V_{\gamma\lambda\mu\eta}$ in (1) imply that $H^{\mathrm{ooa}}_{\gamma\lambda\mu\eta}(0) = H_0 + V^{\mathrm{ooa}}_{\gamma\lambda\mu\eta}$, where

$$[V^{\mathrm{ooa}}_{\gamma\lambda\mu\eta}f](p) \ = \tfrac{\eta}{2\pi^2}(\sin p_1 \sin 2p_2 - \sin p_2 \sin 2p_1)\textstyle\int_{\mathbb{T}^2}(\sin q_1 \sin 2q_2 - \sin q_2 \sin 2q_1)f(q)\,dq.$$

Thus, the restriction $H^{ooa}_{\gamma\lambda\mu\eta}(0)$ of the operator $H_{\gamma\lambda\mu\eta}(0)$ onto the Hilbert space $L^{2,ooa}(\mathbb{T}^2)$ is depending only on η, which is simpler to investigate. Therefore, here and everywhere below we use in place of $H^{ooa}_{\gamma\lambda\mu\eta}(0)$ and $V^{ooa}_{\gamma\lambda\mu\eta}$ the notations $H^{ooa}_{\eta}(0)$ and V^{ooa}_{η}, respectively.

Recall that the main purpose of this article is to study the number and location of the eigenvalues of the operator $H^{ooa}_{\eta}(0)$ lying outside the essential spectrum.

Theorem 1. Let $\eta \in \mathbb{R}$. The following relations hold:

(a) if $\eta < -\frac{150\pi-416}{15\pi}$ then the operator $H^{ooa}_{\eta}(0)$ has exactly one eigenvalue lying below the essential spectrum.

(b) if $\frac{150\pi-416}{15\pi} \le \eta \le \frac{150\pi-416}{15\pi}$ then the operator $H^{ooa}_{\eta}(0)$ has no eigenvalues outside the essential spectrum.

(c) if $\eta > \frac{150\pi-416}{15\pi}$ then the operator $H^{oo}a_{\eta}(0)$ has exactly one eigenvalue lying above the essential spectrum;

We note that, by min-max principle [5, Theorem XIII.3], the number of eigenvalues of the operator $H_{\gamma\lambda\mu\eta}(K)$ lying outside the essential spectrum varies, but never exceeds 13.

Theorem 2. Let $\gamma, \lambda, \mu, \eta \in \mathbb{R}$. The following assertions true:

(a) if $\eta < -\frac{150\pi-416}{15\pi}$ then the operator $H_{\gamma\lambda\mu\eta}(K)$ has at least one eigenvalues lying below the essential spectrum.

(b) if $\eta > \frac{150\pi-416}{15\pi}$ then the operator$H_{\gamma\lambda\mu\eta}(K)$ has at least one eigenvalues lying above the essential spectrum;

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CPA-STRUCTURE ON SMALL-DIMENSIONAL NILPOTENT LIE ALGEBRAS

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Several configurations gave rise to post-Lie algebras and post-Lie algebra structures. In order to characterize simply transitive nil-affine actions of nilpotent Lie group G on another nilpotent Lie group N, we devised these structures in [3]. This is relevant to the theory of complete nil-affinely flat manifolds and nil-affine crystallographic groups. Post-Lie algebras emerge as a natural common generalization of LR-algebras and pre-Lie algebras. On the other hand, Vallette [1] separately presented post-Lie algebras in relation to the study of Koszul operads and the homology of partition posets.

The existence of post-Lie algebra structures for a given pair of Lie algebras is an important topic coming here from geometry. In general terms, this is an extremely difficult subject, and the solution depends significantly on the algebraic characteristics of the two specified Lie algebras. See [4] for a survey on the results and unresolved problems. Commutative structures, or *CPA-structures*, provide an essential class of *post-Lie algebra structures*. In comparison with general post-Lie algebra structures, these structures are much more accessible, and we are able to provide answers to a number of concerns about their existence and classification, which helps us identify areas of study for the larger case of post-Lie algebra structures. Refer to [2] for CPA-structures on some classes filiform Lie algebras.

In this work we construct CPA-structures for three-dimensional nilpotent Lie algebras.

Definition 1. A "*commutative post-Lie algebra structure* or "CPA-structure"on a Lie algebra A is a K-bilinear product $x \cdot y$ satisfying the identities:

$$x \cdot y = y \cdot x \tag{1}$$
$$[x, y] \cdot z = x \cdot (y \cdot z) - y \cdot (x \cdot z) \tag{2}$$
$$x \cdot [y, z] = [x \cdot y, z] + [y, x \cdot z] \tag{3}$$

for all $x, y, z \in A$.

It turns out that certain CPA-structures are related to Poisson algebras and Poisson-admissible algebras.

Consider following three-dimensional algebra with multiplication

$$(L(3), [-,-]): \qquad [e_1, e_2] = e_3,$$

where omitted products are zero.

Proposition 1. There are CPA-structure for 3-dimensional nilpotent Lie algebras:

$$
\begin{aligned}
&(L_1(3),[-,-],\cdot): \quad [e_1,e_2]=e_3, \quad e_1\cdot e_1=e_2;\\
&(L_2(3),[-,-],\cdot): \quad [e_1,e_2]=e_3, \quad e_1\cdot e_1=\alpha e_1 \quad (\alpha\neq 0);\\
&(L_3(3),[-,-],\cdot): \quad [e_1,e_2]=e_3, \quad e_1\cdot e_1=e_2; \quad e_1\cdot e_1=\beta e_2 \quad e_2\cdot e_2=\beta^2 e_2 \quad (\forall \beta\in\mathbb{C})\\
&(L_4(3),[-,-],\cdot): \quad [e_1,e_2]=e_3, \quad e_1\cdot e_1=e_3\\
&(L_5(3),[-,-],\cdot): \quad [e_1,e_2]=e_3, \quad e_1\cdot e_1=e_3 \quad e_1\cdot e_2=\gamma e_3 \quad (\gamma\neq 0),
\end{aligned}
$$

where omitted products are zero.

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LARGE COMPONENTS IN RANDOM DECOMPOSABLE STRUCTURES, AND THE BOSE–EINSTEIN CONDENSATION

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Random decomposable structures (such as integer partitions or permutations) may manifest the emergence of a "giant" component due to the asymptotic loss of mass. Connections of such models with statistical physics are well known [1], [2]. Work by A.M. Vershik [3] stimulated interest in the limit shape of large combinatorial structures; in particular, he proposed an interpretation of limit shape in terms of the Bose–Einstein condensation of ideal quantum gas [4].

In my talk, I will present two results in this direction: one related to a unified derivation of the limit shape of random partitions [5], and the other about the asymptotic statistics of cycles in the model of "surrogate-spatial" permutations [6], proposed as an analytically tractable version of the model of spatial random permutations [7].

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ON THE SPECTRUM OF TWO PARTICLE DISCRETE SCHRÖDINGER OPERATORS

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Let $\mathbb{T}^2 = [-\pi, \pi)^2$ and $L^2(\mathbb{T}^2)$ be the Hilbert space of square-summable on $\mathbb{T}^2$. Let $L^{2,\mathrm{e}}(\mathbb{T}^2)$ be the subspace of essentially even in $L^2(\mathbb{T}^2)$ and let $L^{2,\mathrm{ea}}(\mathbb{T}^2) := \{f \in L^{2,\mathrm{e}}(\mathbb{T}^2) : f(p_1, p_2) = -f(p_2, p_1)$ for a.e $(p_1, p_2) \in \mathbb{T}^2\}$ be the Hilbert subspace of (essentially) even-symmetric functions in $L^2(\mathbb{T}^2)$. The Schrödinger operator H_λ^{ea} acting in $L^{2,\mathrm{ea}}(\mathbb{T}^2)$ as

$$H_\lambda^{\mathrm{ea}}(k) := H_0(k) + V_\lambda^{\mathrm{ea}},$$

where $\lambda > 0$, $k \in \mathbb{T}^2$ and

$$[H_0(k)f](p) = \varepsilon_k(p)f(p).$$

Here $\varepsilon_k(p) = \varepsilon(\frac{k}{2} + p) + \varepsilon(\frac{k}{2} - p)$, $p = (p_1, p_2) \in \mathbb{T}^2$.

Hypothesis.1. *The dispersion relation ε is a real-valued even function, symmetric with respect to coordinate permutations and having a non-degenerate unique maximum at $\vec{\pi} = (\pi, \pi) \in \mathbb{T}^2$.*

V_λ^{ea} acts on $L^{2,\mathrm{ea}}(\mathbb{T}^2)$ as

$$(V_\lambda^{\mathrm{ea}} f)(p) = \frac{\lambda}{(2\pi)^2} (\cos p_1 - \cos p_2) \int_{\mathbb{T}^2} (\cos q_1 - \cos q_2) f(q)\, \mathrm{d}q.$$

The essential spectrum of $H_\lambda(k)$ [1]:

$$\sigma_{\mathrm{ess}}(H_\lambda^{\mathrm{ea}}(k)) = \sigma(H_0(k)) = [\varepsilon_{\min}(k),\, \varepsilon_{\max}(k)],$$

where

$$\varepsilon_{\min}(k) =: \min_{p \in \mathbb{T}^2} \varepsilon_k(p), \qquad \varepsilon_{\max}(k) =: \max_{p \in \mathbb{T}^2} \varepsilon_k(p).$$

Let

$$\lambda_0 := \left[\frac{1}{4\pi^2} \int_{\mathbb{T}^2} \frac{(\cos q_1 - \cos q_2)^2 dq}{\varepsilon_{\max}(0) - \varepsilon_0(q)} \right]^{-1} > 0.$$

Theorem. Assume Hypothesis1. Then the following statements hold 1. For any $\lambda \in [0, +\infty)$ the operator $H_\lambda^{\mathrm{ea}}(0)$ has no eigenvalue in $(-\infty, \varepsilon_{\min}(0))$.

2. For any $\lambda \in [0, \lambda_0]$ the operator $H_\lambda^{\mathrm{ea}}(0)$ has no eigenvalue in $(\varepsilon_{\max}(0), +\infty)$.

3. For any $\lambda \in (\lambda_0, +\infty)$ the operator $H_\lambda^{\mathrm{ea}}(0)$ has unique eigenvalue in $(\varepsilon_{\max}(0), +\infty)$.

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GLOBAL DYNAMICS OF A TWO-DIMENSIONAL MOSQUITO POPULATION

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Define the operator $W : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$\begin{cases} x' = \frac{\beta y^2}{\gamma+y} - \frac{\alpha x}{1+x} + x, \\ y' = \frac{\alpha x}{1+x} - \mu y + y, \end{cases} \tag{1}$$

where $\alpha > 0, \beta > 0, \gamma > 0, \mu > 0$. In the system (1), x', y' means the next state relative to the initial state x, y respectively.

It is easy to see that if

$$\beta > 0,\ \gamma > 0,\ 0 < \alpha \leq 1,\ 0 < \mu \leq 1 \tag{2}$$

then operator (1) maps R^2_+ to itself.

We call the partition into types hereditary if for each possible state $\mathbf{z} = (x, y) \in \mathbb{R}^2_+$ describing the current generation, the state $\mathbf{z}' = (x', y') \in \mathbb{R}^2_+$ is uniquely defined describing the next generation. This means that the association $z \to z'$ defines a map $W : \mathbb{R}^2_+ \to \mathbb{R}^2_+$ called the evolution operator.

The main problem for a given operator W and arbitrarily initial point $\mathbf{z}^{(0)} = (x^{(0)}, y^{(0)}) \in \mathbb{R}^2_+$ is to describe the limit points of the trajectory $\{\mathbf{z}^{(m)}\}_{m=0}^{\infty}$, where $\mathbf{z}^{(m)} = W^m(\mathbf{z}^{(0)})$.

The dynamics of the operator (1) were studied in detail under condition $\gamma = 0$ in [1-4].

The following theorem describes the trajectory of any point $(x^{(0)}, y^{(0)})$ in $\mathbb{R}^2_+$.

Theorem. Assume that (2) holds. For the operator W given by (1) for any initial point $(x^{(0)}, y^{(0)}) \in \mathbb{R}^2_+$ the following hold:

(i) If $y^{(n)} > \frac{\alpha}{\mu}$ for any natural number n then

$$\lim_{n\to\infty} x^{(n)} = +\infty,\ \lim_{n\to\infty} y^{(n)} = \frac{\alpha}{\mu};$$

(ii) If there exists n_0 number such that $y^{(n_0)} \leq \frac{\alpha}{\mu}$ and $\beta \leq \mu(1 + \frac{\gamma\mu}{\alpha})$ then

$$\lim_{n\to\infty} x^{(n)} = 0,\ \lim_{n\to\infty} y^{(n)} = 0;$$

where $(x^{(n)}, y^{(n)}) = W^n(x^{(0)}, y^{(0)})$, with W^n is n-th iteration of W.

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A NOTE ON LIMITING GIBBS DISTRIBUTIONS FOR THE INFINITE TODA CHAIN

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It is well known that the discrete Toda chain is a completely integrable model (see [3]). Consider the following Hamiltonian on phase space R^{2n}

$$H_{m,l} = \sum_{k=-m}^{l} \frac{p_k^2}{2} + \sum_{j=-m}^{l-1} e^{q_{j+1}-q_j}, \quad m, l > 0$$

The system of differential equations on phase space R^{2n}

$$\begin{cases} \dot{q_k} = \frac{\partial H}{\partial p_k}, \\ \dot{p_k} = -\frac{\partial H}{\partial q_k}, \end{cases}$$

defined in a region U of the space $R^{2n}(q_1, q_2, ..., q_n; p_1, p_2, ..., p_n)$ is called a Hamiltonian system. The function H is called **Hamiltonian**, while the region U is called the **phase space** of the system. Let us introduce new variables:

$$a_j = -\frac{1}{2}p_j, \quad j = 1...n;$$

$$b_l = \frac{1}{2}e^{\frac{1}{2}(q_l - q_{l+1})}, \quad l = 1...(n-1)$$

which is called *variables Flaschke*. Hamiltonian system is equivalent to the matrix equation $\dot{L} = [A, L]$, where

$$L = \begin{pmatrix} a_1 & b_1 & 0 & \cdots & 0 \\ b_1 & a_2 & b_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & b_{n-1} \\ 0 & \cdots & 0 & b_{n-1} & a_n \end{pmatrix} \quad A = \begin{pmatrix} 0 & b_1 & 0 & \cdots & 0 \\ -b_1 & 0 & b_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & b_{n-1} \\ 0 & \cdots & 0 & -b_{n-1} & 0 \end{pmatrix}$$

$[A, L] = AL - LA$ is an ordinary commutator. Functions

$$F_k = \frac{1}{k} tr L^k \quad \text{where } k = 1 \dots n$$

are integrals of motion of the Toda chain. It is known that $F_1 = a_{-m} + a_{-m+1} + ... + a_l = -\frac{1}{2}(p_{-m} + p_{-m+1} + ... + p_l)$ is whole impuls and $F_2 = \sum_{k=-m}^{l} \frac{p_k^2}{2} + \sum_{j=-m}^{l-1} e^{q_{j+1}-q_j} = H_{m,l}$ is energy. We will be interested mainly in linear combinations of first integrals F_2 and F_1:

$$\bar{H}_{m,l} := F_2 + a \cdot F_1$$

Definition 1. The conditional Gibbs distribution at inverse temperature $\beta > 0$ and the condition Y^l, Y^r is such a probability distribution on $M(Y^l, \ , Y^r)$ that for $k \geq 0$ the restriction of this distribution to $M_k(Y^l, Y^r)$ has the density

$$P_{m,l}(q_{-m}, p_{-m}, ..., q_l, p_l) = \Xi^{-1}_{\beta,\mu}(s) \exp\left\{-\beta\left(\bar{H}_{m+1,l+1} + \mu k\right)\right\}, \tag{1}$$

Where $(q_{-m-1}, p_{-m-1}) = z_1$, $(q_{-m-2}, p_{-m-2}) = z_1^l$, $(q_{l+1}, p_{l+1}) = z_2$, $(q_{l+2}, p_{l+2}) = z_2^r$, m and l are the numbers of particles to the left and right of the zero particle, respectively, $m + l + 1 = k$, and $\Xi^{-1}_{\beta,\mu}(s)-$normalizing factor.

At by its definition, integration occurs over $q_i \in \bigcup\limits_{j=-2}^{1} \Delta_j$ $(i = \overline{-m, n})$. Parameter $\mu \in \mathbb{R}^1$ called chemical potential.

Definition 2. The limiting Gibbs distribution $\nu_a(\beta, \mu)$ for $\beta > 0$ and $\mu \in \mathbb{R}^1$ is a probability distribution on γ such that for any $M(\Delta_{-2}, \Delta_{-1} : \Delta_0, \Delta_1)$ its induced conditional distribution on The σ-subalgebra $\gamma^t(\Delta_{-2}, \Delta_{-1} : \Delta_0, \Delta_1)$ coincides with $\nu_{a,b,c}(\beta, \mu)-$ a.s. with (1).

Now we formulate the main results of our work.

Theorem 1. Let the numbers $\beta > 0$ and $\mu \in \mathbb{R}$ satisfy the following condition

$$e^{\beta\mu} \int\limits_{-\infty}^{\infty} e^{-\beta\left(\frac{p^2}{2} - ap\right)} dp \int\limits_{-\infty}^{\infty} e^{-\beta(e^y - \frac{y}{2})} dy < 1.$$

Then, for probability measures with density (1) as $s \to +\infty$ there is at least one limit measure $\nu_{\beta,\mu}$ on phase space M.

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PERIODIC GROUND STATES CORRESPONDING TO A NON-NORMAL SUBGROUP OF INDEX FOUR FOR THE ISING MODEL WITH COMPETING INTERACTIONS

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Let $\Gamma^k = (V, L)$, $k \geq 1$ be a Cayley tree (see [1]). For the fixed $x^0 \in V$, called a root, we set

$$W_n = \{x \in V \ \mid \ d(x, x^0) = n\},$$

$$V_n = \{x \in V \ \mid \ d(x, x^0) \leq n\}, \quad L_n = \{l = \langle x, y\rangle \in L \ \mid \ x, y \in V_n\},$$

where $d(x, y)$ is a distance x to y, and $\langle x, y\rangle$-*nearest neighboring vertices*.

It is known (see [1]) that there exists a one-to-one correspondence between the set V of vertices of the Cayley tree of order $k \geq 1$ and the group G_k of the free products of $k+1$ cyclic groups $\{e, a_i\}$, $i = 1, ..., k+1$ of the second order (i.e. $a_i^2 = e$, $a_i \neq e$) with generators $a_1, a_2, ..., a_{k+1}$.

We denote by $S(x)$ the set of "direct successors"of $x \in G_k$ i.e.

$$S(x) = \{y \in W_{n+1} |\ d(y, x) = 1\}, \ x \in W_n.$$

Also, let $S_1(x)$ be the set of all nearest neighboring vertices of $x \in G_k$, i.e. $S_1(x) = \{y \in G_k : \langle x, y\rangle\}$. We consider models where the spin takes values in the set $\Phi = \{-1, 1\}$. For $A \subseteq V$ a spin *configuration* σ_A on A is defined as a function $x \in A \to \sigma_A(x) \in \Phi$; the set of all configurations is denoted by $\Omega_A = \Phi^A$. Put $\Omega = \Omega_V$, $\sigma = \sigma_V$ and $-\sigma_A = \{-\sigma_A(x), x \in A\}$. Let K be a subgroup of G_k, $k \geq 1$ and $G_k : K = \{K_1, ..., K_r\}$ be a family of cosets $r \geq 1$.

Definition 1. A configuration $\sigma(x)$ is called to be K-periodic if $\sigma(x) = \sigma_i$ for all $x \in K_i$, $i = \overline{1, r}$.

The Ising model with competing interactions has the form

$$H(\sigma) = J_1 \sum_{\langle x,y\rangle \in L} \sigma(x)\sigma(y) + J_2 \sum_{\substack{x,y \in V: \\ d(x,y)=2}} \sigma(x)\sigma(y), \tag{1}$$

where $J_1, J_2 \in \mathbb{R}$ are coupling constants and $\sigma \in \Omega$.

Let M be the set of unit balls with vertices in V. We call the restriction of a configuration σ to the ball $b \in M$ a *bounded configuration* σ_b.

The energy of a ball b for configuration σ defined by the following form:

$$U(\sigma_b) \equiv U(\sigma_b, J) = \frac{1}{2} J_1 \sum_{\langle x,y\rangle \in L} \sigma(x)\sigma(y) + J_2 \sum_{d(x,y)=2} \sigma(x)\sigma(y), \quad x, y \in b,$$

where $J = (J_1, J_2) \in \mathbb{R}^2$.

Definition 2. A configuration φ is called a *ground state* for the Hamiltonian (1) if it satisfies the following condition $U(\varphi_b) = \min\{U_0, U_1, ..., U_{k+1}\}$, for any $b \in M$.

Let $B_0 = \{\emptyset\}, B_d = \{d\}, d \in \{1, 2, 3\}$. Put m_j be a minimal element of B_j, $j \in \{1, 2, 3\}$,

$m_j = j, j \in \{1,2,3\}$.
Let

$$u_{B_1B_2B_3}(x) = \begin{cases} e, & \text{if} \quad x = e; \\ a_i, & \text{if} \quad x = a_i,\ i \in \{1,2,3\}. \end{cases}$$

$$\gamma_1(x) = \begin{cases} e, & \text{if} \quad x = e; \\ a_1, & \text{if} \quad x \in \{a_3a_1, a_2a_3\}; \\ a_2, & \text{if} \quad x \in \{a_1a_3, a_3a_2\}; \\ a_3, & \text{if} \quad x \in \{a_1a_2, a_2a_1\}; \\ \gamma_1(a_{i_1}a_{i_2}...a_{i_{n-2}}\gamma_1(a_{i_{n-1}}a_{i_n})), & \text{if} \quad x = a_{i_1}a_{i_2}...a_{i_n}, \quad l(x) > 2s. \end{cases}$$

Then

$$H_0 = \{x \in G_2|\ \gamma_1(u_{B_1B_2B_3}(x)) = e\}.$$

Note that H_0 is a non-normal subgroup of the group G_2 (see [2]). By using H_0 we define a family of cosets:

$$G_2/H_0 = \{H_0, H_1, H_2, H_3\},$$

where

$H_1 = \{x \in G_2|\gamma_1(u_{B_1B_2B_3}(x)) = a_1\}$, $H_2 = \{x \in G_2|\gamma_1(u_{B_1B_2B_3}(x)) = a_2\}$,

$$H_3 = \{x \in G_2|\gamma_1(u_{B_1B_2B_3}(x)) = a_3\}.$$

H_0-periodic configurations have the following forms

$$\sigma(x) = \begin{cases} \sigma_0, & \text{if} \quad x \in H_0; \\ \sigma_1, & \text{if} \quad x \in H_1; \\ \sigma_2, & \text{if} \quad x \in H_2; \\ \sigma_3, & \text{if} \quad x \in H_3, \end{cases}$$

where $\sigma_i \in \Phi, i \in \{0,1,2,3\}$.

Theorem 1. Let $k = 2$.
1) If $(J_1, J_2) \in A_2$, then there are six H_0-periodic (except for translation-invariant) ground states which corresponding to the following configurations

$$\sigma(x) = \pm \begin{cases} \sigma_0, & \text{if} \quad x \in H_0; \\ \sigma_1, & \text{if} \quad x \in H_1; \\ \sigma_2, & \text{if} \quad x \in H_2; \\ \sigma_3, & \text{if} \quad x \in H_3, \end{cases}$$

where $(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \in \{(-1,-1,1,1), (-1,1,-1,1), (-1,1,1,-1)\}$.
2) If $(J_1, J_2) \in \mathbb{R}^2 \backslash A_2$, there are not H_0-periodic (except for translation-invariant) ground states.

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THE FOKAS' METHOD FOR INVERSE CONTROL PROBLEM FOR HEAT EQUATION ON METRIC GRAPH

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Consider simple metric graph Γ with three finite bonds connected on the point O. The point O called to be a vertex of the graph. We label the bonds of the graph as B_j,$j = \overline{1,3}$. Define coordinate x_j on the bond B_j for $j = \overline{1,3}$ corresponding it to the intervals $(0, L_j)$. At each bond the vertex point O has a coordinate 0. Further we will use x instead of x_j.

We consider the equations of heat dissipation in each bond of the given graph

$$V_t^{(j)}(x,t) = V_{xx}^{(j)}(x,t) + f^{(j)}(x),\ j = \overline{1,3} \tag{1}$$

with initial conditions

$$V^{(j)}(x,0) = V_0^{(j)}(x),\ \ x \in \overline{B_j},\ j = \overline{1,3} \tag{2}$$

and boundary conditions

$$V^{(j)}(x,t)|_{x=L_j} = 0,\ \ j = \overline{1,3}. \tag{3}$$

Moreover, we need to define the following gluing conditions for connectivity of the graph

$$V^{(1)}(0,t) = V^{(2)}(0,t) = V^{(3)}(0,t), \sum_{j=1}^{3} \delta_j^2 V_x^{(j)}(0,t) = 0. \tag{4}$$

The last conditions usually called continuity and flux conservation (Kirchhoff) conditions on branching point of the graphs.

We suppose, that initial data are smooth enough functions and they satisfies the conditions (3)-(4).

Our task is to find the functions $\left\{V^{(j)}(x,t),\ f^{(j)}(x),\ j = \overline{1,3}\right\}$ that satisfy the equation (1), the conditions (2)-(4) and additional condition $V^{(j)}(x,T) = V_T^{(j)}(x),\ j = \overline{1,3}$.

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ON DYNAMICS OF INFINITE DIMENSIONAL VOLTERRA OPERATOR

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It is known [2] that nonlinear (in particular, quadratic) mappings appear in various branches of mathematics and their applications: the theory of differential equations, probability theory, the theory of dynamical systems, mathematical economics, mathematical biology, statistical physics, etc.

In the theory of population genetics or game theory, mathematical models are governed by the quadratic differential systems

$$\frac{dx_i}{dt} = \sum_{j,k} a^i_{jk} x_j x_k, \quad i \in \{1, 2, \dots, n\},$$

for a large interacting population of n constituents, where the numbers x_i represent the fraction of constituents of type i, $i \in \{1, 2, \dots, n\}$ and satisfy the conservation law $\sum_i x_i = 1$ and the tensors $\{a^i_{jk}\}$ of order n having n^3 real constants which satisfy

$$a^i_{jk} = a^i_{kj}, \quad \sum_i a^i_{jk} = 1, \quad a^i_{jk} \geq 0 \ \text{ for all } j \neq i, \ k \neq i, \ i, j, k \in \{1, 2, \dots, n\}. \tag{1}$$

We notice that the discrete time system corresponding to (1) is governed by *quadratic stochastic operators (q.s.o.)* which appeared in the works of Bernstein [1]. Such kind of operators is usually employed to describe the time evolution of species in biology. It turns out that quadratic dynamical systems are considered an important source of analysis in the study of dynamical properties and for modeling in various fields, such as population dynamics, physics, economics and mathematics.

First recall that a quadratic stochastic operator V is called *Volterra* if and only if it can be represented as follows:

$$(V(\mathbf{x}))_k = x_k \left(1 + \sum_{i=1}^{\infty} a_{ki} x_i\right), \quad k \in \mathbb{N}, \tag{2}$$

where $\mathbf{x} = (x_1, x_2, \dots) \in S$ and

$$a_{ki} = -a_{ik}, \ |a_{ki}| \leq 1 \ \text{ for every } k, i \in \mathbb{N}. \tag{3}$$

By $\mathcal{V}$ we denote the set of all Volterra operators on infinite dimensional simplex S, and $\mathbb{A}$ denotes the set of all skew-symmetric matrices with (3). The representation (1) establishes a one-to-one correspondence $\mathfrak{f} : \mathcal{V} \to \mathbb{A}$ by $\mathfrak{f}(V) = (a_{ki})$. It is clear that $\mathfrak{f}$ is affine, hence $\mathcal{V}$ is convex, and moreover, this correspondence allows to investigate certain geometric properties of $\mathcal{V}$ by means of structure of the set $\mathbb{A}$

For a given operator V on S, by $\{V^n(\mathbf{x}_0)\}_{n=1}^{\infty}$ we denote the trajectory of a point $\mathbf{x}_0 \in S$ under V. By $\omega_V(\mathbf{x}_0)$ (respectively, $\omega_V^{(w)}(\mathbf{x}_0)$) we denote the set of limit points of $\{V^n(\mathbf{x}_0)\}_{n=1}^{\infty}$ with respect to ℓ^1-norm (respectively, pointwise convergence).

In what follows, by a *fixed point* of V we mean a vector $\mathbf{x} \in S$ such that $V(\mathbf{x}) = \mathbf{x}$. By $Fix(V)$ we denote the set of all fixed points of V.

Obviously, if $\omega_V(\mathbf{x}_0)$ consists of a single point, i.e. $\omega_V(\mathbf{x}_0) = \{\mathbf{x}^*\}$, then the trajectory $\{V^n(\mathbf{x}_0)\}_{n=1}^{\infty}$ converges to $\mathbf{x}^*$. Moreover, $\mathbf{x}^*$ is a fixed point of V.

Denote

$$\mathbb{A}^+ = \{(a_{ki}) \in \mathbb{A} : \ a_{ki} \geq 0 \text{ for all } k < i\},$$

$$\mathbb{A}^- = \{(a_{ki}) \in \mathbb{A} : \ a_{ki} \leq 0 \text{ for all } k < i\}.$$

We define two subclasses of $\mathcal{V}$ as follows:

$$\mathcal{V}^+ = \{V \in \mathcal{V} : \mathfrak{f}(V) \in \mathbb{A}^+\}, \quad \mathcal{V}^- = \{V \in \mathcal{V} : \ \mathfrak{f}(V) \in \mathbb{A}^-\}.$$

Recall that the dynamics of Volterra stochastic operators from the class $\mathcal{V}^+$ was studied in [3].

Theorem 1. Let $V \in \mathcal{V}^-$ and $V \not\equiv Id$. Assume that there exists a number $a \in [-1; 0)$ such that $a_{ki} \leq a$ for all $k < i$. Then $\omega_V(\mathbf{x}) = \varnothing$ for any $\mathbf{x} \in S \setminus \{\mathbf{e}_i\}_{i=1}^{\infty}$. Moreover, $\omega_V^{(w)}(\mathbf{x}) = \{\mathbf{0}\}$ for all $\mathbf{x} \in S \setminus \{\mathbf{e}_i\}_{i=1}^{\infty}$.

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INVERSE SOURCE PROBLEMS FOR THE TIME-FRACTIONAL PSEUDO-PARABOLIC EQUATION WITH THE RIEMANN-LIOUVILLE DERIVATIVE ON A STAR GRAPH

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We consider the star metric graph Γ with n bonds consists of a finite set of vertices $V = \{\nu_k\}_0^n$ and a finite set of edges $E = \{e_k\}_1^n$, where e_k connect the vertices ν_0 and ν_k, $k = \overline{1,n}$. By assigning the interval $(0, l_k)$, to the bond e_k, $k = \overline{1,n}$ of the graph we define the coordinates x_k on each of the bonds. Where the vertex ν_0 of the graph has a coordinate 0 in each bond. Further, without loss of generality, we will use x instead of x_k.

On the each edge of the above defined graph Γ we consider fractional differential equations:

$$D_{0t}^{\alpha} u^{(k)}(x,t) = a u_{xx}^{(k)}(x,t) + b_C D_{0t}^{\alpha} u_{xx}^{(k)}(x,t) + c u^{(k)}(x,t) + f^{(k)}(x,t) + h^{(k)}(x,t), \qquad (1)$$

$x \in B_k$, $0 < t < T$, $k = \overline{1,j}$, where ${}_C D_{0t}^{\alpha}(\cdot)$ is the Riemann-Liouville fractional derivative with $0 < \alpha < 1$ and $f^{(k)}(x,t)$ are given functions.

Subject to the vertex conditions

$$u^{(1)}(0,t) + u^{(2)}(0,t) + ... + u^{(j)}(0,t) = 0, \qquad (2)$$

$$u_x^{(1)}(0,t) = u_x^{(2)}(0,t) = ... = u_x^{(j)}(0,t), t \in [0,T], \qquad (3)$$

boundary conditions

$$u_x^{(k)}(l_k,t) = 0, t \in [0,T], \qquad (4)$$

and initial condition

$$I_{0,t}^{1-\alpha} u^{(k)}(x,0) = \varphi^{(k)}(x), x \in \bar{B}_k, k = \overline{1,j}, \qquad (5)$$

where $\varphi^{(k)}(x)$ are sufficiently smooth given functions.

Inverse problem. Find a couples of functions $\left(u^{(k)}(x,t), h^{(k)}(x,t)\right)$ in the domain $B_k \times (0,T)$, satisfy (1)-(5) and under an over-specified conditions

$$u^{(k)}(x,0) = \varphi^{(k)}(x), x \in \bar{B}_k, k = \overline{1,j}.$$

The solution of the problem is constructed by the method of separating of variables. Using properties of the Mittag-Lefler function, we prove the uniform convergence of the obtained Fourier series in the domain $[0, l_k] \times [0,T]$. To show the uniqueness we rely on the completeness properties of system of eigenfunctions $\left\{X^{(k)}(x)\right\}$ of corresponding spectral problem(see[1-3]).

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EQUIVALENCE OF ELEMENTARY SURFACES WITH RESPECT TO THE MOVEMENTS IN PSEUDO-EUCLIDEAN SPACE

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This work investigates the conditions for equivalence of regular surfaces under the action of a specific subgroup of linear transformations, denoted by $O(n, p, R)$. $O(n, p, R)$ is a pseudo-orthogonal subgroup, meaning it preserves a certain metric structure defined by a matrix with specific sign properties (e_p).

The work focuses on elementary surfaces, which are mappings from a square in the parameter domain $(0, 1) \times (0, 1)$ to an n-dimensional real vector space. Regularity of a surface is defined by a non-zero determinant of a particular matrix derived from the surface's partial derivatives.

The concept of equivalence between surfaces is introduced, where two surfaces are considered equivalent if they can be transformed into each other through a combination of a shift vector and a linear transformation belonging to a specific subgroup H of the group $GL(n, R)$ of all invertible linear transformations of the space R^n.

The main theorem of the work establishes necessary and sufficient conditions for $O(n, p, R)$-equivalence of regular surfaces. These conditions involve equalities between products of matrices constructed from the surfaces' partial derivatives and the pseudo-orthogonal matrix e_p.

The results provide a framework for understanding how regular surfaces can be related under the action of the subgroup $O(n, p, R)$, potentially offering insights into geometric properties and transformations.

THE GENERALIZATION OF PRESTON'S LEMMA

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From some problems of the theory of Gibbs measures we get an equation of the following form:

$$\left(\frac{ax^2+bx+c}{dx^2+ex+f}\right)^k = \alpha \cdot x, \tag{1}$$

where $x \geq 0, a, b, c, d, e, f > 0, k \geq 1$.

Remark 1. If $a = d = 0, b = c = e = 1, f = b$ in the equation (1), then we have the following

$$\left(\frac{x+1}{x+b}\right)^k = \alpha \cdot x. \tag{2}$$

The equation (2) was studied in [1].

Theorem. Let n_k be the number of positive solutions of equation (1). The following assertions are valid for n_k:

1. If the conditions

$$\begin{cases} af < cd, \\ ae < bd, \\ bf < ce \end{cases}$$

are fulfilled, then $n_k = 1$.

2. If the conditions

$$\begin{cases} af > cd, \\ ae > bd, \\ bf > ce \end{cases} \text{ or } \begin{cases} af > cd, \\ ae > bd, \\ bf < ce \end{cases} \text{ or } \begin{cases} af < cd, \\ ae < bd, \\ bf > ce \end{cases} \text{ or } \begin{cases} af > cd, \\ ae < bd \end{cases} \text{ or } \begin{cases} af < cd, \\ ae > bd \end{cases}$$

are fulfilled, then the following cases are fulfilled:

Case 2.1. When there are such numbers $\eta_1 := \eta_1(a, b, c, d, e, f), \eta_2 := \eta_2(a, b, c, d, e, f)(0 < \eta_1 < \eta_2)$, then

$$n_k = \begin{cases} 1, & \text{if } \alpha \notin [\eta_1, \eta_2], \\ 2, & \text{if } \alpha \in \{\eta_1, \eta_2\}, \\ 3, & \text{if } \alpha \in (\eta_1, \eta_2), \end{cases}$$

where

$$\eta_i = \frac{1}{x_i} \cdot \left(\frac{ax_i^2+bx_i+c}{dx_i^2+ex_i+f}\right)^k,$$

x_1, x_2 are the positive solutions of equation

$$adx^4+((k+1)bd-(k-1)ae)x^3+(be+(2k+1)cd-(2k-1)af)x^2+((k+1)ec-(k-1)bf)x+cf=0. \quad (3)$$

Case 2.2. If equation (3) has no positive solutions, then $n_k = 1$.

Remark 2. Using the results obtained in Theorem, it is possible to solve phase transition problems for some models of statistical mechanics.

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DYNAMICS OF A QUASI NON-VOLTERRA CUBIC STOCHASTIC OPERATOR

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Consider a finite set $E = \{1, 2, ..., n\}$ and define the set of all probability distributions on E as:

$$S^{n-1} = \{x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n, \sum_{i=1}^{n} x_i = 1, \;\; x_i > 0, i = 1, ..., n\},$$

which is often referred to as the $(n-1)$-dimensional simplex. Now, moving on to cubic stochastic operators:

Definition 1. A cubic stochastic operator (CSO) is a mapping $W : S^{n-1} \to S^{n-1}$ of the form

$$W : x'_l = \sum_{i,j,k \in E} P_{ijk,l} x_i x_j x_k, \;\; l \in E \tag{1}$$

where

$$P_{ijk,l} \geq 0, P_{ijk,l} = P_{ikj,l} = ... = P_{jki,l}, \text{for all} \;\; i, j, k, l \in E \;\; \text{and} \;\; \sum_{l \in E} P_{ijk,l} = 1. \tag{2}$$

Definition 2. A Volterra CSO $W : S^{n-1} \to S^{n-1}$ is a CSO with the additional assumption

$$P_{ijk,l} = 0 \;\; \text{if} \;\; l \notin \{i, j, k\} \quad i, j, k, l = 1, \ldots, n. \tag{3}$$

Definition 3. A Volterra CSO (3) is called a non-Volterra CSO if it satisfies the condition:

$$P_{ijk,l} = 0 \;\; \text{if} \;\; l \in \{i, j, k\} \quad i, j, k, l = 1, \ldots, n. \tag{4}$$

Definition 4. A non-Volterra CSO (4) is called a quasi non-Volterra CSO if it satisfies the condition:

$$P_{iii,i} > 0 \;\; \text{for any} \;\; i \in E.$$

Consider CSO $W : S^2 \to S^2 = \{(x, y, z) \in R^2 : x \geq 0, y \geq 0, z \geq 0, x + y + z = 1\}$ is

$$W : \begin{cases} x' = a(x^3 + y^3 + z^3) + 3x^2y + 3x^2z, \\ y' = b(x^3 + y^3 + z^3) + 3y^2x + 3y^2z + 3xyz, \\ z' = c(x^3 + y^3 + z^3) + 3z^2x + 3z^2y + 3xyz. \end{cases} \tag{5}$$

where $a \geq 0$, $b \geq 0$, $c \geq 0$ and $a + b + c = 1$

Let $\mathbf{x}^{(0)} \in S^2$ be the initial point and let $\left\{\mathbf{x}^{(n)}\right\}_{n=0,1,2,...}$ be the trajectory of $\mathbf{x}^{(0)}$ under the action of the operator (5), that is $\mathbf{x}^{(n+1)} = W(\mathbf{x}^{(n)})$, where $n = 0, 1, 2, \ldots$

One of the main problems in mathematical biology is to study the asymptotic behavior of the trajectories. Note the this problem is an open problem for quasi non-Volterra CSOs even in the two-dimensional case.

Generally, the study of the quasi non-Volterra CSO (5) is complicated. We consider the case when $a = 0$, $b + c = 1$.

$$W : \begin{cases} x' = 3x^2y + 3x^2z, \\ y' = b(x^3 + y^3 + z^3) + 3y^2x + 3y^2z + 3xyz, \\ z' = c(x^3 + y^3 + z^3) + 3z^2x + 3z^2y + 3xyz. \end{cases} \tag{6}$$

To find the fixed points we consider the equation $W(\mathbf{x}) = \mathbf{x}$, that is the following system of equations

$$x' = x, \ , y' = y \ , z' = z \tag{7}$$

We denote the set of all fixed points of the operator W as $\mathrm{Fix}\,(W)$, which is defined as:

$$\mathrm{Fix}\,(W) = \{\mathbf{x} \in S^2 : W(\mathbf{x}) = \mathbf{x}\}.$$

Consider the first equation of the system (7) and using $x + y + z = 1$ we have $x = 0$.
The second equation of the (7) it follows

$$y = b, \ z = 1 - b, \quad b \in [0, 1].$$

So

$$\mathrm{Fix}\,(W) = \{(0, b, 1 - b)\}.$$

To find the type of a fixed point of (6). The Jacobian of the operator (6) at fixed points has the following eigenvalues $\mid \lambda_1 \mid = 0 < 1$, $\mid \lambda_2 \mid = 3b(1 - b) < 1$. Then the operator (6) has the an attracting fixed point.

In the next Theorem we give description of the set of limit points of the trajectories.

Theorem 1. For the operator (6) the following statement hold:
for any $\lambda \in (x, y, z) \in S^2 \setminus \mathrm{Fix}\,(W)$ we have

$$\lim_{n\to\infty} W^{(n)}(\lambda) = (0, b, 1 - b).$$

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APPLICATION OF THE HANKEL TRANSFORM TO SOLVING CAUCHY PROBLEMS FOR MODEL PARTIAL DIFFERENTIAL EQUATIONS

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The report shows the application of the Hankel (Fourier-Bessel) transform to solving Cauchy problems for model degenerate equations of mathematical physics [1-6].

Cauchy problem for a model degenerate parabolic equation.

We consider in the domain $\Omega_1 = \{(x,t): \ x>0, \ t>0\}$ degenerate parabolic equation

$$u_t(x,t) = u_{xx}(x,t) + \frac{1}{x}u_x(x,t) - \frac{\nu^2}{x^2}u(x,t), \quad |\nu| < \frac{1}{2}. \tag{1}$$

Cauchy problem. Find a restricted function $u(x,t)$ in the domain Ω_1, satisfying the equation (1) and initial condition

$$u(x,t)|_{t=0} = \varphi(x), \ \ x>0, \tag{2}$$

where

$$\varphi(x) \in C^2, \quad \int_0^\infty |\varphi(x)|\sqrt{x}dx < c = const.$$

The solution to the stated Cauchy problem has the following form

$$u(x,t) = \frac{1}{4t}\int_0^\infty \varphi(\rho)\rho e^{-\frac{\rho^2+x^2}{4t}}\left[I_{-\nu}\left(\frac{\rho x}{2t}\right) + I_\nu\left(\frac{\rho x}{2t}\right)\right]d\rho. \tag{3}$$

The constructed solution (3) satisfies the initial condition (2).

Cauchy problem for a model degenerate hyperbolic equation.

Consider in the domain $\Omega_2 = \{(x,t): x>0, t>0, x>t\}$ degenerate hyperbolic equation

$$u_{tt}(x,t) = u_{xx}(x,t) + \frac{1}{x}u_x(x,t) - \frac{\nu^2}{x^2}u(x,t), \ |\nu| < \frac{1}{2}. \tag{4}$$

Cauchy problem. Find a restricted function $u(x,t)$ in the domain Ω_2 satisfying equation (4) and the initial conditions

$$u(x,t)|_{t=0} = \varphi(x), \ 0<x<\infty, \tag{5}$$

$$u_t(x,t)|_{t=0} = \psi(x), \ 0<x<\infty, \tag{6}$$

where

$$\varphi(x), \psi(x) \in C^2(\Omega), \int_0^\infty |\varphi(x)|\sqrt{x}dx < c = const, \int_0^\infty |\psi(x)|\sqrt{x}dx < c = const.$$

The solution to the stated Cauchy problem has the following form

$$u(x,t)=\frac{1}{2}\int\limits_0^\infty \varphi(\rho)\,G_1(x,t,\rho)\,\rho d\rho+\frac{1}{2}\int\limits_0^\infty \psi(\rho)\,G_2(x,t,\rho)\,\rho d\rho, \tag{7}$$

where

$$G_1(x,t,\rho)=\int\limits_0^\infty \lambda\left[J_{-\nu}(\rho\lambda)\,J_{-\nu}(x\lambda)+J_\nu(\rho\lambda)\,J_\nu(x\lambda)\right]\cos(t\lambda)\,d\lambda,$$

$$G_2(x,t,\rho)=\int\limits_0^\infty \left[J_{-\nu}(\rho\lambda)\,J_{-\nu}(x\lambda)+J_\nu(\rho\lambda)\,J_\nu(x\lambda)\right]\sin(t\lambda)\,d\lambda.$$

The constructed solution (7) satisfies the initial conditions (5) and (6).

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PERIODIC GIBBS MEASURES FOR CONTINUOUS SPIN MODELS ON CAYLEY TREES

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This thesis focuses on periodic Gibbs measures with period two for models that have a continuum set of spin values on Cayley trees.r

A Cayley tree $\Gamma^k = (V, L)$ of order $k \geq 1$ is an infinite homogeneous tree, i.e., a graph without cycles, with exactly $k+1$ edges incident to each vertices. Here V is the set of vertices and L that of edges (arcs).

Consider models where the spin takes values in the set $[0,1]$, and is assigned to the vertices of the tree. For $A \subset V$ a configuration σ_A on A is an arbitrary function $\sigma_A : A \to [0,1]$. Denote $\Omega_A = [0,1]^A$ the set of all configurations on A. A configuration σ on V is then defined as a function $x \in V \mapsto \sigma(x) \in [0,1]$; the set of all configurations is $[0,1]^V$. The (formal) Hamiltonian of the model is:

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} \xi_{\sigma(x),\sigma(y)}, \tag{1}$$

where $J \in R \setminus \{0\}$ and $\xi : (u,v) \in [0,1]^2 \to \xi_{u,v} \in R$ is a given bounded, measurable function. As usually, $\langle x,y\rangle$ stands for nearest neighbor vertices (detail in [1-3]).

Denote

$$c_{ij}(m) = \frac{1}{m + 2(i-1) + 2(j-1)}, \quad (n,m,p) \in N \times N \times N_0, \quad 1 \leq i,j \leq n,$$

$A_n^{(m,p)} = \left(\frac{c_{ij}(m)}{4^{p+j+i-2}}\right)_n$ is $n \times n$-square matrix.
If $n \in \{2,3\}$ then it's easy to check $\det(A_n^{(m,p)}) \neq 0$.

Put

$$\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} = \left(A_3^{(1,0)}\right)^{-1} \begin{pmatrix} 0 \\ \frac{1}{6} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} a_{21} \\ a_{22} \\ a_{23} \end{pmatrix} = \left(A_3^{(3,1)}\right)^{-1} \begin{pmatrix} 0 \\ 0 \\ \frac{1}{20} \end{pmatrix}$$

and

$$\begin{pmatrix} b_{11} \\ b_{12} \end{pmatrix} = \left(A_2^{(5,2)}\right)^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} b_{21} \\ b_{22} \end{pmatrix} = \left(A_2^{(7,3)}\right)^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where $\left(A_n^{(m,p)}\right)^{-1}$ is the inverse matrix of $A_n^{(m,p)}$.
We now define following functions:

$$\psi_1(u) = a_{11} + a_{12}u^2 + a_{13}u^4, \quad \psi_2(u) = a_{21}u^2 + a_{22}u^4 + a_{23}u^6,$$
$$\psi_3(u) = b_{11}u + b_{12}u^3, \quad \psi_4(u) = b_{21}u^3 + b_{22}u^5.$$

Finally,

$$K_1(t,u;k) = \psi_1(u)\left(\sqrt[k]{20t^4 + \frac{3}{4}} - 1\right) + \psi_2(u)\left(\sqrt[k]{6t^2 + \frac{1}{2}} - 1\right),$$

$$K_2(t,u;k)=\psi_3(u)\left(\sqrt[k]{t^3+1}-1\right)+\psi_4(u)\left(\sqrt[k]{t^5+1}-1\right),$$

$$\widetilde{K}(t,u;k)=1+K_1(t,u;k)+K_2(t,u;k).$$

Remark 1. There exists $k_0\in N$ such that for all $k\geq k_0$ the following inequality holds

$$\widetilde{K}\left(t-\frac{1}{2},u-\frac{1}{2};k\right)>0,\quad (t,u)\in[0,1]^2.$$

Theorem 1. Let $k\geq k_0$. The model (1)

$$H(\sigma)=-\frac{1}{\beta}\sum_{<x,y>}ln\widetilde{K}\left(\sigma(x)-\frac{1}{2},\sigma(y)-\frac{1}{2};k\right)$$

on the Cayley tree Γ^k has at least four periodic Gibbs measures.

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ON POSITIVE FIXED POINTS OF OPERATOR OF HAMMERSTEIN TYPE WITH DEGENERATE KERNEL AND GIBBS MEASURES

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From [1] it is known that "translation-invariant Gibbs measures"of the model with an uncountable set of spin values can be described by positive fixed points of a nonlinear integral operator of Hammerstein type. In [2, 3], there are main results on positive fixed points of the operator of Hammerstein type with degenerate kernels, but it was not solved the existence of Gibbs measures corresponding to the founded fixed points for constructed kernels. This thesis is an investigation of the scientific works [2] and [3]. In the present thesis, we construct new degenerate kernels of the Hammerstein operator by taking into account problems in the theory of Gibbs measure, i.e. each positive fixed point of the operator gives translational-invariant Gibbs measure.

For every $k \in \mathbb{N}$ we consider an integral operator H_k acting in the cone $C^+[0,1]$ i.e.,

$$(H_k f)(t) = \int_0^1 K(t,u) f^k(u) du, \;\; k \in \mathbb{N}$$

where $C^+[0,1] = \{f \in C[0,1] : f(x) \geq 0\}$. The operator H_k is called Hammerstein's integral operator of order k. Clearly, if $k \geq 2$ then H_k is a nonlinear operator.

A configuration σ on V is then defined as a function $x \in V \mapsto \sigma(x) \in [0,1]$.

The (formal) Hamiltonian of the model is:

$$H(\sigma) = -J \sum_{\langle x,y \rangle \in L} \xi_{\sigma(x)\sigma(y)}, \tag{1}$$

where $J \in R \setminus \{0\}$ and $\xi : (u,v) \in [0,1]^2 \to \xi_{uv} \in R$ is a given bounded, measurable function. As usual, $\langle x,y \rangle$ stands for the nearest neighbor vertices.

Let $h: \; x \in V \mapsto h_x = (h_{t,x}, t \in [0,1]) \in R^{[0,1]}$ be mapping of $x \in V \setminus \{x^0\}$ and $\beta = \frac{1}{T}$ is positive real number. We define a family of probability kernals as

$$\mu^{(n)}(\sigma_n) = Z_n^{-1} \exp\left(-\beta H(\sigma_n) + \sum_{x \in \mathbb{V}_n} h_{\sigma(x),x} \right), \tag{2}$$

where $\sigma_n : x \in V_n \mapsto \sigma(x)$ and Z_n is the corresponding partition function. By Kolmogorov extension theorem, the family of probability kernals can be extension to the whole space of configurations and this measure μ is called splitting Gibbs measure corresponding to Hamiltonian (1) and boundary function $x \mapsto h_x$, $x \neq x^0$.

The following statement describes conditions on h_x guaranteeing compatibility of the corresponding distributions $\mu^{(n)}(\sigma_n)$.

Proposition. The probability distributions $\mu^{(n)}(\sigma_n), n = 1, 2, \ldots,$ in (2) are compatible iff for any $x \in V \setminus \{x^0\}$ the following equation holds:

$$f(t,x) = \prod_{y \in S(x)} \frac{\int_0^1 \exp(J\beta\xi_{tu}) f(u,y) du}{\int_0^1 \exp(J\beta\xi_{0u}) f(u,y) du}. \tag{3}$$

where $f(t,x)=\exp\left(h_{t,x}-h_{0,x}\right), t\in[0,1]$ and $du=\lambda(du)$.

In the class of translational-invariant functions $f(t,x)$, the equation (3) is equivalent to $H_kf=f$, where $K(t,u)=\exp\left(J\beta\xi_{tu}\right), f(t)>0, t,u\in[0,1]$ (e.g. [1-3]).

We define two continuous positive functions $\zeta_1(u)$ and $\zeta_2(u)$ on $[0,1]$

$$\zeta_1(u)=\begin{cases}\frac{1}{2}+\sin 2\pi u, & \text{if } u\in[0,\frac{1}{2}]\\ \frac{1}{2}, & \text{if } u\in[\frac{1}{2},1]\end{cases}, \quad \zeta_2(u)=\begin{cases}\frac{1}{2}, & \text{if } u\in[0,\frac{1}{2}]\\ \frac{1}{2}-\sin 2\pi u, & \text{if } u\in[\frac{1}{2},1].\end{cases}$$

For each positive numbers a,b we define continuous positive functions $F_1(t;a,b)$ and $F_2(t;a,b)$ on $[0,1]$

$$F_1(t;a,b)=\begin{cases}a\cos\pi t+b, & \text{if } t\in[0,\frac{1}{2}]\\ b, & \text{if } t\in[\frac{1}{2},1]\end{cases}, \quad F_2(t;a,b)=\begin{cases}b, & \text{if } t\in[0,\frac{1}{2}]\\ -a\cos\pi t+b, & \text{if } t\in[\frac{1}{2},1].\end{cases}$$

For each positive numbers a,b we denote

$$\tilde{K}(t,u;a,b)=\zeta_1(u)F_1(t;a,b)+\zeta_2(u)F_2(t;a,b), \quad t,u\in[0,1].$$

Theorem. Let $\tilde{K}(t,u;a,b)$ is kernel, then:

(*i*) $a\le\frac{35(44+15\pi)}{318}b$ there exists a unique positive fixed point of the operator H_3 and is:

$$f(t)=\frac{1}{\sqrt{\frac{177a}{35\pi}+\frac{44+15\pi}{6\pi}b}}\zeta_1(t)+\frac{1}{\sqrt{\frac{177a}{35\pi}+\frac{44+15\pi}{6\pi}b}}\zeta_2(t).$$

(*ii*) $a>\frac{35(44+15\pi)}{318}b$ there exist exactly three positive fixed points of the operator H_3.

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INVARIANCE PRINCIPLE FOR A CLASS OF GIBBS RANDOM FIELDS

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Given integer $N > 1$ and a sequence $(\xi_k)_{k\geq 1}$ of independent identically distributed random variables, a random field $(X_m)_{m=1}^N$ is called a balanced periodic field generated by $\{\xi_k\}_{k=1}^N$, if the distribution of its increments, $(X_k - X_{k-1})_{k=1}^N$ (where $X_0 \equiv X_N$), coincides with that of $(\xi_k)_{k=1}^N$ subject to the conditions

$$\xi_1 + \xi_2 + \cdots + \xi_N = 0 \qquad \text{and} \qquad X_1 + X_2 + \cdots + X_N = 0.$$

One can think of $(X_m)_{m=1}^N$ as an N-periodic random field of SOS-type whose trajectories have zero integral (i.e., balanced, $X_1 + X_2 + \cdots + X_N = 0$).

Let $(x_t^{(N)})_{t\in S_1}$ be the continuous periodic random field on the circle S_1 of length 1, defined via $x_{k/N}^{(N)} = X_k$ for $k = 0, 1, \ldots, N$ (recall that $X_0 = X_N$) and linearly interpolated elsewhere. Under suitable conditions on the distribution of ξ_k, as $N \to \infty$, the law of $(x_t^{(N)})_{t\in S_1}$ converges weakly to that of a unique balanced periodic Gaussian Gibbs field on S_1, whose distribution is explicitly described.

Generalisation to non-homogeneous setting and some applications will also be discussed.

THE BOUNDARY CONDITION FOR A MIXED TYPE MODEL ON THE CAYLEY TREE

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Let $\tau^k = (V, L)$ be the Cayley tree of order k, i.e. a $k+1$-regular infinite tree with exactly $k+1$ edges meeting at each vertex. Denote by V the set of vertices and by L the set of edges of the tree. Two vertex $t, s \in V,\ (t \neq s)$ will be called *neighbors* if there is an edge connecting them. In this case for the pair $\{s, t\}$ we set $\langle s, t\rangle$. We can choose an arbitrary vertex of the tree as a root, which we denote as 0, and we write $s \to t$, if $t \neq s$ and the path connecting 0 and t passes through s. If $s \to t$ and s, t are neighbors, then t is called a *direct successor* of s and we write $s \to_1 t$. The set $\partial A = \{t \in V \backslash A;$ there exists a neighbour of t in $A\}$ is the boundary of $A \subset V$.

We consider model mixed type, where the spin takes values in the set $\Phi = \{-1, 0, +1\},\ m \geq 1$. Let A be a finite subset of $V,\ \omega \in \Omega$ and $\sigma \in \Omega_A$. The Ising-Potts interaction energy on A given the inner configuration σ and boundary condition ω is defined by

$$H_A^{\omega}(\sigma) = -\sum_{t,s\in A:\langle t,s\rangle}\Big[\alpha J_1\sigma(t)\sigma(s) + (1-\alpha)J_2\delta_{\sigma(t)\sigma(s)}\Big]$$
$$-\sum_{t\in A, s\in\partial A:\langle t,s\rangle}\Big[\alpha J_1\sigma(t)\omega(s) + (1-\alpha)J_2\delta_{\sigma(t)\omega(s)}\Big], \tag{1}$$

where J_1, J_2, are the coupling real constants and $0 < \alpha < 1$.

We remark that model (1) coincides with the Potts model, if $\alpha = 0$ and coincides with the Ising model, if $\alpha = 1$. For each $t \in V$, we denote the distance of t from the root 0 by $\mid t \mid$. We write $\mid t \mid = n$ if there is a chain $0 \to_1 u_1 \to_1 u_2 \to_1 u_3 \to_1 \ldots \to_1 u_{n-1} \to_1 t$. We define the sequence of balls $V_n = \{t \in V :\mid t \mid \leq n\},\ n \geq 1$. For each $s \in V$ we denote $\tau_s^k = \{s\} \cup \{t \in V : s \to t\}$ and $V_{n,s} = \tau_s^k \cap V_n,\ n \geq 1$. It is apparent that if the graph is cut at 0 then it splits into $k+1$ identical disconnected pieces, we number them by $1, 2, \ldots, k+1$. Each of these is a rooted tree (with root 0). In what follows, we get $V_n = \{0\} \cup \bigcup_{i=1}^{k+1} V_{n,i}$ and $V_{n,i} = \{i\} \cup \bigcup_{j:i\to_1 j} V_{n,j}$. For every $\omega \in \Omega,\ s \in V\backslash\{0\}$ and $n \geq \mid s \mid$ we define

$$W_{n,s}^{\omega}(l) = \sum_{\sigma\in\Omega_{V_{n,s}}:\sigma(s)=l}\exp\Big[-H_{V_{n,s}}^{\omega}(\sigma) - \Big(\alpha J_1 l\omega(t) + (1-\alpha)J_2\delta_{l,\omega(t)}\Big)\Big], \tag{2}$$

$$R_{n,s}^{l}(\omega) = \frac{W_{n,s}^{\omega}(l)}{W_{n,s}^{\omega}(0)}, \tag{3}$$

where for $l = \pm 1$, t is the unique vertex such that $t \to_1 s$. We get $R_{n,s}^{l}(\omega) = R_{n-|s|+1}^{l}(\omega),\ l \in \Phi$ applying a similar method as in [1,2]. Thus, it follows from (2) that

$$Y_{n,l}(\omega) = k\, F_l(Y_{n-1,-1}(\omega), Y_{n-1,+1}(\omega)), \tag{4}$$

where $l = -1, +1,\ n \geq 2,\ Y_{n,l}(\omega) = \ln R_n^l(\omega)$ and $F = (F_{-1}, F_{+1})$ is defined by

$$F_l(x_{-1},x_{+1})=\ln\left[\frac{\sum\limits_{j\in\Phi}\exp\left(J_1lj+J_2\delta_{l,j}\right)\,\exp(x_j)}{\sum\limits_{j\in\Phi}\exp(J_2\delta_{0,j})\,\exp(x_j)}\right].$$

It is clear that if $\lim\limits_{n\to\infty}Y_{n,i}(\omega)=Y_i(\omega)$ then from (4) we obtain

$$Y_l(\omega)=k\,F_l(Y_{-1}(\omega),Y_{+1}(\omega)),\quad l=\pm1. \tag{5}$$

Denoting $\eta=\exp(J_1),\nu=\exp(J_2),\ h_{l,\omega}=Y_l(\omega),\ l=\pm1$, for $k=2$ and $\eta=\frac{1}{2}$ the equation (5) is reduced to

$$\begin{cases} h_{-1}=2\,\ln\frac{\frac{1}{2}\nu\exp(h_{-1})+2\exp(h_{+1})+1}{exp(h_{-1})+\exp(h_{+1})+\nu},\\ \\ h_{+1}=2\,\ln\frac{\frac{1}{2}\nu\exp(h_{+1})+2\exp(h_{-1})+1}{exp(h_{-1})+\exp(h_{+1})+\nu}. \end{cases} \tag{6}$$

It is important to note that the following set is invariant with respect to the operator $Q:\mathbb{R}^2\to\mathbb{R}^2$ which is defined by $Q(h)=2\,F(h)$, $I_1=\{(h_{-1},h_{+1})\in\mathbb{R}^2:h_{-1}=h_{+1}\}$. The system of equations (6) on the set I_1 is reduced to the following equation $h_{-1}=f(h_{-1})$. For a given initial state $u^{(0)}=(u_0^{(0)},u_0^{(0)})$ we observe the limit $\lim\limits_{n\to\infty}Q^{(n)}(u^{(0)})$. The following equalities can be obtained in much the same way as in [1-3]. $c_1^l(\omega)=\sum_{s:t\to_1 s}l\omega(s)$ and $c_2^l(\omega)=\sum_{s:t\to_1 s}\delta_{l,\omega(s)}\{l=\pm1\}$ are independent of $t\in V\backslash\{0\}$. Due to (4) the asymptotic behaviour of the vector $Y_{n,l}(w)=(Y_l(\omega),Y_l(\omega))$ depends only on the vector $Y_l(\omega)$, where $Y_l(\omega)=J_1(c_1^l(\omega)-c_1^0(\omega))+J_2(c_2^l(\omega)-c_2^0(\omega)),\ l=\pm1$, where $J_1=-\ln 2$ and $c_i^l(\omega),i=1,2$.

We denote the following sets: $\mathbb{D}=\{\omega\in\Omega:c_i^{(-1)}(\omega)=c_i^{(+1)}(\omega),i=1,2\}$, $\mathbb{D}^+=\{\omega\in\mathbb{D}:S>2\ln\gamma_2\}$, $\mathbb{D}^0=\{\omega\in\mathbb{D}:S=2\ln\gamma_2\}$, $\mathbb{D}^-=\{\omega\in\mathbb{D}:S<2\ln\gamma_2\}$, where $S=S(\omega,J_1,J_2):=J_1(c_1^{(+1)}(\omega)-c_1^0(\omega))+J_2(c_2^{(+1)}(\omega)-c_2^0(\omega))$

Theorem 1. (i) If $a<a_c$ and $\omega\in\mathbb{D}$ then $P^\omega=\mu_1(a)$.

(ii) If $a=a_c$ then

$$P^\omega=\begin{cases}\mu_2(a),\ \text{if }\omega\in\mathbb{D}^0\cup\mathbb{D}^+,\\ \mu_1(a),\ \text{if }\omega\in\mathbb{D}^-.\end{cases}$$

(iii) If $a>a_c$ then

$$P^\omega=\begin{cases}\mu_3(a),\ \text{if }\omega\in\mathbb{D}^+,\\ \mu_2(a),\ \text{if }\omega\in\mathbb{D}^0,\\ \mu_1(a),\ \text{if }\omega\in\mathbb{D}^-.\end{cases}$$

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THE DYNAMICS OF SUPERPOSITION OF VOLTERRA AND PERMUTED VOLTERRA QUADRATIC OPERATORS

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The concept of a quadratic stochastic operator and its application in a biological context were first established by Bernstein in [1]. Since then, the theory of quadratic stochastic operators has been further deepened motivated by their frequent occurrence in mathematical models of genetics, where quadratic stochastic operators serve as a tool for the study of dynamical properties and modelling; see e.g. [2, 3, 4].

Let $E = \{1, \dots, m\}$ be a finite set and set of all probability distributions on E

$$S^{m-1} = \{\mathbf{x} = (x_1, x_2, \dots, x_m) \in \mathbb{R}^m : \ x_i \geq 0, \ \text{ for any } \ i \ \text{ and } \ \sum_{i=1}^{m} x_i = 1\},$$

the $(m-1)$- dimensional simplex.

A mapping $V : \mathbb{R}^m_+ \to \mathbb{R}^m_+$ is called *stochastic operator* (SO) if $V(S^{m-1}) \subset S^{m-1}$. The simplest non-linear SOs are quadratic stochastic operators.

A map $V : S^{m-1} \to S^{m-1}$ is called a quadratic stochastic operator (QSO) if

$$(V\mathbf{x})_k = \sum_{i,j=1}^{m} p_{ij,k} x_i x_j$$

for any $\mathbf{x} \in S^{m-1}$ and for all $k = 1, \dots, m$, where

$$p_{ij,k} = p_{ji,k} \geq 0, \qquad \text{for all } i, j; \quad \sum_{k=1}^{m} p_{ij,k} = 1.$$

A QSO is said to be a *Volterra QSO* if the coefficients of heredity satisfy the following additional assumption $p_{ij,k} = 0, \ \text{ for } \ k \notin \{i, j\}, \qquad i, j, k = 1, \dots, m.$

The trajectory $\{\mathbf{x}^{(n)}\}_{n=0}^{\infty}$ of an operator V for an initial point $\mathbf{x}^{(0)} \in S^{m-1}$ is defined by

$$\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)}) = V^{n+1}(\mathbf{x}^{(0)}), \ n = 0, 1, 2, \dots$$

Denote by $\omega_V(\mathbf{x}^{(0)})$ the set of limit points of the trajectory $\{\mathbf{x}^{(n)}\}_{n=0}^{\infty}$. One of the main problems in mathematical biology consists to study of asymptotical behaviour of the trajectories for a given QSO. In other words, the main task is the description of the set $\omega_V(\mathbf{x}^{(0)})$ for any initial point $\mathbf{x}^{(0)} \in S^{m-1}$ for a given QSO.

We let a face of the simplex S^{m-1} be the set $\Gamma_\alpha = \{\mathbf{x} \in S^{m-1} : x_i = 0, \quad i \notin \alpha \subset E\}$ and let $\mathbf{e}_i = (\delta_{1i}, \delta_{2i}, \dots, \delta_{mi}) \in S^{m-1}, \ i = 1, \dots, m$, the vertices of the simplex S^{m-1}, where δ_{ij} is the Kronecker delta.

Consider the following Volterra and permuted Volterra QSOs

$$V : \begin{cases} x_1' = x_1 (1 - x_2 - x_3), \\ x_2' = x_2 (1 + x_1 - x_3), \\ x_3' = x_3 (1 + x_1 + x_2). \end{cases} \qquad V_\pi : \begin{cases} x_1' = x_3 (1 + x_1 + x_2), \\ x_2' = x_2 (1 + x_1 - x_3), \\ x_3' = x_1 (1 - x_2 - x_3). \end{cases}$$

We shall study the dynamics of superposition of the QSOs V and V_π. It is easy to verify that the operator $W = V \circ V_\pi$ has form

$$W : \begin{cases} x_1' = x_3\,(1 + x_1 + x_2)\,(1 - x_1 - x_2 + x_1x_3 + x_2x_3)\,, \\ x_2' = x_2\,(1 + x_1 - x_3)\,(1 + x_3 - x_1 + x_1x_2 + 2x_1x_3 + x_2x_3)\,, \\ x_3' = x_1\,(1 - x_2 - x_3)\,(1 + x_2 + x_3 + x_1x_2 + x_1x_3)\,. \end{cases}$$

Denote $\mathbf{x}^* = (0, 1 - x^*, x^*)$, $\mathbf{y}^* = (x^*, 0, 1 - x^*)$, $\mathbf{z}^* = (1 - x^*, x^*, 0)$, where x^* is a real solution of the equation $x^4 - 2x^2 - x + 1 = 0$ in the segment $[0, 1]$.

In the following theorem we describe dynamical properties the stochastic operator W.

Theorem. For the stochastic operator W the following assertions are hold:

i) the face Γ_{13} is an invariant set;

ii) Fix $(W) = \{\mathbf{e}_2, \mathbf{y}^*\}$ and the vertex $\mathbf{e}_2$ is an attracting point and $\mathbf{y}^*$ is a repelling point;

iii) for any $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_2, \mathbf{y}^*\}$ we have

$$\omega_W\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_2\}\,, & \text{if} \quad x_1^{(0)} < x^*, x_3^{(0)} < 1 - x^*, \\ \{\mathbf{x}^*, \mathbf{z}^*\}\,, & \text{if} \quad x_1^{(0)} = x^* \ \text{ or } \ x_3^{(0)} = 1 - x^*, \\ \{\mathbf{e}_1, \mathbf{e}_3\}\,, & \text{otherwise.} \end{cases}$$

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THE ASYMPTOTICAL BEHAVIOR OF TRAJECTORIES OF A SEPARABLE CUBIC OPERATOR

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Let $E = \{1, 2, \dots, m\}$ be a finite set and the set of all probability distributions on E

$$S^{m-1} = \{\mathbf{x} = (x_1, x_2, \dots, x_m) \in \mathbb{R}^m : \ x_i \geq 0, \text{ for any } i \text{ and } \sum_{i=1}^{m} x_i = 1\},$$

be the $(m-1)$-dimensional simplex. Cubic stochastic operator (CSO) is a mapping $W : S^{m-1} \to S^{m-1}$ of the form

$$W : x'_l = \sum_{i,j,k \in E} P_{ijk,l} x_i x_j x_k, \quad l \in E = \{1, ..., m\}, \tag{1}$$

where $P_{ijk,l}$ are the coefficients of heredity such that

$$P_{ijk,l} \geq 0, \quad \sum_{l \in E} P_{ijk,l} = 1, \quad \forall i, j, k, l \in E \tag{2}$$

Consider the following CSO defined on a simplex S^{m-1} which has the form

$$W : \begin{cases} x'_k = x_{\pi(k)} \left(\sum\limits_{i=1}^{m-1} x_i + c x_m \right)^2, \ \ 1 \leq k \leq m-1, \\ x'_m = x_m \left(a \sum\limits_{i=1}^{m-1} x_i + x_m \right) \left(b \sum\limits_{i=1}^{m-1} x_i + x_m \right), \end{cases} \tag{3}$$

where $a + b + c^2 = 3, \ ab + 2c = 3, \ a, b \in [0, 1], \ c \in [1, 1.5]$ and π is a permutation of the set $\{1, \dots, m-1\}$. The CSO (3) is called separable CSO (SCSO). Let $\pi = \tau_1 \tau_2 \cdots \tau_q$ be a permutation of the set $\{1, \dots, m-1\}$, where $\tau_1, \dots, \tau_q$ are disjoint cycles and we denote by $ord(\tau_i)$ the order of a cycle τ_i. The set of all periodic points of period n by $Per_n(W)$.

Theorem 1. For the SCSO W the following statements are true:

i) a) The set $M_0 = \{\mathbf{x} \in S^{m-1} : x_1 \cdots x_{m-1} = 0\}$ is invariant set.
b) If $\mathrm{ord}(\tau_i) = \mathrm{ord}(\tau_j)$ and $i \neq j$ then the set

$$M_\eta = \Big\{ \mathbf{x} \in S^{m-1} : \prod_{k \in \mathrm{supp}(\tau_i)} x_k = \eta \prod_{k \in \mathrm{supp}(\tau_j)} x_k \vee \prod_{k \in \mathrm{supp}(\tau_i)} x_k = (1/\eta) \prod_{k \in \mathrm{supp}(\tau_j)} x_k \Big\}$$

is invariant set, where $\eta > 0$;

ii) a) If $a \neq 1$ and $\pi \neq Id$ then we have $\mathrm{Fix}(W) = \widehat{X} \cup \{\mathbf{e}_m\}$, where

$$\widehat{X} = \{\mathbf{x} \in S^{m-1} : \ x_k = x_l, \ \text{ for all } \ k, l \in \mathrm{supp}(\tau_i), \ \ i = 1, \dots, q, \ \ x_m = 0\}$$

and $\mathbf{e}_m = (0, \dots, 0, 1)$.

b) If $a = 1$ and $\pi \neq Id$ then we have $\mathrm{Fix}(W) = \widetilde{X}$, where

$$\widetilde{X} = \{\mathbf{x} \in S^{m-1} : \ x_k = x_l, \ \text{ for all } \ k, l \in \mathrm{supp}(\tau_i), \ \ i = 1, \dots, q, \ \ x_m = x^*\}$$

and $x^* \in (0, 1)$;

iii) a) If $a \neq 1$ and $\pi \neq Id$ then the points of the set

$$\mathrm{Per}_s(W) = \Big\{\mathbf{x} \in S^{m-1} : x_1 + \cdots + x_{m-1} = 1, \ x_m = 0\Big\}$$

are periodic points of the SCSO W with period $s = \mathrm{lcm}\big(\mathrm{ord}(\tau_1), \dots, \mathrm{ord}(\tau_q)\big)$.

b) If $a = 1$ and $\pi \neq Id$ then for any $x^* \in (0, 1)$ the points of the set

$$\mathrm{Per}_s(W) = \Big\{\mathbf{x} \in S^{m-1} : x_1 + \cdots + x_{m-1} = 1 - x^*, \ x_m = x^*\Big\}$$

are periodic points of the operator W with period $s = \mathrm{lcm}\big(\mathrm{ord}(\tau_1), \dots, \mathrm{ord}(\tau_q)\big)$;

iv) $\mathrm{Per}_n(W) = \emptyset$ for $n > s$.

Theorem 2. For the operator W the following statements are true:

i) if $a = 1$ and $\pi \neq \mathbf{I}d$, then $\omega_W\big(\mathbf{x}^{(0)}\big) = \{\mathbf{x}^{(0)}, W(\mathbf{x}^{(0)}), \dots, W^{s-1}(\mathbf{x}^{(0)})\}$ for any $\mathbf{x}^{(0)} \in S^{m-1} \setminus \{\mathbf{e}_m\}$;

ii) if $a \neq 1$ and $\pi \neq \mathbf{I}d$, then $\omega_W\big(\mathbf{x}^{(0)}\big) = \{\mathbf{x}_\xi, \mathbf{x}_\xi^1, \dots, \mathbf{x}_\xi^{s-1}\}$ for any $\mathbf{x}^{(0)} \in S^{m-1} \setminus \{\mathbf{e}_m\}$;

iii) if $a \neq 1$ and $\pi = \mathbf{I}d$, then

$$\omega_W\big(\mathbf{x}^{(0)}\big) = \begin{cases} \Big\{(x_1^{(0)}, x_2^{(0)}, \dots, x_{m-1}^{(0)}, 0)\Big\} & \text{if } \ x_m^{(0)} = 0, \\ \big\{(x_1^*, x_2^*, \dots, x_{m-1}^*, 0)\big\} & \text{if } \ x_m^{(0)} > 0, \end{cases}$$

for any $\mathbf{x}^{(0)} \in S^{m-1} \setminus \{\mathbf{e}_m\}$.

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ON AUTOMORPHISMS OF A THREE-DIMENSIONAL NON-VOLTERRA GENETIC ALGEBRA

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The work of Bernstein [1] has pioneered the application of quadratic stochastic operators (QSO), and in general, these operators are employed within the domain of biology especially in the presentation of time evolution of countless species. There are various established classes of non-associative algebras and so forth, and all these have significantly contributed to the theoretical population genetics (see e.g. [2]). One class is named by genetic algebra which is used in the description of some models in genetic. The application of a type of genetic algebra called evolution algebra was demonstrated in Tian [3].

Recall the definition of an evolution algebra of a free population following [2]. Let $E = \{1, \dots, m\}$. By $\mathbf{e}_i$, $i = 1, \dots, m$ we denote the standard basis in $\mathbb{R}^m$. The convex hull of these basis elements is $(m-1)-$ dimensional simplex, that is

$$S^{m-1} = \left\{ \mathbf{x} = (x_i) \in \mathbb{R}^m \ : \ x_i \geq 0, \ \forall i \in E, \quad \sum_{i=1}^{m} x_i = 1 \right\}. \tag{1}$$

A *quadratic stochastic operator (QSO)* is a mapping $V : S^{m-1} \to S^{m-1}$ of the form

$$V : x'_k = \sum_{i,j=1}^{m} p_{ij,k} x_i x_j, \quad k = 1, 2, \dots, m \tag{2}$$

where $p_{ij,k}$ are heredity coefficients, which satisfy the following conditions:

$$p_{ij,k} \geq 0, \quad p_{ij,k} = p_{ji,k}, \quad \sum_{k=1}^{m} p_{ij,k} = 1, \quad i, j, k \in \{1, 2, \dots, m\} \tag{3}$$

Let V be a QSO and suppose that $\mathbf{x} = (x_1, \dots, x_m), \mathbf{y} = (y_1, \dots, y_m) \in \mathbb{R}^m$ are arbitrary vectors, we introduce a multiplication rule (see [2]) on $\mathbb{R}^m$ by

$$(\mathbf{x} \circ_V \mathbf{y})_k = \sum_{i,j=1}^{m} p_{ij,k} x_i y_j. \tag{4}$$

Using (3) it is easy to see that $\mathbf{x} \circ_V \mathbf{y} = \mathbf{y} \circ_V \mathbf{x}$, i.e. the multiplication is commutative. In general, the genetic algebra is not necessarily to be associative.

Definition. Linear operator $\mathbf{A} : \mathcal{A}_V \to \mathcal{A}_V$ is called an *automorphism* if conditions $\mathbf{A}(\mathbf{x} \circ \mathbf{y}) = \mathbf{A}(\mathbf{x}) \circ \mathbf{A}(\mathbf{y})$ holds for all $\mathbf{x}, \mathbf{y} \in \mathcal{A}_V$.

Let us consider the following Volterra QSO defined on the S^2

$$V : \begin{cases} x'_1 = x_1^2 + (1-a)x_1x_2 + (1+c)x_1x_3, \\ x'_2 = x_2^2 + (1+a)x_1x_2 + (1-b)x_2x_3, \\ x'_3 = x_3^2 + (1-c)x_1x_3 + (1+b)x_2x_3, \end{cases} \tag{5}$$

where $-1 \leq a, b, c \leq 1$. Let us fix the following permutations of the set $E = \{1, 2, 3\}$

$$\pi = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}.$$

From [4] we known that the operator $V_\pi = T_\pi V$ is a quadratic automorphisms of S^2, where $T_\pi(\mathbf{x}) = \big(x_{\pi(1)}, x_{\pi(2)}, x_{\pi(3)}\big)$. Obviously that if we take $\pi = Id$, then V_π will be a Volterra quadratic operator. Another cases the operator V_π will be non-Volterra operator.

Now, by (5) V_π is defined as follows:

$$V_\pi : \begin{cases} x_1' = x_2^2 + (1+a)x_1x_2 + (1-b)x_2x_3, \\ x_2' = x_1^2 + (1-a)x_1x_2 + (1+c)x_1x_3, \\ x_3' = x_3^2 + (1-c)x_1x_3 + (1+b)x_2x_3, \end{cases} \tag{6}$$

where $-1 \leq a, b, c \leq 1$.

Then, by (6) the corresponding algebra $\mathcal{A}_{V_\pi}$ has the following multiplication table:

$$\mathbf{e}_1 \circ \mathbf{e}_1 = \mathbf{e}_2 \qquad \mathbf{e}_1 \circ \mathbf{e}_2 = \mathbf{e}_2 \circ \mathbf{e}_1 = \frac{1+a}{2}\mathbf{e}_1 + \frac{1-a}{2}\mathbf{e}_2$$

$$\mathbf{e}_2 \circ \mathbf{e}_2 = \mathbf{e}_1 \qquad \mathbf{e}_1 \circ \mathbf{e}_3 = \mathbf{e}_3 \circ \mathbf{e}_1 = \frac{1+c}{2}\mathbf{e}_2 + \frac{1-c}{2}\mathbf{e}_3$$

$$\mathbf{e}_3 \circ \mathbf{e}_3 = \mathbf{e}_3 \qquad \mathbf{e}_2 \circ \mathbf{e}_3 = \mathbf{e}_3 \circ \mathbf{e}_2 = \frac{1-b}{2}\mathbf{e}_1 + \frac{1+b}{2}\mathbf{e}_3$$

Theorem. Let $\mathbf{A} : \mathcal{A}_{V_\pi} \to \mathcal{A}_{V_\pi}$ be a non-zero linear map of $\mathcal{A}_{V_\pi}$ defined as

$$\mathbf{A}(\mathbf{e}_1) = \alpha\mathbf{e}_1 + \beta\mathbf{e}_2 - (1-\alpha-\beta)\mathbf{e}_3,$$

$$\mathbf{A}(\mathbf{e}_2) = \beta\mathbf{e}_1 + \alpha\mathbf{e}_2 - (1-\alpha-\beta)\mathbf{e}_3,$$

$$\mathbf{A}(\mathbf{e}_3) = \gamma\mathbf{e}_1 + \gamma\mathbf{e}_2 - (1-2\gamma)\mathbf{e}_3.$$

where $\alpha, \beta, \gamma \in \mathbb{R}$. Then $\mathbf{A}$ is an automorphism on $\mathcal{A}_{V_\pi}$ if and only if $a = b = c = 0$.

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A REGULAR NON-VOLTERRA QUADRATIC OPERATOR

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Let

$$S^{m-1}=\left\{\mathbf{x}=(x_1,x_2,\dots,x_m)\in R^m:\ x_i\geq 0,\ \sum_{i=1}^{m}x_i=1\right\}$$

be the $(m-1)$-dimensional simplex. A map V from S^{m-1} into itself is called a *quadratic stochastic operator* if

$$(V\mathbf{x})_k=\sum_{i,j=1}^{m}p_{ij,k}x_ix_j \tag{1}$$

for any $\mathbf{x}\in S^{m-1}$ and for all $k=1,\dots,m$, where

$$p_{ij,k}\geq 0,\quad p_{ij,k}=p_{ji,k}\ \text{ for all }\ i,j,k,\ \ \sum_{k=1}^{m}p_{ij,k}=1. \tag{2}$$

Assume $\left\{V^n\left(\mathbf{x}^{(0)}\right)\right\}_{n=0}^{\infty}$ is the trajectory (orbit) of the initial point $\mathbf{x}^{(0)}\in S^{m-1}$, where $\mathbf{x}^{(n+1)}=V(\mathbf{x}^{(n)})$ for all $n=0,1,2,\dots$ We denote by $\omega_V\left(\mathbf{x}^{(0)}\right)$ the set of ω- limiting points of the trajectory $\left\{V^n\left(\mathbf{x}^{(0)}\right)\right\}_{n=0}^{\infty}$. The main problem is investigation of the asymptotical behaviour of the trajectories for a given QSO. In other words the main task is the description of the set $\omega\left(\mathbf{x}^{(0)}\right)$ for any initial point $\mathbf{x}^{(0)}\in S^{m-1}$ for a given QSO. A point $\mathbf{x}\in S^{m-1}$ is called a *fixed point* of a QSO V if $V(\mathbf{x})=\mathbf{x}$.

Definiton 1. A QSO V is called *regular* if for any initial point $\mathbf{x}\in S^{m-1}$, the limit $\lim\limits_{n\to\infty}V^n(\mathbf{x})$ exists.

Definiton 2. A quadratic stochastic operator (),(2) is called Volterra if

$$p_{ij,k}=0,\quad \text{for any}\ \ k\notin\{i,j\},\quad i,j,k=1,\dots,m.$$

Thus, the fixed points of a QSO describe limit or long run behavior of the trajectories for any initial point. The limit behavior of trajectories and fixed points play an important role in many applied problems.

The following notations will be used throughout this paper. We let ∂S^{m-1} denote the boundary of S^{m-1}, $\partial S^{m-1}=\{\mathbf{x}\in S^{m-1}:\ x_i=0 \text{ for at least one } i\in\{1,2,\dots,m\}\}$; the interior of S^{m-1} be the set $\operatorname{int}S^{m-1}=\{\mathbf{x}\in S^{m-1}:\ x_1x_2\cdots x_m>0\}$; a face of the simplex S^{m-1} be the set $\Gamma_\alpha=\{\mathbf{x}\in S^{m-1}:\ x_i=0,\quad i\notin\alpha\subset\{1,\dots,m\}\}$ and interior of Γ_α be the set $\operatorname{int}\Gamma_\alpha=\{\mathbf{x}\in S^{m-1}:\ \prod\limits_{i\in\alpha}x_i>0,\quad \alpha\subset\{1,\dots,m\}\}$. Let $\mathbf{e}_i=(\delta_{1i},\delta_{2i},\dots,\delta_{mi})\in S^{m-1},\ i=1,\dots,m$, denote the vertices of the simplex S^{m-1}, where δ_{ij} is the Kronecker delta.

Definiton 3. A fixed point $\mathbf{x}^*$ is called hyperbolic if its Jacobian $DV(\mathbf{x}^*)$ has no eigenvalues on the unit circle.

Definiton 4. A hyperbolic fixed point $\mathbf{x}^*$ is called:

i) attracting if all the eigenvalues of the Jacobian $DV(\mathbf{x}^*)$ are less than 1 in absolute value;

ii) repelling if all the eigenvalues of the Jacobian $DV(\mathbf{x}^*)$ are greater than 1 in absolute value;

iii) a saddle otherwise.

Let us consider a non-Volterra QSO defined on the three-dimensional simplex which has the form

$$V:\begin{cases} x_1' = 2x_1x_4 + (1-a)x_2x_4, \\ x_2' = (1+a)x_2x_4, \\ x_3' = 2x_3x_4, \\ x_4' = x_4^2 + (x_1+x_2+x_3)^2, \end{cases} \tag{3}$$

where $a \in [-1,1]$.

We denote

$$A = \left\{\mathbf{x} \in S^3 : x_1+x_2+x_3 = \frac{1}{2},\ x_4 = \frac{1}{2}\right\}, B = \left\{\mathbf{x} \in S^3 :\ x_1+x_3 = \frac{1}{2},\ x_4 = \frac{1}{2}\right\},$$

$$\mathbf{m}_1 = \big(1/2,0,0,1/2\big), \mathbf{m}_2 = \big(0,1/2,0,1/2\big), \mathbf{m}_3 = \big(0,0,1/2,1/2\big).$$

$$\lambda_1 = \left(0, \alpha x_2^{(0)}, \alpha x_3^{(0)}, \frac{1}{2}\right),\ \lambda_2 = \left(\beta x_1^{(0)}, 0, \beta x_3^{(0)}, \frac{1}{2}\right) \text{ and } \lambda_3 = \left(\gamma x_1^{(0)}, \beta x_2^{(0)}, 0, \frac{1}{2}\right).$$

where

$$\alpha = \left(2(x_2^{(0)} + x_3^{(0)})\right)^{-1},\quad \beta = \left(2(x_1^{(0)} + x_3^{(0)})\right)^{-1},\quad \gamma = \left(2(x_1^{(0)} + x_2^{(0)})\right)^{-1}.$$

In the next Theorem we give the description of the set of fixed points, their type and the set of limit points of the operator V.

Theorem 1. For the operator V the following statements are hold:

i)

$$\mathrm{Fix}\big(V\big) = \begin{cases} \big\{\mathbf{e}_4\big\} \cup A, & \text{if } \ a = 1, \\ \big\{\mathbf{e}_4\big\} \cup B, & \text{if } \ -1 \le a \le 1; \end{cases}$$

ii) if $a = 1$ then the fixed points from the set A are non-hyperbolic points, the points set B is a non-hyperbolic when $-1 \le a < 1$ and the vertex

$$\mathbf{e}_4 \ \text{ has the type a (an) } \begin{cases} \text{saddle}, & \text{if } \ a < 0, \\ \text{non-hyperbolic}, & \text{if } \ a = 0, \\ \text{repelling}, & \text{if } \ a > 0. \end{cases}$$

Theorem 2. For the operator V the following statements are hold:

i) if $-1 \le a < 1$ then

$$\omega\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_4\}, & \text{if } \ \mathbf{x}^{(0)} \in \Gamma_{123} \cup \{\mathbf{e}_4\}, \\ \{\mathbf{m}_3\}, & \text{if } \ \mathbf{x}^{(0)} \in \Gamma_{34} \setminus \{\mathbf{e}_3, \mathbf{e}_4\}, \\ \{\mathbf{m}_1\}, & \text{if } \ \mathbf{x}^{(0)} \in \Gamma_{124} \setminus \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_4\}, \\ \{(x_1^*, 0, x_3^*, x_4^*)\}, & \text{if } \ \mathbf{x}^{(0)} \in \Gamma_{134} \setminus \{\mathbf{e}_1, \mathbf{e}_3, \mathbf{e}_4\}, \\ \{(x_1^*, 0, x_3^*, x_4^*)\}, & \text{if } \ \mathbf{x}^{(0)} \in (int\Gamma_{234} \cup intS^3) \setminus Fix\,(V)\,; \end{cases}$$

ii) if $a = 1$ then

$$\omega\left(\mathbf{x}^{(0)}\right) = \begin{cases} \{\mathbf{e}_4\}, & \text{if } \mathbf{x}^{(0)} \in \Gamma_{123} \cup \{\mathbf{e}_4\}, \\ \{\mathbf{m}_1\}, & \text{if } \mathbf{x}^{(0)} \in \Gamma_{14} \setminus \{\mathbf{e}_1, \mathbf{e}_4\}, \\ \{\mathbf{m}_2\}, & \text{if } \mathbf{x}^{(0)} \in \Gamma_{24} \setminus \{\mathbf{e}_2, \mathbf{e}_4\}, \\ \{\mathbf{m}_3\}, & \text{if } \mathbf{x}^{(0)} \in \Gamma_{34} \setminus \{\mathbf{e}_3, \mathbf{e}_4\}, \\ \{\lambda_1\}, & \text{if } \mathbf{x}^{(0)} \in int\Gamma_{234}, \\ \{\lambda_2\}, & \text{if } \mathbf{x}^{(0)} \in int\Gamma_{134}, \\ \{\lambda_3\}, & \text{if } \mathbf{x}^{(0)} \in int\Gamma_{124}, \\ \{\widehat{\mathbf{x}}\}, & \text{if } \mathbf{x}^{(0)} \in \operatorname{int} S^3 \setminus M, \end{cases}$$

where

$$\widehat{\mathbf{x}} = \left(\frac{x_1^{(0)}}{2\left(x_1^{(0)} + x_2^{(0)} + x_3^{(0)}\right)}, \frac{x_2^{(0)}}{2\left(x_1^{(0)} + x_2^{(0)} + x_3^{(0)}\right)}, \frac{x_3^{(0)}}{2\left(x_1^{(0)} + x_2^{(0)} + x_3^{(0)}\right)}, \frac{1}{2} \right).$$

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ON CONVEX COMBINATION OF NON-VOLTERRA QUADRATIC OPERATORS

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Let S^{m-1} be the $(m-1)$-dimensional simplex. A map V from S^{m-1} into itself is called a *quadratic stochastic operator* if

$$(V\mathbf{x})_k = \sum_{i,j=1}^{m} p_{ij,k} x_i x_j, \tag{1}$$

for any $\mathbf{x} \in S^{m-1}$ and for all $k = 1, \dots, m$, where

$$p_{ij,k} \geq 0, \quad p_{ij,k} = p_{ji,k} \ \text{ for all } \ i,j,k, \quad \sum_{k=1}^{m} p_{ij,k} = 1. \tag{2}$$

Assume $\left\{V^n\left(\mathbf{x}^{(0)}\right)\right\}_{n=0}^{\infty}$ is the trajectory (orbit) of the initial point $\mathbf{x}^{(0)} \in S^{m-1}$, where $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$ for all $n = 0, 1, 2, \dots$ We denote by $\omega_V\left(\mathbf{x}^{(0)}\right)$ the set of ω- limiting points of the trajectory $\left\{V^n\left(\mathbf{x}^{(0)}\right)\right\}_{n=0}^{\infty}$.

One of the main problems is investigation of the asymptotical behaviour of the trajectories for a given QSO. In other words the main task is the description of the set $\omega\left(\mathbf{x}^{(0)}\right)$ for any initial point $\mathbf{x}^{(0)} \in S^{m-1}$ for a given QSO.

A QSO V is called *regular* if for any initial point $\mathbf{x} \in S^{m-1}$, the limit $\lim\limits_{n\to\infty} V^n(\mathbf{x})$ exists.

A quadratic stochastic operator (1), (2) is called Volterra if

$$p_{ij,k} = 0, \quad \text{for any } \ k \notin \{i,j\}, \quad i,j,k = 1, \dots, m.$$

The following notations will be used throughout this paper. We let ∂S^{m-1} denote the boundary of S^{m-1}, $\partial S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_i = 0 \text{ for at least one } i \in \{1, 2, \dots, m\}\}$; the interior of S^{m-1} be the set $\operatorname{int} S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_1 x_2 \cdots x_m > 0\}$; a face of the simplex S^{m-1} be the set $\Gamma_\alpha = \{\mathbf{x} \in S^{m-1} : x_i = 0, \quad i \notin \alpha \subset \{1, \dots, m\}\}$. Let $\mathbf{e}_i = (\delta_{1i}, \delta_{2i}, \dots, \delta_{mi}) \in S^{m-1}$, $i = 1, \dots, m$, denote the vertices of the simplex S^{m-1}, where δ_{ij} is the Kronecker delta.

A point $\mathbf{x}^* \in S^{m-1}$ is called a *fixed point* of a QSO V if $V(\mathbf{x}^*) = \mathbf{x}^*$.

Hence, the fixed points of a QSO describe limit or long run behavior of the trajectories for any initial point. The limit behavior of trajectories and fixed points play an important role in many applied problems.

A fixed point $\mathbf{x}^*$ is called hyperbolic if its Jacobian $DV(\mathbf{x}^*)$ has no eigenvalues on the unit circle.

A hyperbolic fixed point $\mathbf{x}^*$ is called:

i) attracting if all the eigenvalues of the Jacobian $DV(\mathbf{x}^*)$ are less than 1 in absolute value;

ii) repelling if all the eigenvalues of the Jacobian $DV(\mathbf{x}^*)$ are greater than 1 in absolute value;

iii) a saddle otherwise.

Consider the following non-Volterra QSO defined on the two-dimensional simplex

$$V_1: \begin{cases} x_1' = x_1^2, \\ x_2' = x_2^2 + x_3^2 + 2x_1x_2, \\ x_3' = 2x_1x_3 + 2x_2x_3, \end{cases} \quad (3) \qquad V_2: \begin{cases} x_1' = x_1^2 + x_3^2 + 2x_1x_2, \\ x_2' = x_2^2, \\ x_3' = 2x_1x_3 + 2x_2x_3. \end{cases} \quad (4)$$

In the next theorem we describe dynamical properties of the operators (3) and (4).

Theorem 1. For the QSOs (3) and (4) the following assertions are hold:

i) $\mathrm{Fix}(V_1) = \{\mathbf{e}_1, \mathbf{e}_2, (0, 1/2, 1/2)\}$;

ii) $\mathrm{Fix}(V_2) = \{\mathbf{e}_1, \mathbf{e}_2, (1/2, 0, 1/2)\}$;

iii) $\lim\limits_{n\to\infty} V_1^n\left(\mathbf{x}^{(0)}\right) = \left(0, \frac{1}{2}, \frac{1}{2}\right)$ for any $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_1, \mathbf{e}_2\}$;

iv) $\lim\limits_{n\to\infty} V_2^n\left(\mathbf{x}^{(0)}\right) = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$ for any $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_1, \mathbf{e}_2\}$.

Let us consider the convex combination of given QSOs (3) and (4), that is

$$V_\lambda = \lambda V_1 + (1-\lambda)V_2, \quad \lambda \in [0,1].$$

It is easy to verify that V_λ has the form

$$V_\lambda: \begin{cases} x_1' = x_1^2 + (1-\lambda)x_3^2 + 2(1-\lambda)x_1x_2, \\ x_2' = x_2^2 + \lambda x_3^2 + 2\lambda x_1x_2, \\ x_3' = 2x_1x_3 + 2x_2x_3. \end{cases} \quad (5)$$

Since the QSOs (3) and (4) are regular transformations it is natural to expect that the QSO V_λ also is a regular transformation. Below we will check this hypothesis.

Setting

$$\lambda = \frac{1}{2} + \tau, \quad \tau \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

one can rewrite the QSO (5) in the following form

$$V_\tau: \begin{cases} x_1' = x_1^2 + \left(\frac{1}{2} - \tau\right)x_3^2 + (1-2\tau)x_1x_2, \\ x_2' = x_2^2 + \left(\frac{1}{2} + \tau\right)x_3^2 + (1+2\tau)x_1x_2, \\ x_3' = 2x_1x_3 + 2x_2x_3. \end{cases} \quad (6)$$

In the next Theorem we give description of the set of fixed points and set of limit points of the operator V_τ.

Theorem 2. Let $\tau = 0$. Then one has

i) $\mathrm{Fix}(V_0) = \{(1/4, 1/4, 1/2)\} \cup \Gamma_{\{1,2\}}$, where $\Gamma_{\{1,2\}} = \{\mathbf{x} \in S^2 : x_3 = 0\}$;

ii) the fixed point $(1/4, 1/4, 1/2)$ is an attracting point and arbitrary fixed point $\mathbf{x} \in \Gamma_{\{1,2\}}$ is a non-hyperbolic point;

iii) $\lim\limits_{n\to\infty} V_0^n\big(\mathbf{x}^{(0)}\big) = (1/4, 1/4, 1/2)$ for any $\mathbf{x}^{(0)} \in S^2 \setminus \Gamma_{\{1,2\}}$.

Theorem 3. Let $\tau \neq 0$. Then one has

i) $\mathrm{Fix}\,(V_\tau) = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{x}^*\}$, where $\mathbf{e}^* = \left(x_1^*, x_2^*, \dfrac{1}{2}\right)$

$$x_1^* = \frac{1 + 2\tau - \sqrt{1 + 12\tau^2}}{8\tau} \quad \text{and} \quad x_2^* = \frac{2\tau - 1 + \sqrt{1 + 12\tau^2}}{8\tau};$$

ii) The vertex

$$\mathbf{e}_2 \ \text{ is } \ \begin{cases} \text{saddle}, & \text{if} \quad \tau > 0, \\ \text{repelling}, & \text{if} \quad \tau < 0; \end{cases}$$

The vertex

$$\mathbf{e}_1 \ \text{ is } \ \begin{cases} \text{saddle}, & \text{if} \quad \tau \in \left[-\dfrac{1}{2}, 0\right) \cup \left(0, \dfrac{\sqrt{3}-1}{4}\right), \\ \text{non-hyperbolic}, & \text{if} \quad \tau = \dfrac{\sqrt{3}-1}{4}, \\ \text{repelling}, & \text{if} \quad \tau \in \left(\dfrac{\sqrt{3}-1}{4}, \dfrac{1}{2}\right]; \end{cases}$$

iii) $\lim\limits_{n\to\infty} V_\tau^n\big(\mathbf{x}^{(0)}\big) = \mathbf{x}^*$ for any $\mathbf{x}^{(0)} \in S^2 \setminus \{\mathbf{e}_1, \mathbf{e}_2\}$.

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A REGULAR NON-VOLTERRA QUADRATIC STOCHASTIC OPERATOR

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Let

$$S^{m-1}=\{\mathbf{x}=(x_1,x_2,\dots,x_m)\in R^m:\ x_i\geq 0,\sum_{i=1}^{m}x_i=1\}$$

the $(m-1)$-dimensional simplex. A map V of S^{m-1} into itself is called a *quadratic stochastic operator* (QSO) if

$$(Vx)_k=\sum_{i,j=1}^{m}P_{ij,k}x_ix_j$$

for any $x\in S^{m-1}$ and for all $k=1,...,m$ where

$$P_{ij,k}\geq 0,\quad P_{ij,k}=P_{ji,k},\quad \sum_{k=1}^{m}P_{ij,k}=1.$$

The trajectory $\{x^{(n)}\}_{n\geq 0}$ of an operator V for a point $x\in S^{m-1}$ is defined by $x^{(n+1)}=V(x^{(n)})$ for all $n=0,1,2,...,$. Denote by $\omega_V(x^{(0)})$ the set of limit points of the trajectory $\{x^{(n)}\}_{n\geq 0}$.

Definition 1. A point $x\in S^{m-1}$ is called a *periodic point* of V if there exists an n so that $V^n(x)=x$. The smallest positive integer n satisfying the above is called the prime period or least period of the point x. A period-one point is called a *fixed point* of V, denote the set of all fixed points by $Fix(V)$. Let $DW(\mathbf{x}^*)=\left(\partial W_i/\partial x_j\right)(\mathbf{x}^*)$ be the Jacobian of V at the point $\mathbf{x}^*$.

Definition 2. A fixed point x^* is called hyperbolic if its Jacobian $D(V(x^*))$ has no eigenvalues on the unit circle. A hyperbolic fixed point x^* is called:

- *attracting*, if all the eigenvalues of the Jacobian $D(V(x^*))$ are less than 1 in absolute value;
- *repelling*, if all the eigenvalues of the Jacobian $D(V(x^*))$ are greater than 1 in absolute value;
- a *saddle*, otherwise.

Let us consider a non-Volterra QSO defined on the two-dimensional simplex which has the form

$$V:\begin{cases}x_1'=bx_1^2+\beta x_2^2+x_1x_2+x_1x_3,\\ x_2'=\alpha x_2^2+cx_3^2+x_1x_2+x_2x_3,\\ x_3'=ax_1^2+dx_3^2+x_1x_3+x_2x_3,\end{cases}\tag{1}$$

where $0\leq a,b,\alpha,\beta,c,d\leq 1$ and $a+b=\alpha+\beta=c+d=1$.

Note that the operator (1) is a non-Volterra QSO.

For $\sqrt{\beta c}+\sqrt{ac}+\sqrt{a\beta}>0$ we denote $\mathbf{x}^*:=\mathbf{x}^*(a,\beta,c)=(x_1^*,x_2^*,x_3^*)$, where

$$x_1^*=\frac{\sqrt{c\beta}}{\sqrt{\beta c}+\sqrt{ac}+\sqrt{a\beta}},\quad x_2^*=\frac{\sqrt{ac}}{\sqrt{\beta c}+\sqrt{ac}+\sqrt{a\beta}},\quad x_3^*=\frac{\sqrt{a\beta}}{\sqrt{\beta c}+\sqrt{ac}+\sqrt{a\beta}}.$$

Theorem 1. For the QSO (1) the following statements are hold:

i)
$$\mathrm{Fix}\,(V)=\begin{cases}\Gamma_{23}, & a>0,\ \beta=c=0,\\ \Gamma_{13}, & \beta>0,\ a=c=0,\\ \Gamma_{12}, & c>0,\ a=\beta=0,\\ \{\mathbf{e}_1\}, & a=0,\ c,\beta>0,\\ \{\mathbf{e}_2\}, & \beta=0,\ a,c>0,\\ \{\mathbf{e}_3\}, & c=0,\ a,\beta>0,\\ \{\mathbf{x}^*\}, & a\beta+ac+\beta c>0.\end{cases}$$

ii) If $a\beta+ac+\beta c\le 0$ then any vertex $\mathbf{e}_i$, $i=1,2,3$ is a non-hyperbolic point. Also the fixed points from the faces $\Gamma_{12},\Gamma_{13},\Gamma_{23}$ are a non-hyperbolic points when they exist;

iii) Let $a\beta+ac+\beta c>0$. If one of the parameters a,β,c is zero then the fixed point $\mathbf{x}^*$ is a non-hyperbolic point and it is an attracting points when $a>0,\beta>0,c>0$.

iv) If $a=\beta=c=1$, then $V^3(e_3)=V^2(e_2)=V(e_1)=e_3$.

Theorem 2. For the QSO (1) we have

$$\omega_V\left(\mathbf{x}^{(0)}\right)=\begin{cases}\left\{\left(x_1^{(0)},1-x_1^{(0)},0\right)\right\}, & \text{if } a=\beta=0,\ c>0, \quad \mathbf{x}^{(0)}\in S^2\setminus\left(\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}\cup\Gamma_{12}\right),\\ \left\{\left(1-x_3^{(0)},0,x_3^{(0)}\right)\right\}, & \text{if } a=c=0,\ \beta>0, \quad \mathbf{x}^{(0)}\in S^2\setminus\left(\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}\cup\Gamma_{13}\right),\\ \left\{\left(0,x_2^{(0)},1-x_2^{(0)}\right)\right\}, & \text{if } \beta=c=0,\ a>0, \quad \mathbf{x}^{(0)}\in S^2\setminus\left(\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}\cup\Gamma_{23}\right),\\ \{\mathbf{e}_1\}, & \text{if } a=0,\beta>0,c>0, \quad \mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3,\mathbf{x}^*\},\\ \{\mathbf{e}_2\}, & \text{if } \beta=0,a>0,c>0, \quad \mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3,\mathbf{x}^*\},\\ \{\mathbf{e}_3\}, & \text{if } c=0,a>0,\beta>0, \quad \mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3,\mathbf{x}^*\},\\ \{\mathbf{x}^*\}, & \text{if } a>0,\beta>0,c>0, \quad \mathbf{x}^{(0)}\in S^2\setminus\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}.\end{cases}$$

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FIXED POINTS FOR A TYPE OF VOLTERRA QUADRATIC OPERATORS

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In this paper, we consider the simplex I_n of idempotent measures on $\{1, 2, \ldots, n\}$, where

$$I_n = \{(x_1, x_2, ..., x_n) \in R^n_{\max} : \max_{1\le i\le n} x_i = 0\} =$$

$$= \{(x_1, x_2, ..., x_n) \in R^n_{\max} : x_1 \oplus x_2 \oplus ... \oplus x_n = 1\}$$

We are going to solve problem of defining fixed points Volterra type quadratic operators which map I_n to itself.

Assume $P = \{p_{ij,k}\}$ – set of numbers which holds following conditions:

$$p_{ij,k} \le 0, \quad p_{ij,k} = p_{ji,k}, \quad i, j, k = 1,\ 2,\ \ldots,\ \text{n}. \tag{1}$$

Definition 1. The map $V : x \to x'$ called quadratic if it satisfies following equality:

$$x'_k = \sum_{i,j=1}^{n} p_{ij,k} x_i x_j, k = 1,\ 2,\ \ldots,\ \text{n}. \tag{2}$$

Definition 2. Quadratic operator corresponding to matrix $P = \{p_{ij,k}\}$ which holds condition (1) called quadratic Volterra operator on I_n if it satisfies condition:

$$p_{ij,k} = 0, k \notin \{i, j\}. \tag{3}$$

Theorem 1. A quadratic Volterra operator, given by matrix $P = \{p_{ij,k}\}$ maps I_n to itself.

Definition 3. Quadratic Volterra operator on I_n with condition $\sum_{k=1}^{n} p_{ij,k} = -1$ called Quadratic absolute stochastic (QAS) Volterra operator.

Definition 4. Recall that fixed points of $V : I_n \to I_n$ are solutions to $V(x) = x$. Denote by $Fix\,(V)$ the set of all fixed point of V.

Let's find fixed points of (QAS) Volterra operator $V : I_2 \to I_2$.

Proposition 1. Assume $n = 2$, the set of all fixed points for the (QAS) Volterra operator $Fix(V)$ is

$$Fix(V) = \begin{cases} (-1, 0), \\ (0, -1), \\ (0, 0). \end{cases}$$

Now, let's find fixed points for quadratic absolute stochastic Volterra operator $V : I_3 \to I_3$. By using (1) and (3) we get following system of equation:

$$\begin{cases} x_1 = -x_1^2 + 2ax_1x_2 + 2bx_1x_3, \\ x_2 = -x_2^2 - 2\,(1 + a)\,x_1x_2 + 2cx_2x_3, \\ x_3 = -x_3^2 - 2\,(1 + b)\,x_1x_3 - 2\,(1 + c)\,x_2x_3. \end{cases}$$

By assigning variables and performing elementary algebraic operations, we obtain the following proposition.

Proposition 2. Consider $n = 3$, the set of all fixed points for the (QAS) Volterra operator $Fix(V)$ is:

$$Fix(V) = \begin{cases} (-1,0,0)\,, \\ (0,-1,0)\,, \\ (0,0,-1)\,, \\ \textit{all points of lines } x_i + x_j = -1,\ \textit{if } p_{ij,i} = p_{ij,j} = -\frac{1}{2},\ i \neq j; \\ (0,0,0)\,,\ \textit{for all other cases.} \end{cases}$$

In conclusion, studying prepositions which given above and using method mathematical induction we get theorem for the set of all fixed points of every $(n-1)-$dimensional simplex of idempotent measures as:

Theorem 2. The set of all fixed points Fix(P) for the (QAS) Volterra operator $V : I_n \to I_n$ defined on a $(n-1)-$dimensional simplex of idempotent measures is:

$$Fix(V) = \begin{cases} x = (x_1, x_2, ..., x_n) \text{ where } \sum_{i=1}^{n} x_i = -1,\ \textit{if } p_{ij,i} = p_{ij,j} = -\frac{1}{2},\ i \neq j; \\ (0,0,...,0) \text{ for all other cases.} \end{cases}$$

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MATHEMATICAL MODELING OF PREVALENCE OF HIV USING CONTINUOUS DYNAMICAL SYSTEMS

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The research study on the prevalence of HIV in Kazakhstan from 2010 to 2020 and its forecasting for the next 10 years provides crucial insights into the evolving landscape of HIV/AIDS in the region. The study aims to analyze the current situation of HIV in Kazakhstan, explore the trends over the past decade, and predict future prevalence rates using advanced modeling techniques. This is a joint work [1].

The analysis of HIV prevalence data from 2010 to 2020 revealed a concerning increase in the number of cases, highlighting the persistent public health burden posed by HIV in Kazakhstan. By employing sophisticated forecasting models such as ARIMA and SI, the study predicts a significant rise in HIV prevalence over the next decade, emphasizing the urgent need for targeted interventions and prevention strategies.

The Susceptible-Infected (SI) model can be represented by the following system of ordinary differential equations:

$$\frac{dS}{dt} = -\beta \cdot \frac{S \cdot I}{N}$$
$$\frac{dI}{dt} = \beta \cdot \frac{S \cdot I}{N}$$

where:

S : Number of susceptible individuals
I : Number of infected individuals
N : Total population size
β : Transmission rate

This system describes the dynamics of disease transmission in a population where individuals move from being susceptible to being infected at a rate determined by the transmission parameter β.

The Susceptible-Infected (SI) model with Pre-Exposure Prophylaxis (PrEP) can be represented by the following system of ordinary differential equations:

$$\frac{dS}{dt} = -\beta \cdot \frac{S \cdot I}{N} - k \cdot S_p$$
$$\frac{dI}{dt} = \beta \cdot \frac{S \cdot I}{N} + k \cdot S_p$$

where S, I, N, β as before and

S_p : Number of susceptible individuals using PrEP
k : Proportion of PrEP users becoming sexually active

In this model, the introduction of Pre-Exposure Prophylaxis (PrEP) is considered, where a proportion of susceptible individuals use PrEP to reduce their susceptibility to infection. The parameter k represents the rate at which susceptible individuals using PrEP become sexually active and are exposed to the infection.

The findings of this research have significant implications for HIV/AIDS research and palliative care in Kazakhstan and beyond. The projected increase in HIV prevalence underscores the importance of implementing effective control measures, such as PrEP, to mitigate the spread of the infection (see Figure 1). By leveraging the insights provided by the forecasting models, healthcare professionals and policymakers can develop proactive strategies to address the evolving HIV epidemic in Kazakhstan.

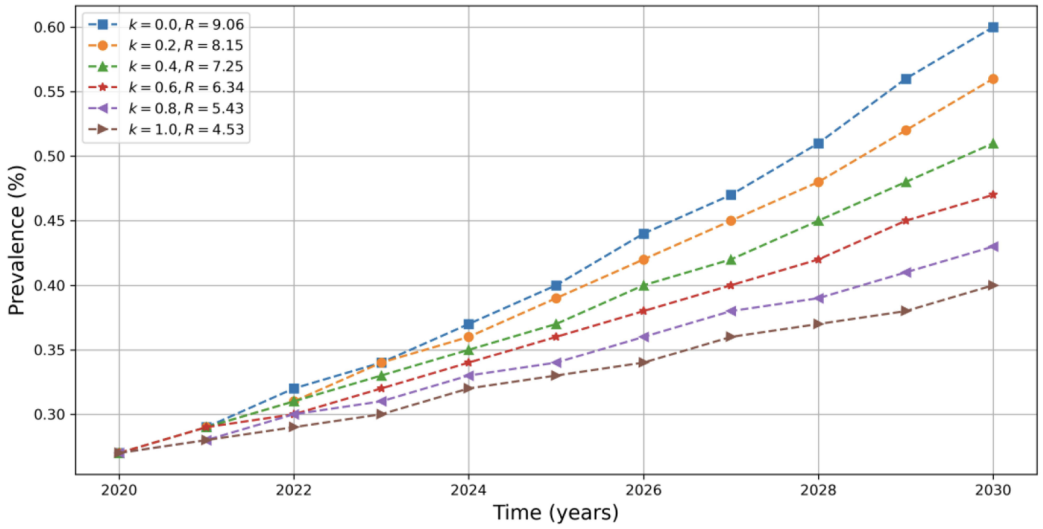


Рис. 1: HIV prevalence forecasting with PrEP for various scenarios.

In conclusion, the study sheds light on the evolving landscape of HIV in Kazakhstan and offers valuable predictions for the future trajectory of the epidemic. By combining historical data analysis with advanced forecasting techniques, the research provides a comprehensive understanding of the challenges posed by HIV/AIDS in the region and underscores the importance of proactive intervention strategies to combat the spread of the infection.

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PERIODIC GIBBS MEASURES FOR THE KITTEL'S ZIPPER MODEL ON THE CAYLEY TREE

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The zipper model is considered in [1] by Kittel for 1-D case (see also [2] for more details). In [3], Rozikov considers this model on the general order Cayley tree, which the explicit critical temperature for occurrence a phase transition (non-uniqueness of Gibbs measures) is obtained. In the present work, we study two-periodic Gibbs measures of the model on the semi-infinite Cayley tree of order k and we show that they coincide with translation-invariant Gibbs measures.

Let $\Gamma_0^k = (V_0, L_0)$ be a semi-infinite Cayley tree of order $k \geq 1$ with the root x^0, where V_0 is the set of vertices and L_0 is the set of edges. The vertices x and y are called nearest neighbours and they are denoted by $l = \langle x, y\rangle$ if there exists an edge connecting them. A collection of the pairs $\langle x, x_1\rangle$, $\langle x_1, x_2\rangle$, $\ldots$, $\langle x_{d-1}, y\rangle$ is called a *path* from the point x to the point y. The distance $d(x, y)$, $x, y \in V_0$, on the semi-Cayley tree, is the length of the shortest path from x to y. Let $x_\downarrow$ be unique vertex in $S_1(x)$ (where $x \in V_0\backslash\{x^0\}$) which is closer to x^0 than other elements of $S_1(x)$. For the fixed $x^0 \in V_0$ and $n \geq 1$ we set

$$| x | = d(x, x^0), W_n = \{x \in V_0 \,||\, x | = n\},$$

$$V_n = \{x \in V_0 \,||\, x | \leq n\}, L_n = \{l = \langle x, y\rangle \in L_0 \mid x, y \in V_n\}.$$

Consider spin values from $\Phi = \{0, 1, \cdots, q\}$, where $q \geq 1$. A configuration is any mapping $\sigma : x \in V \to \sigma(x) \in \Phi$. The vertex x with $\sigma(x) = 0$ means that the vertex is closed. Each $\sigma(x) = 1, 2, \cdots, q$ means that the vertex x is open in one of possible q states. For any $x \in V_0$ denote by π_x the unique path connection x^0 and x. A configuration $\sigma \in \Omega$ is said to be zipper-admissible if $\sigma(x^0) = 0$ and from $\sigma(x) = 0$ it follows that $\sigma(y) = 0$ for all $y \in \pi_x$. In this work, we only consider zipper-admissible configurations. Denote by $\Omega = \Phi^{V_0}$ the set of all such configurations.

We consider the following model of the energy of the zipper-admissible configuration $\sigma \in \Omega$:

$$H(\sigma) = \epsilon \sum_{x \in W_1} (1 - \delta_{0,\sigma(x)}) + \sum_{x \in V_0 \backslash V_1} (\epsilon + J\delta_{0,\sigma(x_\downarrow)})(1 - \delta_{0,\sigma(x)}),$$

where $\epsilon \in \mathbb{R}, J \in \mathbb{R}$ are parameters, δ is the Kronecker delta.

Let Ω_n be the set of all zipper-admissible configurations on V_n. Define a finite-dimensional distribution of a probability measure μ on Ω_n as

$$\mu_n(\sigma_n) = Z_n^{-1} \exp\left\{-\beta H_n(\sigma_n) + \sum_{y \in W_n} h_{\sigma(y),y}\right\}, \tag{1}$$

where $\{h_{a,x} \in \mathbb{R} : a \in \Phi, x \in V_0\}$ is a collection of real numbers.

We say that the probability distributions (1) are compatible if for all $n \geq 1$ and $s_{n-1} \in \Omega_{n-1}$:

$$\sum_{\omega_n \in \Omega_{W_n}} \mu_n(s_{n-1} \vee \omega_n) = \mu_{n-1}(s_{n-1}).$$

The compatibility condition is satisfied if and only if $\forall x \in V_0 \backslash \{x^0\}$ the following equation holds [3,4]

$$z_x = \prod_{y \in S(x)} (\theta z_y + \eta) \tag{2}$$

where $\theta = \frac{1}{q}\exp(\beta\epsilon), \eta = \exp(-\beta J)$, and $z_x = \exp(h_{0,x} - h_{i,x})$ (independent on $i = 1, \cdots, q$).

We define a coordinate structure in Γ_0^k: every vertex x (except for x^0) of Γ_0^k has coordinates $(i_1, \ldots, i_n)$, here $i_m \in \{1, \ldots, k\}$, $1 \leq m \leq n$ and for the vertex x^0 we put (0). Let us define on Γ_0^k binary operation $\circ : \Gamma_0^k \times \Gamma_0^k \to \Gamma_0^k$ as follows: for any two elements $x = (i_1, \ldots, i_n)$ and $y = (j_1, \ldots, j_m)$ put

$$x \circ y = (i_1, \ldots, i_n) \circ (j_1, \ldots, j_m) = (i_1, \ldots, i_n, j_1, \ldots, j_m),$$

$$x \circ x^0 = x^0 \circ x = (i_1, \ldots, i_n) \circ (0) = (i_1, \ldots, i_n).$$

By means of the defined operation Γ_0^k becomes a noncommutative semigroup with a unit. Let us denote this group $(G^k, \circ)$. Using this semigroup structure one defines translations $\tau_g : G^k \to G^k$, $g \in G^k$ by $\tau_g(x) = g \circ x$. Let $G \subset G^k$ be a sub-semigroup of G^k and $h : G^k \to Y$ be a Y-valued function defined on G^k. We say that h is *G-periodic* if $h(\tau_g(x)) = h(x)$ for all $g \in G$ and $x \in G^k$. Any G^k-periodic function is called *translation-invariant*. We put $G_2 = \{x \in G^k : d(x, x^0) \equiv 0 (\mathrm{mod}\, 2)\}$. One can check that G_2 is a sub-semigroup of G^k. Finding the G_2-periodic solutions of the equation (2) is reduced to the following system of equations

$$\begin{cases} z = (\theta\widetilde{z} + \eta)^k \\ \widetilde{z} = (\theta z + \eta)^k. \end{cases} \tag{3}$$

We remark that the solution (z, z) (i.e., $z = \widetilde{z}$) in (3) corresponds to the translation-invariant Gibbs measure which is studied in [3]. After introducing the notations $x = \sqrt[k]{z}$, $y = \sqrt[k]{\widetilde{z}}$, subtracting the second equation from the first in the system, and after some algebra, we obtain $x = y$. Summarising, we have the following result

Theorem. Let $J < \infty$. For the Zipper model on the semi-infinite Cayley tree of order k, all G_2-periodic GMs coincide with TIGMs.

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TRANSPOSED POISSON ALGEBRAS

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Recently, a dual notion of the Poisson algebra (transposed Poisson algebra), by exchanging the roles of the two binary operations in the Leibniz rule defining the Poisson algebra, has been introduced in the paper of Bai, Bai, Guo, and Wu. They have shown that the transposed Poisson algebra defined in this way not only shares common properties with the Poisson algebra, including the closure undertaking tensor products and the Koszul self-duality as an operad but also admits a rich class of identities. More significantly, a transposed Poisson algebra naturally arises from a Novikov-Poisson algebra by taking the commutator Lie algebra of the Novikov algebra. The talk is about new results related to transposed Poisson algebras and their relations with other types of algebraic systems.

k_m-TRANSLATION-INVARIANT SPLITTING GIBBS MEASURES IN THE CASE $G = wand$

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Let $\Im^k = (V, L)$ is Cayley tree of order $k \geq 2$. Let $\Im^k = (V, L)$, where V is the set of vertices $\Im^k$ and L is the set of its edges.

For fixed $x^0 \in V$ we set $W_n = \{x \in V \,|\, d(x, x^0) = n\}$, $V_n = \bigcup_{m=0}^{n} W_m$, where $d(x, y)$ is the distance between vertices x and y on the Cayley tree.

Denote the direct successors of the vertex x by $S(x)$, i.e. if $x \in W_n$, then

$$S(x) = \{y_i \in W_{n+1} | d(x, y_i) = 1, i = 1, 2, \ldots, k\}.$$

Let $\Phi = \{0, 1, 2\}$ and $\sigma \in \Omega = \Phi^V$ be the configuration, i.e., $\sigma = \{\sigma(x) \in \Phi : x \in V\}$. In other words, in this model, each vertex x is assigned to one of the values $\sigma(x) \in \Phi = \{0, 1, 2\}$. The values of $\sigma(x) \neq 0$ mean that the vertex x is 'occupied', and the value $\sigma(x) = 0$ means that the vertex x is 'vacant'. The set of all configurations on V (V_n) is denoted by Ω (Ω_{V_n}).

Consider the set Φ as the set of vertices of some graph G. The σ configuration is called *G-admissible configuration* on the Cayley tree (in V_n) if $\{\sigma(x), \sigma(y)\}$ is edge of the graph G for any the nearest pair of neighbors x, y from V (from V_n). Denote the set of G-admissible configurations via $\Omega^{a,G}$ ($\Omega^{a,G}_{V_n}$).

The activity set for the graph G is the function $\lambda : G \to R^3_+$. Value λ_i of function λ at vertex $i \in \{0, 1, 2\}$ is called its "activity" (see [1]).

For given G and λ, we define the Hamiltonian of the $G-HC$-model as

$$H^{\lambda}_G(\sigma) = \begin{cases} -\sum\limits_{x \in V} \log \lambda_{\sigma(x)}, & \text{if } \sigma \in \Omega^{a,G}, \\ +\infty, & \text{if } \sigma \notin \Omega^{a,G}. \end{cases}$$

In the case graph $G = wand$ there exists a one-to-one correspondence between Gibbs measures for the three state HC model and the following solution of system of equations

$$\begin{cases} z_{1,x} = \lambda \prod_{y \in S(x)} \frac{1+z_{1,y}}{z_{1,y}+z_{2,y}}, \\ z_{2,x} = \lambda \prod_{y \in S(x)} \frac{1+z_{2,y}}{z_{1,y}+z_{2,y}}, \end{cases} \tag{1}$$

where $S(x)$ the set of direct successors of x (see [2]).

The reader can find the definition of the Gibbs measure and of other subjects related to Gibbs measure theory, for example, in [2]. In [3]-[5] translation-invariant and periodic splitting Gibbs measures studied.

In this paper we study new solution to the system of equation (1). i.e. take m to be non-negative integers such that $0 \leq m \leq k$.

It is known from [5] that for $0 < \lambda < 1$ there exist $G^{(2)}_k$-periodic splitting Gibbs measures μ_1, μ_2 on the second-order Cayley tree, which correspond to the solutions $(\tilde{z}, \tilde{t})$

and $(\tilde{t}, \tilde{z})$ of the system of equations (3.7) of [5], where

$$\tilde{z} = \frac{(1+\sqrt{1-\lambda})^2}{\lambda},\ \tilde{t} = \frac{\lambda}{(1+\sqrt{1-\lambda})^2}.$$

Let be $(\tilde{z}, \tilde{t}) = h, (\tilde{t}, \tilde{z}) = l$. The boundary condition $z = \{z_x, x \in G_k\}$, with fields taking values h, l is defined by the following steps:

• If at vertex x we have $z_x = z$, then the function z_y, which gives real values to each vertex $y \in S(x)$ and h on m vertices of $S(x)$, l on m vertices of $S(x)$, z on $k-2m$ remaining vertices,

• If at vertex x we have $z_x = h$, then the function z_y, which gives real values to each vertex $y \in S(x)$ and h on m vertices of $S(x)$, l on m vertices of $S(x)$, $\tilde{h}$ on $k-2m$ remaining vertices,

• If at vertex x we have $z_x = l$, then the function z_y, which gives real values to each vertex $y \in S(x)$ and h on m vertices of $S(x)$, l on m vertices of $S(x)$, $\tilde{l}$ on $k-2m$ remaining vertices. Here $\tilde{h} = (\tilde{h}_1, \tilde{h}_2)$, $\tilde{l} = (\tilde{l}_1, \tilde{l}_2)$ and $2m < k$.

It is easy to prove that $\tilde{h}_1 = \tilde{l}_2 > 0$. Let $A(k,m) = \{\lambda : 0 < \lambda < 1, \tilde{h}_2 = \tilde{l}_1 > 0\}$. We consider the set $B(k,m) = A(k,m) \cap (0,1)$.

The measure corresponding to z_x, determined by the above method, is called the k_m-periodic Gibbs measure.

Lemma. The following statements holds:

1. $B(3,1) = \{\lambda : \lambda_1^* < \lambda < 1\}$, where $\lambda_1^* \approx 0.95867$.
2. $B(4,1) = \{\lambda : \lambda_2^* < \lambda < 1\}$, where

$$\lambda_2^* = -\sqrt[3]{\frac{8}{27} + \frac{8}{45}\sqrt{\frac{21}{5}}} - \sqrt[3]{\frac{8}{27} - \frac{8}{45}\sqrt{\frac{21}{5}}} + \frac{4}{3}.$$

3. $B(5,1) = \{\lambda : \lambda_3^* < \lambda < 1\}$, where $\lambda_3^* \approx 0.76284$.

The following theorem is true.

Theorem. Let $k \geq 2$. Then the following statements are true for the HC model in the case $G = wand$:

1. If $\lambda \in B(3,1)$ or $\lambda \in B(4,1)$, then there exists one k_m-periodic Gibbs measure μ_0^*.

2. Let $\lambda \in B(5,1)$ and $\lambda_{cr} = \frac{2(\sqrt[6]{54} - \sqrt[3]{4})}{3 - \sqrt[3]{4}}$. Then, for $\lambda_3^* < \lambda < \lambda_{cr}$ there exists one k_m-periodic Gibbs measure μ_1^* and for $\lambda_{cr} < \lambda < 1$ there are three k_m-periodic Gibbs measures μ_1^*, μ_2^*, μ_3^*.

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ON THE TRANSLATION-INVARIANT P-ADIC GIBBS MEASURE FOR THE HC MODEL WITH FOUR STATES ON THE CAYLEY TREE: IN THE "STICK" CASE

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We study a four-state p-adic Hard-Core model on a Cayley tree. Conditions are found that must be imposed on the order of the Cayley tree k and the prime number p for the existence of a translation-invariant p-adic Gibbs measure.

p-Adic numbers. Let $\mathbb{Q}$ be the field of rational numbers. For a fixed prime number p, every rational number $x \neq 0$ can be represented in the form $x = p^r \frac{n}{m}$, where $r, n \in \mathbb{Z}$, m is a positive integer, and n and m are relatively prime with p: $(p, n) = 1$, $(p, m) = 1$. The p-adic norm of x is given by $|x|_p = p^{-r}$, for $x \neq 0$, $|x|_p = 0$, for $x = 0$.

Since this norm satisfies the so-called strong triangle inequality $|x + y|_p \leq \max\{|x|_p, |y|_p\}$, it is a non-Archimedean norm. The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$. Detailed information about the analysis in the field $\mathbb{Q}_p$ can be obtained from the literature [1-4]. Denote $\mathcal{E}_p = \left\{x \in \mathbb{Q}_p : |x-1|_p < p^{-1/(p-1)}\right\}$.

The Cayley tree. The Cayley tree $\Gamma^k = (V, L)$ of order $k \geq 1$ is an infinity tree (a grafh without cycles) to each vertex of which exactly $k+1$ edges are insident. Here V is the set of vertices and L the set of edges. We call two vertices x and y nearest neighbords if there exists an edge $l \in L$ connecting these vertices (denoted by $l = \langle x, y\rangle$). The distance $d(x, y)$ is equal to the number of edges constituting the shortest path connecting x and y. Let $x^0 \in V$ be a fixed point. Denote $W_n = \{x \in V : d(x, x^0) = n\}$, $V_n = \bigcup_{k=1}^{n} W_k$, $S(x) = \{y \in W_{n+1} : d(x, y) = 1\}$ for $x \in W_n$.

The HC model. We consider the HC model of four states on the Cayley tree. In this model, we set one of the values $\sigma(x) \in \{0, 1, 2, 3\}$ into correspondence with each vertex $x \in V$. If $\sigma(x) \in \{1, 2, 3\}$, then this means that the vertex $x \in V$ is occupied, and if $\sigma(x) = 0$, then the vertex $x \in V$ is vacant. We consider the fertile graph "stick" : $G = \{\{0, 1\}, \ \{1, 2\}, \ \{2, 3\}\}$, one of the four-state fertile graphs presented in [5].

We say that a configuration σ is an G-admissible on the Cayley tree if $\{\sigma(x), \sigma(y)\}$ is an edge of the graph G for any pair of nearest neighbors x and y in V. We let Ω^G denote the set of all G-admissible configurations on Cayley tree.

For $\sigma_n \in \Omega^G_{V_n}$, we let $\#\sigma_n$ denote the number of occupied vertices in a configuration σ_n. Let $\mathbf{z} : \ x \longrightarrow z_x = (z_{0,x}, z_{1,x}, z_{2,x}, z_{3,x}) \in \mathcal{E}_p^3$ be a vector-valued function on V. Let $\lambda \in \mathcal{E}_p$. We consider the p-adic probability distribution $\mu^{(n)}$ on the set $\Omega^G_{V_n}$, which is defined as

$$\mu^{(n)}(\sigma_n) = Z_n^{-1} \lambda^{\sharp\sigma_n} \prod_{x \in W_n} z_{\sigma_n(x),x}, \quad n = 1, 2, ..., \tag{1}$$

where Z_n is the normalizing constant. A p-adic probability distribution $\mu^{(n)}$ is said to be consistent if for all $n \neq 1$ and $\sigma_{n-1} \in \Omega^G_{V_{n-1}}$, we have

$$\sum_{\omega_n \in \Omega_{W_n}} \mu^{(n)}(\sigma_{n-1} \vee \omega_n)\mathbf{1}(\sigma_{n-1} \vee \omega_n \in \Omega^G_{V_n}) = \mu^{(n-1)}(\sigma_{n-1}). \quad (2)$$

In this case, by the Kolmogorov theorem [16], we have a unique measure μ on the set Ω^G such that $\mu(\{\sigma|_{V_n} = \sigma_n\}) = \mu^{(n)}(\sigma_n)$ for all n and $\sigma_n \in \Omega^G_{V_n}$. The measure $\mu^{(n)}$ defined in (1) and satisfying condition (2) is called a p-adic $G-HC$ Gibbs measure corresponding to the function $z : x \to z_x, x \in V \setminus \{x^0\}$.

Let $L(G)$ be the set of edges of a graph G. We let $A \equiv A^G = (a_{ij})_{i,j=0.1.2.3}$ denote the incidence matrix of the graph G, i.e.,

$$a_{ij} \equiv a^G_{ij} = \begin{cases} 1, \{i,j\} \in L(G), \\ 0, \{i,j\} \notin L(G). \end{cases}$$

The following theorem provides the condition on z_x ensuring the consistency of a distribution $\mu^{(n)}$.

Theorem 1. A probability distribution $\mu^{(n)}, n = 1, 2, ...,$ determined by formula (1) is consistent if and only if for any $x \in V$, we have the equation

$$z'_{i,x} = \lambda \prod_{y \in S(x)} \frac{a_{i0}z'_{0,y} + a_{i1}z'_{1,y} + a_{i2}z'_{2,y} + a_{i3}}{a_{30}z'_{0,y} + a_{31}z'_{1,y} + a_{32}z'_{2,y} + a_{33}}, \quad i = 0, 1, 2, \quad (3)$$

where $z'_{i,x} = \lambda z'_{i,x}/z_{3,x} \in \mathcal{E}_p$.

Theorem 2. If $p \nmid 2^k - 1$, then system (3) has no solution in $\mathcal{E}^3_p$.

Remark 1. The condition that $2^k - 1$ is divisible by p is necessary for the existence of p-adic Gibbs measures for the $G - HC$-model. From this condition for $k \geq 1$, we can find the values of the prime p for which system of equations (3) can have solutions (see following table for small values of k).

k	1	2	3	4	5	6	7	8	...
p	$-$	3	7	3; 5	31	3; 7	127	5; 51	...

Let's look for a translation-invariant solution of (3), i.e., $z_{i,x} = z_i$ for $\forall x \in V$. In that case we have

Theorem 3. If $p \mid 2^k - 1$ and $p \nmid k^2 - 4$, then system (3) has unique translation-invariant solution in $\mathcal{E}^3_p$.

Corollary. When the conditions of Theorem 3 are fulfilled, the p-adic $G - HC$ translation-invariant Gibbs measure is unique for the fertile graph G under consideration.

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NON UNIQUENESS CONDITIONS OF TRANSLATION-INVARIANT GIBBS MEASURES FOR THE HARD-CORE MODEL WITH A COUNTABLE SET OF SPIN VALUES

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Let $\Im^k = (V, L)$ is Cayley tree of order $k \geq 2$. Let $\Phi = \{\ldots, -1, 0, 1, \ldots\}$ and $\sigma \in \Omega = \Phi^V$ be a configuration. We consider the set Φ as the set of vertices of a graph G. A configuration σ is called a *G-admissible configuration* on the Cayley tree, if $\{\sigma(x), \sigma(y)\}$ is the edge of the graph G for any pair of nearest neighbors x, y in V.

The activity set [1] for a graph G is a function $\lambda : G \to \mathbb{R}_+$. For given G and λ we define the Hamiltonian of the G-HC model as

$$H_G^{\lambda}(\sigma) = -\sum_{x \in V} \ln \lambda_{\sigma(x)},\ \sigma \in \Omega^G.$$

The reader can find the definition of the Gibbs measure and of other subjects related to Gibbs measure theory, for example, in [2],[3].

Let $L(G)$ be the set of edges of a graph G, and let $A \equiv A^G = (a_{ij})_{i,j \in \mathbb{Z}}$ denote the adjacency matrix of G, i.e

$$a_{ij} = a_{ij}^G = \begin{cases} 1, & \text{if } \{i, j\} \in L(G) \\ 0, & \text{if } \{i, j\} \notin L(G). \end{cases}$$

For any function $z : x \to z_x = (..., z_{-1,x}, z_{0,x}, z_{1,x}, ...) \in \mathbb{R}_+^{\infty}$ satisfying the equalities

$$z_{i,x} = \lambda_i \prod_{y \in S(x)} \frac{a_{i0} + \sum_{j \in \mathbb{Z}_0} a_{ij} z_{i,y}}{a_{00} + \sum_{j \in \mathbb{Z}_0} a_{0j} z_{i,y}}, \quad \mathbb{Z}_0 = \mathbb{Z} \setminus \{0\} \tag{1},$$

there exists a unique HC-Gibbs measure μ, and vice versa, where $S(x)$ the set of direct successors of x (see [4]).

We consider the graph G with $a_{i0} = 1$ and $a_{0i} = 1$ for any $i \in \mathbb{Z}$ and

$$\begin{cases} a_{ij} = 1, & \text{if } i + j \text{ even}, \\ a_{ij} = 0, & \text{otherwise}. \end{cases}$$

In the case of G, from (1) we obtain

$$\begin{cases} z_{2i,x} = \lambda_{2i} \prod\limits_{y \in S(x)} \frac{1 + \sum_{j \in \mathbb{Z}_0} z_{2j,y}}{1 + \sum_{j \in \mathbb{Z}_0} z_{j,y}}, & i \in \mathbb{Z}_0, \\ z_{2i+1,x} = \lambda_{2i+1} \prod\limits_{y \in S(x)} \frac{1 + \sum_{j \in \mathbb{Z}} z_{2j+1,y}}{1 + \sum_{j \in \mathbb{Z}_0} z_{j,y}}, & i \in \mathbb{Z}. \end{cases} \tag{2}$$

We study translation-invariant solutions (TI) of system (2), i.e., $z_x = z \in \mathbb{R}_+^{\infty}$.

Let series $\sum_{j\in\mathbb{Z}_0} z_j$ and $\sum_{j\in\mathbb{Z}_0} \lambda_j$ obtained respectively from $\{z_i\}_{i\in\mathbb{Z}_0}$ and $\{\lambda_j\}_{j\in\mathbb{Z}_0}$ converge, i.e.,

$$\sum_{j\in\mathbb{Z}_0} \lambda_{2j} = \Delta, \qquad \sum_{j\in\mathbb{Z}} \lambda_{2j+1} = \Psi.$$

In the case $\Delta = \Psi$ the following theorem is true.

Theorem 1. Let $k = 2, 3, 4, 5$ and $\Lambda_{cr}(k) = \frac{1}{k-1}\left(\frac{k+1}{k}\right)^k$. Then for an HC model with a countable set of spin values (corresponding to the graph G) for $\Lambda \le \Lambda_{cr}(k)$, there is exactly one TIGM, for $\Lambda > \Lambda_{cr}(k)$ there are exactly three TIGMs.

In the case $\Delta \neq \Psi$ the following theorem is true.

Theorem 2. Let $k = 2$. Then for an HC model with a countable set of spin values (corresponding to the graph G), the following statements are true:

(i) For $0 < \Psi < \Lambda_1(\Delta)$, there is exactly one TIGM;

(ii) For $\Psi = \Lambda_1(\Delta)$, there are exactly two TIGMs;

(iii) For $\Lambda_1(\Delta) < \Psi < \Lambda_2(\Delta)$, there are exactly three TIGMs;

(iv) For $\Psi = \Lambda_2(\Delta)$, there are exactly two TIGMs;

(v) For $\Psi > \Lambda_2(\Delta)$, there is exactly one TIGM.

Where x_1 and x_2 are the positive roots of the following equation:

$$\sqrt{\Delta}x^{5/2} + (6-\Delta)x^2 - 5\sqrt{\Delta}x^{3/2} + \Delta = 0$$

and

$$\Lambda_1(\Delta) = \frac{\Delta(1+x_1)^3\left(\sqrt{\Delta}-\sqrt{x_1}\right)}{\sqrt{x}\left(x_1\sqrt{\Delta}-x_1\sqrt{x_1}+\sqrt{\Delta}\right)^2}, \qquad \Lambda_2(\Delta) = \frac{\Delta(1+x_2)^3\left(\sqrt{\Delta}-\sqrt{x_2}\right)}{\sqrt{x_2}\left(x_2\sqrt{\Delta}-x_2\sqrt{x_2}+\sqrt{\Delta}\right)^2}.$$

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THREE STATE HARD-CORE MODELS ON THE BINARY TREE: TRANSLATION INVARIANT GIBBS MEASURES

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Let $\Im^k = (V, L)$ is Cayley tree of order $k \geq 2$. Let $\Im^k = (V, L)$, where V is the set of vertices $\Im^k$ and L is the set of its edges.

For fixed $x^0 \in V$ we set

$$W_n = \{x \in V \,|\, d(x, x^0) = n\}, \qquad V_n = \bigcup_{m=0}^{n} W_m,$$

where $d(x, y)$ is the distance between vertices x and y on the Cayley tree.

Let $\Phi = \{0, 1, 2\}$ and $\sigma \in \Omega = \Phi^V$ be the configuration, i.e., $\sigma = \{\sigma(x) \in \Phi : x \in V\}$. In other words, in this model, each vertex x is assigned to one of the values $\sigma(x) \in \Phi = \{0, 1, 2\}$.

Consider the set Φ as the set of vertices of some graph G. The σ configuration is called *G-admissible configuration* on the Cayley tree (in V_n) if $\{\sigma(x), \sigma(y)\}$ is edge of the graph G for any the nearest pair of neighbors x, y from V (from V_n). Let us denote the set of G-admissible configurations via $\Omega^{a,G}$ ($\Omega^{a,G}_{V_n}$).

The activity set for the graph G is the function $\lambda : G \to R^3_+$. Value λ_i of function λ at vertex $i \in \{0, 1, 2\}$ is called its "activity" (see [1]).

For given G and λ, we define the Hamiltonian of the $G-HC$-model as

$$H^{\lambda}_G(\sigma) = \begin{cases} -\sum\limits_{x \in V} \log \lambda_{\sigma(x)}, & \text{if } \sigma \in \Omega^{a,G}, \\ +\infty, & \text{if } \sigma \notin \Omega^{a,G}. \end{cases}$$

Let $L(G)$ be the set of edges of a graph G. We let $A \equiv A^G = \big(a_{ij}\big)_{i,j}$ denote the adjacency matrix of the graph G, i.e.,

$$a_{ij} = a^G_{ij} = \begin{cases} 1, & \text{if } \{i, j\} \in L(G), \\ 0, & \text{if } \{i, j\} \notin L(G). \end{cases}$$

Let $z : \; x \mapsto z_x = (z_{0,x}, z_{1,x}, z_{2,x}) \in R^3_+$ vector-valued function on V. For $n \geq 1$ and $\lambda > 0$ consider the probability measure $\mu^{(n)}$ on $\Omega^{a,G}_{V_n}$, defined as

$$\mu^{(n)}(\sigma_n) = \frac{1}{Z_n} \prod_{x \in V_n} \lambda_{\sigma_n(x)} \prod_{x \in W_n} z_{\sigma_n(x),x}. \tag{1}$$

Here Z_n is the normalizing divisor:

$$Z_n = \sum_{\varphi_n \in \Omega^{a,G}_{V_n}} \prod_{x \in V_n} \lambda_{\varphi_n(x)} \prod_{x \in W_n} z_{\varphi_n(x),x}.$$

Definition 1. The measure μ that is the limit of a sequence $\mu^{(n)}$ defined by (1) with consistency condition is called the splitting HC-Gibbs measure (SGM) with $\lambda > 0$ corresponding to the function $z : x \in V \setminus \{x^0\} \mapsto z_x$. Moreover, an HC-Gibbs measure corresponding to a constant function $z_x \equiv z$ is said to be translation-invariant (TI).

Theorem 1. Probability measures $\mu^{(n)}$, $n = 1, 2, \ldots$, given by the formula (1), are consistent if and only if for any $x \in V$ the following equation holds:

$$\begin{cases} z'_{1,x} = \lambda'_1 \prod_{y \in S(x)} \frac{a_{10}+a_{11}z'_{1,y}+a_{12}z'_{2,y}}{a_{00}+a_{01}z'_{1,y}+a_{02}z'_{2,y}}, \\ z'_{2,x} = \lambda'_2 \prod_{y \in S(x)} \frac{a_{20}+a_{21}z'_{1,y}+a_{22}z'_{2,y}}{a_{00}+a_{01}z'_{1,y}+a_{02}z'_{2,y}}, \end{cases} \tag{2}$$

where $z'_{i,x} = \lambda'_i \frac{z_{i,x}}{z_{0,x}}$, $\lambda'_i = \frac{\lambda_i}{\lambda_0}$, $i = 1, 2$.

We study translation-invariant solutions (TI) of system (2), i.e., $z_x = z \in R^3_+$.

In this paper we consider the case $\lambda_1 \neq \lambda_2$ and we study TISGMs in the case fertile graph $G = hinge$ (see [2], [3]):

$$hinge: \quad \{0,0\}\{0,1\}\{0,2\}\{1,1\}\{2,2\}.$$

Let $k = 2$. In the case $G = hinge$ from (2) we have

$$\begin{cases} z_1 = \lambda_1 \left(\frac{1+z_1}{1+z_1+z_2}\right)^2, \\ z_2 = \lambda_2 \left(\frac{1+z_2}{1+z_1+z_2}\right)^2. \end{cases} \tag{3}$$

Analysis of the system of equations (3) gives the following theorem.

Theorem 2. Let $k = 2$. Then for the three state Hard-Core model in the case hinge, the number of TISGMs is

$$N = \begin{cases} 1, & \text{if } 0 < \lambda_1 < \gamma_1(\lambda_2) \text{ or } \lambda_1 > \gamma_2(\lambda_2), \\ 2, & \text{if } \lambda_1 = \gamma_1(\lambda_2) \text{ or } \lambda_1 = \gamma_2(\lambda_2), \\ 3, & \text{if } \gamma_1(\lambda_2) < \lambda_1 < \gamma_2(\lambda_2). \end{cases}$$

Here

$$\gamma_1(\lambda_2) = \frac{\lambda_2(1+x_1)^3(\sqrt{\lambda_2}-\sqrt{x_1})}{\sqrt{x}(x_1\sqrt{\lambda_2}-x_1\sqrt{x_1}+\sqrt{\lambda_2})^2}, \quad \gamma_2(\lambda_2) = \frac{\lambda_2(1+x_2)^3(\sqrt{\lambda_2}-\sqrt{x_2})}{\sqrt{x_2}\left(x_2\sqrt{\lambda_2}-x_2\sqrt{x_2}+\sqrt{\lambda_2}\right)^2},$$

where x_1 and x_2 are the positive roots of the following equation:

$$\sqrt{\lambda_2}\sqrt{x^3} + (6-\lambda_2)x^2 - 5\sqrt{\lambda_2}\sqrt{x^3} + \lambda_2 = 0.$$

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A NOTE HOMOTOPY PROPERTIES OF THE SPACE OF COMPACT MAXIMAL LINKED SYSTEMS

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A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called linked, if any two elements of ξ intersect.

Any linked system can be upgraded to a maximal linked system (MLS). But such upgrade, as a rule, is not one valued.

A linked system of space is MLS, if and only if it possesses the following completeness property (see [1]):

If a closed set $A \subset X$ intersects with every element of ξ, then $A \in \xi$. We denote λX as the set of all MLS of the space X.

For the close set $A \in \lambda X$ we consider $A^+ = \{\xi \in \lambda X : A \in \xi\}$.

For the open set $U \subset X$ we consider $O(U) = \{\xi \in \lambda X :$ there exists $F \in \xi$ such that $F \subset U\}$.

A maximal linked system μ of a space X is called a *compact maximal linked system* if μ contains at least one compact element.

We denote by $\lambda_c X$ the set of all thin maximal linked systems of the space X (see [2]).

A continuous mapping $f : [0,1] \to X$ is called a *path* in X. The point $f(0)$ is called the initial point and $f(1)$ is called the final or terminal point of this path. If $x \in X$, then one defines $e_x : I \to X$ as the constant path, i.e. $e_x(t) = x$ for any $t \in I$. A topological space X is said to be *path-connected* if given any two points x_0, x_1 in X there is a path in X from x_0 to x_1.

Theorem 1. If the mapping $f : I \to X$ is path in X from the point x_0 to the point x_1, then a mapping $\lambda_c f : I \to \lambda_c X$ defined by $\lambda_c f(t) = \{\mu \in \lambda_c X : f(t) \in \mu, \ \ \forall t \in [0,1]\}$ is a path from the point $\xi_0 = \{L_0 \in \mu : f(0) \in L_0, \ \ \mu \in \lambda_c X\}$ to the point $\xi_1 = \{L_1 \in \mu : f(1) \in L_1, \ \ \mu \in \lambda_c X\}$ in $\lambda_c X$.

From Theorem 1 we get the following corollary.

Corollary 1. If a topological space X is path-connected, then the space $\lambda_c X$ is also path-connected.

Continuous mappings $f, g : X \to Y$ are said to be *homotopic* if there is a continuous mapping $H : X \times I \to Y$ such that $H(x,0) = f(x)$ and $H(x,1) = g(x)$. The mapping H is called a *homotopy* between f and g and we write $f \simeq g$ (see [3]).

Theorem 2. If mappings $f, g : X \to Y$ are homotopic, then the mappings $\lambda_c f, \lambda_c g : \lambda_c X \to \lambda_c Y$ are also homotopic.

From Theorem 2 we get the following corollary.

Corollary 2. If the mapping H is homotopy between f and g, then the mapping $\lambda_c H$ is also a homotopy between $\lambda_c f$ and $\lambda_c g$.

A continuous mapping $f : X \to Y$ is said to be a *homotopy equivalence* if there exists a continuous mapping $g : Y \to X$ such that the compositions $g \circ f$ and $f \circ g$ are homotopic to the identity mappings on X and Y respectively. Two topological spaces X and Y is said to be *homotopically equivalent* (notation, $X \simeq Y$) if there exists a homotopy equivalence $f : X \to Y$ (see [3]).

Proposition 1. If the mapping $f : X \to Y$ is homotopy equivalence, then the mapping $\lambda_c f : \lambda_c X \to \lambda_c Y$ is also homotopy equivalence.

Corollary 3. If spaces X and Y are homotopically equivalent, then the spaces $\lambda_c X$ and $\lambda_c Y$ are also homotopically equivalent.

A space X is said to be *contractible* if it is homotopy equivalent to a point (see [4]).

Proposition 2. If a topological space X is contractible, then the space $\lambda_c X$ is also contractible.

By a *covariant homotopy functor* we mean a functor $\mathcal{F}$ in the category **Top** of topological spaces and their continuous mappings satisfying

(*) $\mathcal{F}$ preserves homotopy, that is, if a mapping $H(x,t)$ is a homotopy between continuous mappings $f, g : X \to Y$, then $F(H(x,t))$ is also a homotopy between mappings $F(f), F(g) : F(X) \to F(Y)$.

Theorem 3. The functor of complete linked systems λ_c is a covariant homotopy functor.

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LIMIT THEOREM OF RESCALED HITTING TIME FUNCTIONS FOR QUADRATIC IRRATIONAL ROTATIONS

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Let $(\Omega, \mathcal{F}, \mu)$ be a probability space and $T : \Omega \to \Omega$ be a $\mu-$ measure preserving transformation. For the measurable subset of $A \in \mathcal{F}\,(\mu(A) > 0)$, the return time function $R_A : A \to \mathbb{N}$ is defined by

$$R_A(x) = \inf\left\{j \geq 1 :\ T^j x \in A\right\}.$$

An important consequence of the existence of an invariant measure was obtained by A.Poincare

Theorem 1. [1] Let $(\Omega, \mathcal{F}, \mu)$ be a probability space, $T : \Omega \to \Omega$ be a $\mu-$ invariant transformation and $A \in \mathcal{F}$, $\mu(A) > 0$. Then for almost every point x of A there exists an increasing sequence $\{n_k, k = 1, \infty\}$, for which $T^{n_k} x \in A$.

It is obvious that for $\mu(A) > 0$, $R_A(x)$ is almost everywhere finite. Note that if $\{A_n\}_{n=1}^{\infty}, A_n \supset A_{n+1}, n \geq 1$ sequence of subsets contained x, then $R_{A_n}(x)$ is an increasing sequence. A great number of papers have been published on estimating the asymptotic behaviuor of return times. Let $\mathcal{P}$ be a measurable partition of the probability space Ω. Let $\mathcal{P}_n$ be the sequence of partitions of Ω which defined by following equality

$$\mathcal{P}_n = \mathcal{P} \vee T^{-1}\mathcal{P} \vee ... \vee T^{-n+1}\mathcal{P}.$$

Where $\mathcal{P} \vee \mathcal{Q} = \{P \cap Q : P \in \mathcal{P}, Q \in \mathcal{Q}\}$. Ornstein and Weiss showed the return time property which says that if T is ergodic, then

$$\lim_{n\to\infty} \frac{\log R_{P_n(x)}(x)}{n} = h(T, \mathcal{P}), \qquad a.e.$$

Let $T_\rho = \{x + \rho\}$ irrational rotation and $\rho = [k_1, k_2, \ldots, k_n, \ldots]$. Z. Coelho and E. de Faria investigated return time problems for C^1-diffeomorphisms in their work cited as [3]. They focused on a set A within the sequence of renormalization intervals $V_n = [x_{q_n}, x_{q_n+1}]$ of the homeomorphism. Their research led to the establishment of a necessary and sufficient condition on a subsequence of the V_n's, ensuring the convergence of the corresponding rescaled return time point process.

Consider the sequence of dynamical partitions $\mathcal{D}_n$ of the interval $[0, 1)$ which defined by $\mathcal{D}_n = \bigcup_{j=1}^{q_{n-1}+q_n} T^j\mathcal{D}$. Where $\mathcal{D} = \{[0,\, \rho) \cup [\rho, 1)\}$ is measurable partition of $[0, 1)$. We define the function $H_n(\alpha_n; x, y) : [0, 1) \times [0, 1) \to \mathbb{N}$

$$H_n(\alpha_n; x, y) = \min\{j :\ T_\rho^j y \in I_n(x)\},$$

where $I_n(x)$ is an element of the partition α_n containing the point x. The function $H_n(\alpha_n; x, y)$ is called **hitting time function** with respect to partition α_n. Now define the rescaled hitting time as

$$\tilde{H}_n(\alpha_n; x, y) = \frac{H_n(\alpha_n; x, y)}{q_{n+1} + q_n}.$$

We define the distribution functions of $\tilde{H}_n(\mathcal{D}_n; x, y)$ as

$$\Phi_{\theta,n}(t) = \mu_2\left((x,y) \in [0,1) \times [0,1) : \tilde{H}_n(\mathcal{D}_n; x, y) \leq t\right), \quad t \in \mathbb{R}^1.$$

where $\mu_2(\cdot)$ is the Lebesgue measure on the torus $[0,1) \times [0,1)$.

Now we formulate our main result.

Theorem 2. Let T_ρ be irrational rotation of the circle. Suppose the rotation number $\rho = [a_1, a_2, \cdots, a_m, \, k_1, k_2, ..., k_s, \, k_1, k_2, ..., k_s, ...]$ is a quadratic irrational. The distribution functions $\Phi_{ns+i}(t), \; 1 \leq i \leq s$ of $\tilde{H}_{ns+i}(\mathcal{D}_{ns+i}; x, y)$ converge uniformly to the continuous piecewise linear distribution function on interval $[0,1]$.

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ABOUT DYNAMIC SYSTEMS OF A QnSO

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Let $I = \{1, 2, \dots, m\}$. A distribution of the set I is a probability measure $x = (x_1, \dots, x_m)$, i.e. an element of the simplex:

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \geq 0,\ \sum_{i=1}^{m} x_i = 1 \right\}. \tag{1}$$

The quadratic stochastic operator (QSO) is a mapping of the simplex S^{m-1} into itself, of the form

$$V : x'_k = \sum_{i,j=1}^{m} P_{ij,k} x_i x_j,\ \ k = 1, \dots, m. \tag{2}$$

where $P_{ij,k}$ are coefficients of heredity and

$$P_{ij,k} \geq 0,\ \ P_{ij,k} = P_{ji,k},\ \ \sum_{k=1}^{m} P_{ij,k} = 1,\ \ i, j, k = 1, \dots, m. \tag{3}$$

Thus, each quadratic stochastic operator V can be uniquely defined by a cubic matrix $(P_{ij,k})_{i,j,k=1}^{m}$ with conditions (3).

For a given $x^{(0)} \in S^{m-1}$ the trajectory (orbit) $\{x^{(n)}\}$ of $x^{(0)}$ under the action of QSO (2) is defined by

$$x^{(n+1)} = V(x^{(n)}),\ \ n = 0, 1, 2...$$

In general, a quadratic operator V, $V : x \in \mathbb{R}^m \to x' = V(x) \in \mathbb{R}^m$ corresponding to a cubic matrix $(P_{ij,k})_{i,j,k=1}^{m}$ is defined by (2):

Definition.[1] A quadratic operator (2) preserving a simplex, is called non-stochastic (QnSO) if at least one of its coefficients $P_{ij,k},\ i \neq j$ is negative.

Theorem 1. [1] For a quadratic operator V (given by (2)), to preserve a simplex S^{m-1} it is **sufficient** that

i) $\sum\limits_{k=1}^{m} P_{ij,k} = 1,\ \ i, j = 1, \dots, m;$

ii) $0 \leq P_{ii,k} \leq 1,\ \ i, k = 1, \dots, m;$

iii) $-\frac{1}{m-1}\sqrt{P_{ii,k}P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$

and **necessary** that the conditions (i), (ii) and

iii′) $-\sqrt{P_{ii,k}P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$

are satisfied.

Consider the following QnSO on S^2:

$$W_0 : \begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2 \\ z' = z(2-z). \end{cases} \tag{4}$$

where

$$a, c \in [0,1], \quad b \in [-\sqrt{ac}, \quad 1+\sqrt{(1-a)(1-c)}]. \tag{5}$$

Theorem 2. For the initial point $(x_0, y_0, z_0) \in S^2$, $z_0 \neq 0$ the trajectory (x_n, y_n, z_n) approaches the point $e_1 = (0,0,1)$.

Denote

$$\mathcal{Z} = \{(x,y,z) \in S^2 : z = 0\}.$$

It is easy to see that $\mathcal{Z}$ is an edge of the simplex and it is invariant: $z = 0 \quad \Rightarrow z' = 0$.

Remark. If $z = 0$, then the restriction of operator (4) to $\mathcal{Z}$ is an arbitrary one-dimensional QnSO in S^1:

$$\begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2. \end{cases} \tag{6}$$

Some special cases a operator the form (6) were studied in [1].

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ALMOST INNER DERIVATIONS OF FINITE DIMENSIONAL SIMPLE LEIBNIZ ALGEBRAS

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Definition 1. An algebra $\mathcal{L}$ over a field $\mathbb{F}$ is called a *Leibniz algebra* if for any $x, y, z \in \mathcal{L}$, the Leibniz identity

$$[x,[y,z]] = [[x,y],z] - [[x,z],y]$$

is satisfied, where $[-,-]$ is the multiplication in $\mathcal{L}$.

Let $\mathfrak{g}$ a simple Lie algebra. For a Leibniz algebra $\mathcal{L}$, a subspace generated by its squares $\mathcal{I} = \text{span}\{[x,\, x] : x \in \mathcal{L}\}$ due to Leibniz identity becomes an ideal, and the quotient $\mathfrak{g}_{\mathcal{L}} = \mathcal{L}/\mathcal{I}$ is a Lie algebra called liazation of $\mathcal{L}$. Moreover, $[\mathcal{L}, \mathcal{I}] = 0$. In general, $[\mathcal{I}, \mathcal{L}] \neq 0$. Since we are interested in Leibniz algebras which are not Lie algebras, we will always assume that $\mathcal{I} \neq 0$. Then $[\mathcal{I}, \mathfrak{g}] = \mathcal{I}$.

Definition 2. A Leibniz algebra $\mathcal{L}$ is called *simple* if its liezation is a simple Lie algebra and ideal $\mathcal{I}$ is a simple ideal. Equivalently, $\mathcal{L}$ is simple if $\mathcal{I}$ is the only non-trivial ideal of $\mathcal{L}$.

A *derivation* of Leibniz algebra $\mathcal{L}$ is a linear map $D : \mathcal{L} \to \mathcal{L}$ which satisfies the Leibniz law, that is,

$$D([x,y]) = [D(x), y] + [x, D(y)]$$

for all $x, y \in \mathcal{L}$.

The set of all derivations of $\mathcal{L}$ with respect to the commutation operation is a Lie algebra and it is denoted by $Der(\mathcal{L})$.

Let $\mathcal{L}$ be a Leibniz algebra. For each $a \in \mathcal{L}$, the operator $R_a : \mathcal{L} \to \mathcal{L}$ which is called the *right multiplication*, such that $R_a(y) = [y, a],\ y \in \mathcal{L}$, is a derivation. This derivation is called an *inner derivation* of $\mathcal{L}$, and we denote the space of all inner derivations by $InDer(\mathcal{L})$.

Now let us give the definitions of the almost inner derivations for the Leibniz algebras.

Definition 3. The derivation $D \in Der(\mathcal{L})$ of the Leibniz algebra $\mathcal{L}$ is called *almost inner derivation*, if $D(x) \in [x, \mathcal{L}]$ holds for all $x \in \mathcal{L}$; in other words, there exists $a_x \in \mathcal{L}$ such that $D(x) = [x, a_x]$. The space of all almost inner derivations of $\mathcal{L}$ is denoted by $AID(\mathcal{L})$.

Let $\mathcal{L} = \mathfrak{g} \ltimes \mathcal{I}$ be a simple Leibniz algebras, where $\mathfrak{g}$ - simple Lie algebra, $\mathcal{I}$ - Leibniz ideal.

The following result is the main result of this paper.

Theorem 1. Any almost inner derivations of simple Leibniz algebras $\mathcal{L}$ is inner.

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THRESHOLD EFFECT OF THE GENERALIZED FRIEDRICHS MODEL

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Let $\mathbb{T}^3 = (\mathbb{R}/2\pi\mathbb{Z})^3 = (-\pi, \pi]^3$ is the three-dimensional torus and $L^2(\mathbb{T}^3)$ is the Hilbert space of square-integrable functions defined on the torus $\mathbb{T}^1$. We consider the generalized Friedrichs model $H_{\lambda_1\lambda_2\lambda_3}(p) := H_0(p) + \lambda_1 V_1 + \lambda_2 V_2 + \lambda_3 V_3$, depending on the parameters $p \in \mathbb{T}^3$ and $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0$, with the rank three perturbation associated to a system of two arbitrary or identical quantum mechanical particles moving on the three-dimensional lattice, where $H_0(K)$, $p \in \mathbb{T}^3$ is a multiplication operator by the function $\varepsilon_K(\cdot) := \varepsilon(K, \cdot)$:

$$(H_0(K)f)(q) = \varepsilon_K(q)f(q), \quad f \in L^2(\mathbb{T}^3),$$

and $V_i : L^2(\mathbb{T}^3) \to L^2(\mathbb{T}^3)$ are the operators of the form

$$(V_i f)(q) = \varphi_i(q_i)(f, \varphi_i), \quad i = 1, 2, 3,$$

where $(\cdot, \cdot)$ stands for the inner product in $L^2(\mathbb{T}^3)$.

Let us introduce a hypothesis, which we always assume to hold in presenting our results.

Hypothesis 1.: The functions $\varphi_i(\cdot)$, $i = 1, 2, 3$ and $\varepsilon(\cdot, \cdot)$ used in the definition of the operator $H_{\lambda_1\lambda_2\lambda_3}(p)$ satisfy the following conditions:

(i) $\varphi_i(\cdot)$, $i = 1, 2, 3$ are nontrivial, real-analytic on $\mathbb{T}^1$ and

(ii) $\varepsilon(\cdot, \cdot)$ is real-analytic on $(\mathbb{T}^3)^2$ and have a unique non-degenerate maximum at $(\hat{\pi}, \hat{\pi}) \in (\mathbb{T}^3)^2$, $\hat{\pi} = (\pi, \pi, \pi)$.

The perturbation $V = \lambda_1 V_1 + \lambda_2 V_2 + \lambda_3 V_3$ of the non-perturbed operator $H_0(K), K \in \mathbb{T}^1$ is an operator of rank three. Consequently, by the well-known Weyl theorem [1. Theorem XIII.14] on the compact perturbations

$$\sigma_{\text{ess}}(H_{\lambda_1\lambda_2\lambda_3}(K)) = \sigma_{\text{ess}}(H_0(K)) = \sigma(H_0(K)) = [m(K),\, M(K)],$$

where

$$m(K) = \min_{q\in\mathbb{T}^1} \varepsilon_K(q),\; M(K) = \max_{q\in\mathbb{T}^1} \varepsilon_K(q).$$

Hypothesis implies the existence of a δ-neighborhood $U_\delta(\hat{\pi}) \subset \mathbb{T}^3$ of the point $\hat{\pi} \in \mathbb{T}^3$ and an analytic function $q_1 : U_\delta(\hat{\pi}) \to \mathbb{T}^3$ that for any $K \in U_\delta(\hat{\pi})$ the point $q_1(K) \in \mathbb{T}^3$ is a unique non-degenerate maximum of the function $w_p(\cdot)$.

Note that under Hypothesis 1 and for any $K \in U_\delta(\hat{\pi})$ the following integrals

$$\lambda_i(K) = \int\limits_{\mathbb{T}^3} \frac{\varphi_i^2(s_i)ds}{w_K(s) - M(K)} < 0, \quad i = 1, 2, 3$$

exist.

Definition. The number $z = M(K)$ is called a virtual level of the operator $H_{\lambda_1\lambda_2\lambda_3}(K)$, if the equation $H_{\lambda_1\lambda_2\lambda_3}(K)f = M(K)f$ has a nonzero solution $f \in L_1(\mathbb{T}^3) \setminus L_2(\mathbb{T}^3)$, where $L_1(\mathbb{T}^3)$ is the Banach space of integrable functions on $\mathbb{T}^3$. The corresponding solution f is called a virtual state of the operator $H_{\lambda_1\lambda_2\lambda_3}(K)$.

Hypothesis 2.: For any $z > M(K)$ and for any $K \in U_\delta(\hat{\pi})$ the following equality hold

$$\int\limits_{\mathbb{T}^3} \frac{\varphi_i(s_i)\varphi_j(s_j)}{w_K(s) - z} ds = 0. \quad i \neq j. \quad i, j = 1, 2, 3.$$

Theorem. Assume Hypothesis 1 and 2 hold. If $\lambda_i = \lambda_i(K), \lambda_j \neq \lambda_j(K), i, j = 1, 2, 3, i \neq j, K \in U_\delta(\hat{\pi})$ then the number $M(K)$ is virtual lavel or eigenvalue of the operator $H_{\lambda_1\lambda_2\lambda_3}(K)$.

The generalized Friedrichs model also considered in [2,3]

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THE DISCRETE SPECTRUM OF THE GENERALIZED FRIEDRICHS MODEL WITH A RANK-TWO PERTURBATION

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Let $\mathbb{T}^1 = (-\pi, \pi]$ be one-dimensional torus and $L^2(\mathbb{T}^1)$ be the Hilbert space of square-integrable functions defined on the torus $\mathbb{T}^1$.

We consider the generalized Friedrichs model $H_{\lambda_1\lambda_2}(p)$, $p \in \mathbb{T}^1$ acting in $L^2(\mathbb{T}^1)$ defined as

$$H_{\lambda_1\lambda_2}(p) = H_0(p) + \lambda_1 V_1 + \lambda_2 V_2, \ \lambda_1, \lambda_2 \in \mathbb{R},$$

where $H_0(p)$, $p \in \mathbb{T}^1$ is a multiplication operator by the function $w_p(\cdot) := w(p, \cdot)$:

$$(H_0(p)f)(q) = w_p(q)f(q), \quad f \in L^2(\mathbb{T}^1)$$

and $V_i : L^2(\mathbb{T}^1) \to L^2(\mathbb{T}^1)$ are the operators of the form

$$(V_i f)(q) = \varphi_i(q)(f, \varphi_i), \quad i = 1, 2,$$

where $(\cdot, \cdot)$ stands for the inner product in $L^2(\mathbb{T}^1)$.

The perturbation $V = \lambda_1 V_1 + \lambda_2 V_2$ of the non-perturbed operator $H_0(p), p \in \mathbb{T}^1$ is an operator of rank two. Consequently, by the well-known Weyl theorem [1. Theorem XIII.14] on the compact perturbations

$$\sigma_{\text{ess}}(H_{\lambda_1\lambda_2}(p)) = \sigma_{\text{ess}}(H_0(p)) = \sigma(H_0(p)) = [m(p),\, M(p)],$$

where

$$m(p) = \min_{q\in\mathbb{T}^1} w_p(q), \ M(p) = \max_{q\in\mathbb{T}^1} w_p(q).$$

Hypothesis 1. The functions $\varphi_i(\cdot)$, $i = 1, 2$ and $w(\cdot, \cdot)$ used in the definition of the operator $H_{\lambda_1\lambda_2}(p)$ satisfy the following conditions:

(i) $\varphi_i(\cdot)$, $i = 1, 2$ are nontrivial, real-analytic on $\mathbb{T}^1$ and $(\varphi_1, \varphi_2) = 0$;

(ii) $w(\cdot, \cdot)$ is real-analytic on $(\mathbb{T}^1)^2 = \mathbb{T}^1 \times \mathbb{T}^1$ and have a unique non-degenerate minimum and maximum at $(p_{min}, q_{min}) \in (\mathbb{T}^1)^2$ and $(p_{max}, q_{max}) \in (\mathbb{T}^1)^2$ respectively.

Hypothesis 1 implies the existence of a δ-neighborhood $U_\delta(p_{min}) \subset \mathbb{T}^1$(resp. $U_\delta(p_{max}) \subset \mathbb{T}^1$) of the point $p_{min} \in \mathbb{T}^1$ (resp. $p_{max} \in \mathbb{T}^1$) and an analytic function $q_0 : U_\delta(p_{min}) \to \mathbb{T}^1$ (resp. $q_1 : U_\delta(p_{max}) \to \mathbb{T}^1$) that for any $p \in U_\delta(p_{min})$ (resp. $p \in U_\delta(p_{max})$) the point $q_0(p) \in \mathbb{T}^1$ (resp. $q_1(p) \in \mathbb{T}^1$) is a unique non-degenerate minimum (resp. maximum) of the function $w_p(\cdot)$.

Let us consider the polynomials $A(\lambda_1, \lambda_2; p), p \in U_\delta(p_{\min})$ and $B(\lambda_1, \lambda_2; p), p \in U_\delta(p_{\max})$ of two-variable defined by

$$A(\lambda_1, \lambda_2; p) = \lambda_1 + \lambda_2 + \lambda_1\lambda_2 a(p), \ a(p) > 0$$

$$B(\lambda_1,\lambda_2;p)=\lambda_1+\lambda_2+\lambda_1\lambda_2 b(p),\ b(p)<0.$$

The curves (hyperbolas) $A(\lambda_1,\lambda_2;p)=0, p\in U_\delta(p_{\min})$ and $B(\lambda_1,\lambda_2;p)=0, p\in U_\delta(p_{\max})$ divide the (λ_1,λ_2)-plane into several connected components. Let us denote those components with A_α^- and $B_\alpha^+, \alpha=0,1,2$ as:

$$A_0^-:=\{(\lambda_1,\lambda_2)\in\mathbb{R}^2: A(\lambda_1,\lambda_2;p)>0, \lambda_1>-\frac{1}{a(p)}\},$$

$$A_1^-:=\{(\lambda_1,\lambda_2)\in\mathbb{R}^2: A(\lambda_1,\lambda_2;p)<0\},$$

$$A_2^-:=\{(\lambda_1,\lambda_2)\in\mathbb{R}^2: A(\lambda_1,\lambda_2;p)>0, \lambda_1<-\frac{1}{a(p)}\},$$

$$B_0^+:=\{(\lambda_1,\lambda_2)\in\mathbb{R}^2: B(\lambda_1,\lambda_2;p)>0, \lambda_1<-\frac{1}{b(p)}\},$$

$$B_1^+:=\{(\lambda_1,\lambda_2)\in\mathbb{R}^2: B(\lambda_1,\lambda_2;p)<0\},$$

$$B_2^+:=\{(\lambda_1,\lambda_2)\in\mathbb{R}^2: B(\lambda_1,\lambda_2;p)>0, \lambda_1>-\frac{1}{b(p)}\}.$$

Theorem 1. Let Hypothesis 1 holds and $\alpha=0,1,2$.

(i) If for any fixed $p\in U_\delta(p_{\min})$, $\varphi_1(q_0(p))=\varphi_2(q_0(p))\neq 0$, then there exists such a number $a(p)>0$ that for any $(\lambda_1,\lambda_2)\in A_\alpha^-$ the operator $H_{\lambda_1\lambda_2}(p)$ has α eigenvalue(s) in the interval $(-\infty,m(p))$.

(ii) If for any fixed $p\in U_\delta(p_{\min})$, $\varphi_1(q_1(p))=\varphi_2(q_1(p))\neq 0$, then there exists such a number $b(p)<0$ that for any $(\lambda_1,\lambda_2)\in B_\alpha^+$ the operator $H_{\lambda_1\lambda_2}(p)$ has α eigenvalue(s) in the interval $(M(p),+\infty)$.

The generalized Friedrichs model was considired in [2-4].

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THE NUMBER OF EIGENVALUES OF THE TWO PARTICLE DISCRETE SCHRÖDINGER OPERATORS

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Let $\mathbb{T}^2 = [-\pi, \pi)^2$ and $L^2(\mathbb{T}^2)$ be the Hilbert space of square-summable on $\mathbb{T}^2$. Let $L^{2,\mathrm{e}}(\mathbb{T}^2)$ be the subspace of essentially even in $L^2(\mathbb{T}^2)$ and let $L^{2,\mathrm{es}}(\mathbb{T}^2) := \{f \in L^{2,\mathrm{e}}(\mathbb{T}^2) : f(p_1, p_2) = f(p_2, p_1)$ for a.e $(p_1, p_2) \in \mathbb{T}^2\}$ be the (Hilbert) subspace of (essentially) even-symmetric functions in $L^2(\mathbb{T}^2)$. For any $\mu, \lambda > 0$ and $k \in \mathbb{T}^2$ the Schrödinger operator $H^{es}_{\mu\lambda}$ is bounded and self-adjoint acting in $L^{2,\mathrm{es}}(\mathbb{T}^2)$ as

$$H^{\mathrm{es}}_{\mu\lambda}(k) := H_0(k) + V^{\mathrm{es}}_{\mu\lambda},$$

where

$$[H_0(k)f](p) = \varepsilon_k(p)f(p).$$

Here $\varepsilon_k(p) = \varepsilon(\frac{k}{2} + p) + \varepsilon(\frac{k}{2} - p), \quad p = (p_1, p_2) \in \mathbb{T}^2$.

Hypothesis.1. The dispersion relation ε is a real-valued even function, symmetric with respect to coordinate permutations and having a non-degenerate unique maximum at $\vec{\pi} = (\pi, \pi) \in \mathbb{T}^2$. Moreover, ε is analytic near $\vec{\pi}$.

$V^{\mathrm{es}}_{\mu,\lambda}$ acts on $L^{2,\mathrm{es}}(\mathbb{T}^2)$ as

$$(V^{\mathrm{es}}_{\mu\lambda}f)(p) = \frac{\mu}{(2\pi)^2}\int_{\mathbb{T}^2} f(q)\,\mathrm{d}q + \frac{\lambda}{(2\pi)^2}(\cos p_1 + \cos p_2)\int_{\mathbb{T}^2}(\cos q_1 + \cos q_2)f(q)\,\mathrm{d}q$$

The essential spectrum of $H_{\mu\lambda}(k)$:

$$\sigma_{\mathrm{ess}}(H^{\mathrm{es}}_{\mu\lambda}(k)) = \sigma(H_0(k)) = [\varepsilon_{\min}(k),\, \varepsilon_{\max}(k)],$$

where

$$\varepsilon_{\min}(k) =: \min_{p\in\mathbb{T}^2}\varepsilon_k(p), \qquad \varepsilon_{\max}(k) =: \max_{p\in\mathbb{T}^2}\varepsilon_k(p).$$

Let

$$B_0 := \frac{1}{4\pi^2}\int_{\mathbb{T}^2}\frac{(2+\cos q_1+\cos q_2)^2 dq}{\varepsilon_{\max}(0) - \varepsilon(q)} > 0.$$

We denote by

$\mathbb{G}_1 = \{(\mu, \lambda) \in \mathbb{R}^2_+ : \quad \mu + 4\lambda - \mu\lambda B_0 > 0\}$,
$\mathbb{G}_2 = \{(\mu, \lambda) \in \mathbb{R}^2_+ : \quad \mu + 4\lambda - \mu\lambda B_0 < 0,\ \lambda > \frac{1}{B_0}\}$

the connected components of the quadrant $\mathbb{R}^2_+ := \{(\mu, \lambda) \in \mathbb{R}^2 : \mu > 0, \lambda > 0\}$, in which the number of eigenvalues of the operator $H^{\mathrm{es}}_{\mu\lambda}(0)$, lying outside of the essential spectrum, is constant.

Theorem.

1. For any $(\mu, \lambda) \in \mathbb{G}_1$ the operator $H^{\mathrm{es}}_{\mu\lambda}(0)$ has unique eigenvalue in $(\varepsilon_{\max}(0), +\infty)$.

2. For any $(\mu, \lambda) \in \mathbb{G}_2$ the operator $H^{\mathrm{es}}_{\mu\lambda}(0)$ has exactly two eigenvalues in $(\varepsilon_{\max}(0), +\infty)$.

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EXPANDING PROPERTY AND STATISTICAL LAWS FOR P-ADIC SUBHYPERBOLIC MAPS

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Let K be a finite extension of the field $\mathbb{Q}_p$ of p-adic numbers. A rational map $\phi \in K(z)$ of degree at least 2 is called subhyperbolic if each critical point in the $\mathbb{C}_p$-Julia set of ϕ is eventually periodic. We show that subhyperbolic maps in $K(z)$ exhibit expanding property with respect to some singular metric. As an application, under a mild assumption, we establish several statistical laws for such maps in $K(z)$ with compact $\mathbb{C}_p$-Julia sets. This is a joint work with Shilei Fan, Hongming Nie and Yuefei Wang.

COMPLETENESS TYPE PROPERTIES OF MODIFICATION OF THE NIEMYTZKI PLANE

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We use the technique of Hattori [1, 2] to generate new topologies on the closed upper half plane which lie between the usual metric topology and the Niemytzki topology. We study some of their completeness type properties.

We denote an order pair by $\langle x, y\rangle$, the set of real numbers by $\mathbb{R}$, the natural numbers by $\mathbb{N}$, and the rationals by $\mathbb{Q}$.

Let $X = \{\langle x, y\rangle \in \mathbb{R}^2 : y \geq 0\}$ and $P = \{\langle x, y\rangle \in \mathbb{R}^2 : y > 0\}$, so the $x-$axis is $L = X \setminus P$. Denote the usual metric topology on X by $\mathcal{U}$ and the Niemytzki topology on X by $\mathcal{N}$.

Let X be a non-empty proper subset of the $x-$axis L. For each point of A we give as its neighborhood open set from the usual topology, and for each point of the $L \setminus A$ we give as its neighborhood open set from the Niemytzki topology. We call this topology on X the $H-$generated topology on X from $\mathcal{U}$ and $\mathcal{N}$ and denote it by $\mathcal{U}_A\mathcal{N}$. We call X with this $H-$generated topology an $H-$space and denote it by $(X,\ \mathcal{U}_A\mathcal{N})$ [3].

We received the following results.

Proposition 1. If $(X,\ \mathcal{U}_A\mathcal{N})$ is Čech-analytic, then $L \setminus A$ is countable.

Proposition 2. If $L \setminus A$ is countable, then $(X,\ \mathcal{U}_A\mathcal{N})$ is a Polish space.

Proposition 3. If $(X,\ \mathcal{U}_A\mathcal{N})$ is Čech-complete if and only if $L \setminus A$ is countable.

Proposition 4. If $(X,\ \mathcal{U}_A\mathcal{N})$ is locally compact if and only if $L \setminus A$ is closed in $\mathbb{R}$ and discrete in $(X,\ \mathcal{N})$.

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A NON-LOCAL PROBLEM FOR A TIME FRACTIONAL WAVE EQUATION ON METRIC GRAPH

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Let us consider a graph $\mathbf{\Gamma} = V \bigcup E$ be a connected metric graph [33], where $E = \{B_k\}_{k=1}^{j}$ is the set of its edges, and $V = \{\nu_k\}_{k=1}^{n}$ is the set of vertices. Let us determine the coordinates x_k on the edges of the graph using isometric mapping of these edges onto the intervals $(0, L_k)$, $k = 1, 2, ..., j$.

We will say that a vertex ν is in contact with an edge B_k if it is the end of this edge, and denote this as $B_k \sim \nu$. The number of elements of the set $\{b : b \sim \nu, b \in E\}$ is called vertex valency ν. If the valency of a vertex is equal to one, then it is called boundary. Let $\{\gamma_1, \gamma_2, ..., \gamma_{m_1}\} = \partial\Gamma \subset V$ be the boundary vertices of the graph. Further,for simplicity, we will use x instead of x_k .

On the each edges of the overdefined graph, we consider fractional differential equations

$$D_{0t}^{(\alpha)} u^{(k)}(x,t) - u_{xx}^{(k)}(x,t) = f^{(k)}(x,t), \quad (x,t) \in (B_k \times (0,T)), k = \overline{1,j} \tag{1}$$

where D_{0t}^{α} is the Riemann - Liouville fractional differential operator, $1 < \alpha < 2$ and $f^{(k)}(x,t)$ are known functions.

We will study the following problem for equation (1) on $\mathbf{\Gamma}$.

Problem. To find functions $u^{(k)}(x,t)$ in the domain $B_k \times (0,T)$, satisfy equation (1) with the following properties:

$$u^{(k)}(x,t) \in C([0,L_k] \times (0,T]), u_x^{(k)}(x,t) \in C([0,L_k) \times (0,T]),$$
$$\frac{d}{dt} I_{0t}^{(2-\alpha)} u^{(k)}(x,t) \in C\left((0,L_k) \times (0,T]\right), k = \overline{1,j},$$
$$u_{xx}^{(k)}(x,t), D_{0t}^{\alpha} u^{(k)}(x,t) \in C\left((0,L_k) \times (0,T)\right),$$

non-local and initial conditions:

$$I_{0t}^{2-\alpha} u^{(k)}(x,t)\Big|_{t=0} = M I_{0t}^{2-\alpha} u^{(k)}(x,t)\Big|_{t=T}, \ \gamma = \alpha + l\mu - \alpha\mu \ \ x \in \overline{B_k},$$
$$\frac{d}{dt} I_{0t}^{2-\alpha} u^{(k)}(x,t)\Big|_{t=0} = \varphi^{(k)}(x), \quad x \in B_k, k = \overline{1,j};$$

At branching points (i.e., at internal vertices) of the graph the solution must satisfy the following δ' *type conditions* which are at a vertex ν as follows:

$$\begin{cases} u_x^{(k)}(x,t) \text{ are continuous at } \nu, \\ \sum\limits_{B_k \sim \nu} u^{(k)}(x,t)\Big|_{\nu} = 0, \end{cases}$$

and boundary conditions

$$u^{(k)}(x,t)|_{\gamma_k} = 0, \qquad \gamma_k \in \partial\Gamma, t \in (0,T], \ k = \overline{1,m_1}.$$

where $\varphi^{(k)}(x)$ are sufficiently smooth given functions.

SOME HOMOTOPY PROPERTIES OF THE SPACE OF THIN COMPLETE LINKED SYSTEMS

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A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called linked, if any two elements of ξ intersect.

A *complete linked system* is a linked system η of closed subsets of a space X such that for any closed set $F \subset X$ the condition $(*)$ implies $F \in \eta$ (see [1]).

The *space of complete linked systems* of a space X is the set $\mathcal{N}X$ of all complete linked systems endowed with the topology whose open base is formed by sets of the form

$$\mathtt{E} = O(U_1, U_2, \dots, U_n)\langle V_1, V_2, \dots, V_s\rangle = \{\mathfrak{M} \in \mathcal{N}X : \forall i \le n\, \exists F_i \in \mathfrak{M} \text{ such that } F_i \subset U_i, \text{ and } \forall j \le s \text{ and } \forall F \in \mathfrak{M}, F \cap V_j \neq \emptyset\},$$

where $U_1, U_2, \dots, U_n$; $V_1, V_2, \dots V_s$ are nonempty open sets in X.

For a basic set $\mathtt{E} = O(U_1, U_2, \dots, U_n)\langle V_1, V_2, \dots, V_s\rangle$ in $\mathcal{N}X$ one defines the *pairwise trace* of $\mathtt{E}$ in X, denoted by $S(\mathtt{E})$, in the following way:

$$S(\mathtt{E}) := \{U_k \cap U_m : k, m \le n\} \cup \{U_i \cap V_j : i \le n, j \le s\}.$$

A complete linked system $\mathfrak{M}$ of a space X is called a *thin complete linked system* if $\mathfrak{M}$ contains at least one finite element.

We denote by $\mathcal{N}^*X$ the set of all thin complete linked systems of the space X (see [2]).

Continuous mappings $f, g : X \to Y$ are said to be *homotopic* if there is a continuous mapping $H : X \times I \to Y$ such that $H(x, 0) = f(x)$ and $H(x, 1) = g(x)$. The mapping H is called a *homotopy* between f and g and we write $f \simeq g$ (see [3]).

Theorem 1. If mappings $f, g : X \to Y$ are homotopic, then the mappings $\mathcal{N}^*f, \mathcal{N}^*g : \mathcal{N}^*X \to \mathcal{N}^*Y$ are also homotopic.

From Theorem 1 we get the following corollary.

Corollary 1. If the mapping H is homotopy between f and g, then the mapping $\mathcal{N}^*H$ is also a homotopy between $\mathcal{N}^*f$ and $\mathcal{N}^*g$.

A continuous mapping $f : X \to Y$ is said to be a *homotopy equivalence* if there exists a continuous mapping $g : Y \to X$ such that the compositions $g \circ f$ and $f \circ g$ are homotopic to the identity mappings on X and Y respectively. Two topological spaces X and Y is said to be *homotopically equivalent* (notation, $X \simeq Y$) if there exists a homotopy equivalence $f : X \to Y$ (see [3]).

Proposition 1. If the mapping $f : X \to Y$ is homotopy equivalence, then the mapping $\mathcal{N}^*f : \mathcal{N}^*X \to \mathcal{N}^*Y$ is also homotopy equivalence.

Corollary 2. If spaces X and Y are homotopically equivalent, then the spaces $\mathcal{N}^*X$ and $\mathcal{N}^*Y$ are also homotopically equivalent.

A space X is said to be *contractible* if it is homotopy equivalent to a point (see [4]).

Proposition 2. If a topological space X is contractible, then the space $\mathcal{N}^*X$ is also contractible.

By a *covariant homotopy functor* we mean a functor $\mathcal{F}$ in the category **Top** of topological spaces and their continuous mappings satisfying

(*) $\mathcal{F}$ preserves homotopy, that is, if a mapping $H(x,t)$ is a homotopy between continuous mappings $f, g : X \to Y$, then $F(H(x,t))$ is also a homotopy between mappings $F(f), F(g) : F(X) \to F(Y)$.

Theorem 2. The functor of complete linked systems $\mathcal{N}^*$ is a covariant homotopy functor.

A continuous mapping $f : [0,1] \to X$ is called a *path* in X. The point $f(0)$ is called the initial point and $f(1)$ is called the final or terminal point of this path. If $x \in X$, then one defines $e_x : I \to X$ as the constant path, i.e. $e_x(t) = x$ for any $t \in I$. A topological space X is said to be *path-connected* if given any two points x_0, x_1 in X there is a path in X from x_0 to x_1.

Theorem 3. If the mapping $f : I \to X$ is path in X from the point x_0 to the point x_1, then a mapping $\mathcal{N}^* f : I \to \mathcal{N}^* X$ defined by $\mathcal{N}^* f(t) = \{\mathfrak{M} \in \mathcal{N}^* X : f(t) \in \mathfrak{M}, \ \ \forall t \in [0,1]\}$ is a path from the point $\xi_0 = \{L_0 \in \mathfrak{M} : f(0) \in L_0, \ \ \mathfrak{M} \in \mathcal{N}^* X\}$ to the point $\xi_1 = \{L_1 \in \mathfrak{M} : f(1) \in L_1, \ \ \mathfrak{M} \in \mathcal{N}^* X\}$ in $\mathcal{N}^* X$.

Corollary 3. If a topological space X is path-connected, then the space $\mathcal{N}^* X$ is also path-connected.

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APPLICATION OF CUBIC MATRICES TO CRYPTOGRAPHY

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Three-dimensional matrices, commonly referred to as **3D matrices**, are mathematical structures that extend the concept of two-dimensional matrices (which have rows and columns) to three dimensions. The dimension of a 3D matrix is typically described as $m \times n \times p$, where m represents the number of rows (height), n the number of columns (width), and p the number of layers (depth).

A **cubic matrix** of dimension n is 3D matrix of dimension $n \times n \times n$ with entries in $\mathbb{K}$. The set of all n-dimensional cubic matrices is denoted by $\mathfrak{C}_n$. A multiplication in algebra $\mathcal{A}$ is defined by the structure constants ω_{ijk} determining the equalities

$$e_i e_j = \sum_{k=1}^{n} \omega_{ijk} e_k.$$

or, if $\mathcal{A}$ is an algebra, then the product in $\mathcal{A}$ is determined by the 3D matrix $M = (M_1|...|M_n)$ where

$$M_k = \begin{pmatrix} \omega_{11k} & \cdots & \omega_{1nk} \\ & & \\ \omega_{n1k} & & \omega_{nnk} \end{pmatrix} \quad k = 1, ..., n.$$

We consider the following multiplication to arbitrary cubic matrices

$$A = (a_{ijk})_{i,j,k=1}^{n}, \quad B = (b_{ijk})_{i,j,k=1}^{n}, \quad C = (c_{ijk})_{i,j,k=1}^{n},$$

we get that the entries of $C = AB$ can be written as

$$c_{ijr} = \sum_{k=1}^{n} a_{ijk} b_{kjr}.$$

In summary, although 3D matrices and cubic matrices are more commonly associated with Computer Graphics, they also play a significant role in the field of Cryptography. They are useful both for manipulating 3D objects in virtual environments and for data encryption.

We define a **chain of 3D matrices** in $\mathfrak{C}_n$, associated to the time-range $[a, b]$, for $a, b \in \mathbb{R}_0^+$ with $a < b$, as is a family of cubic matrices over $\mathbb{K} = \mathbb{R}$

$$\mathcal{CH}_{[a,b]}^{n,*} := \{M^{[s,t]} \in \mathfrak{C}_n : a \leq s < t \leq b\}$$

satisfying the Chapman-Kolmogorov equation, which means that

$$\mathcal{M}^{[s,t]} = \mathcal{M}^{[s,\tau]} * \mathcal{M}^{[\tau,t]},$$

for all $a \leq s < \tau < t \leq b$, where $*$ is a predefined product of cubic matrices in $\mathfrak{C}_n$.

The matrices $M^{[s,t]}$ given by

$$M^{[s,t]} = \left(\begin{array}{cc|cc} \cos(t-s) & \cos(t-s) & \sin(t-s) & -\sin(t-s) \\ -\sin(t-s) & \sin(t-s) & \cos(t-s) & \cos(t-s) \end{array} \right),$$

for $s,t \in \mathbb{R}_0^+$ with $s<t$, define a chain

$$\mathcal{CH}^{2,*}_{[0,+\infty[} := \{M^{[s,t]} : 0 \le s < t < +\infty\},$$

of 3D matrices in $\mathfrak{C}_2$ (with $\mathbb{K}=\mathbb{R}$) with respect to the product given above and this chain is called a rotation chain.

Encrypting and decrypting with the 3D rotation chain. When Amin requests to use the encryption system managed by Bakir, he inputs his data $a=\alpha e_1+\beta e_2$ into the system at a specific instant of time t. Then we obtain a double first encryption of $a=(\alpha,\beta)$ as follows:

$$(\alpha,\beta)\left(\begin{array}{cc} c^{[0,t]}_{111} & c^{[0,t]}_{112} \\ c^{[0,t]}_{211} & c^{[0,t]}_{212} \end{array} \right) = (u^a_1(t),u^a_2(t)), \quad (\alpha,\beta)\left(\begin{array}{cc} c^{[0,t]}_{121} & c^{[0,t]}_{122} \\ c^{[0,t]}_{221} & c^{[0,t]}_{222} \end{array} \right) = (v^a_1(t),v^a_2(t)),$$

where $c^{[0,t]}_{ijk}$ are elements of rotation chain.

To decrypt this double encryption of $a=\alpha e_1+\beta e_2$ (equivalently, of (α,β)), we consider the following

$$(w^a_1(s),w^a_2(s)) := (u^a_1(t),u^a_2(t))\left(\begin{array}{cc} c^{[t,s]}_{111} & c^{[t,s]}_{112} \\ c^{[t,s]}_{211} & c^{[t,s]}_{212} \end{array} \right) \text{ and}$$

$$(x^a_1(s),x^a_2(s)) := (v^a_1(t),v^a_2(t))\left(\begin{array}{cc} c^{[t,s]}_{121} & c^{[t,s]}_{122} \\ c^{[t,s]}_{221} & c^{[t,s]}_{222} \end{array} \right),$$

and we use properties of the rotation chain and some important algorithms.

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ON HOMOTOPY PROPERTIES OF THE $n-$FOLD SYMMETRIC PRODUCT OF TOPOLOGICAL SPACES

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All of our space are Hausdorff unless otherwise indicated. The symbol N stands for the set of positive integers and R stands for the set of real numbers.

Given a space X, we define its hyperspaces as the following sets:

1) $CL(X) = \{A \subset X \mid A \text{ is closed and nonempty }\}$;

2) $2^X = \{A \in CL(X) \mid A \text{ is compact }\}$;

3) $\mathcal{F}_n(X) = \{A \in 2^X \mid A \text{ has at most } n \text{ points }\}$, $n \in N$ (see [1, 2]).

$CL(X)$ is topologized by the Vietoris topology defined as the topology generated by $\beta = \{\langle U_1, ..., U_k\rangle \mid U_1, ..., U_k \text{ are open subsets of } X,\ k \in N\}$,
where $\langle U_1, ..., U_k\rangle = \{A \in CL(X) \mid A \subset \bigcup U_j \text{ and } A \cap U_j \neq \emptyset \text{ for each } j \in \{1, ..., k\}\}$.

Note that, by definition, 2^X, $\mathcal{F}_n(X)$ are subsets of $CL(X)$. Hence, they are topologized with the appropriate restriction of the Vietoris topology. $CL(X)$ is called the hyperspace of nonempty closed subsets of X. 2^X is called the hyperspace of nonempty compact subsets of X, $\mathcal{F}_n(X)$ is called the $n-$fold symmetric product of X (see [1]).

Continuous mappings $f, g : X \to Y$ are said to be *homotopic* if there is a continuous mapping $H : X \times I \to Y$ such that $H(x, 0) = f(x)$ and $H(x, 1) = g(x)$. The mapping H is called a *homotopy* between f and g and we write $f \simeq g$ (see [3]).

Theorem 1. If mappings $f, g : X \to Y$ are homotopic, then the mappings $\mathcal{F}_n f, \mathcal{F}_n g : \mathcal{F}_n X \to \mathcal{F}_n Y$ are also homotopic.

From Theorem 1 we get the following corollary.

Corollary 1. If the mapping H is homotopy between f and g, then the mapping $\mathcal{F}_n H$ is also a homotopy between $\mathcal{F}_n f$ and $\mathcal{F}_n g$.

A continuous mapping $f : X \to Y$ is said to be a *homotopy equivalence* if there exists a continuous mapping $g : Y \to X$ such that the compositions $g \circ f$ and $f \circ g$ are homotopic to the identity mappings on X and Y respectively. Two topological spaces X and Y is said to be *homotopically equivalent* (notation, $X \simeq Y$) if there exists a homotopy equivalence $f : X \to Y$ (see [3]).

Proposition 1. If the mapping $f : X \to Y$ is homotopy equivalence, then the mapping $\mathcal{F}_n f : \mathcal{F}_n X \to \mathcal{F}_n Y$ is also homotopy equivalence.

Corollary 2. If spaces X and Y are homotopically equivalent, then the spaces $\mathcal{F}_n X$ and $\mathcal{F}_n Y$ are also homotopically equivalent.

A space X is said to be *contractible* if it is homotopy equivalent to a point (see [4]).

Proposition 2. If a topological space X is contractible, then the space $\mathcal{F}_n X$ is also contractible.

By a *covariant homotopy functor* we mean a functor $\mathcal{F}$ in the category **Top** of topological spaces and their continuous mappings satisfying

(*) $\mathcal{F}$ preserves homotopy, that is, if a mapping $H(x, t)$ is a homotopy between continuous mappings $f, g : X \to Y$, then $F(H(x, t))$ is also a homotopy between mappings $F(f), F(g) : F(X) \to F(Y)$.

Theorem 2. The functor of complete linked systems $\mathcal{F}_n$ is a covariant homotopy functor.

A continuous mapping $f : [0,1] \to X$ is called a *path* in X. The point $f(0)$ is called the initial point and $f(1)$ is called the final or terminal point of this path. If $x \in X$, then one defines $e_x : I \to X$ as the constant path, i.e. $e_x(t) = x$ for any $t \in I$. A topological space X is said to be *path-connected* if given any two points x_0, x_1 in X there is a path in X from x_0 to x_1.

Theorem 3. If the mapping $f : I \to X$ is path in X from the point x_0 to the point x_1, then a mapping $\mathcal{F}_n f : I \to \mathcal{F}_n X$ defined by $\mathcal{F}_n f(t) = \bigcap\{A \in \mathcal{F}_n X : f(t) \in A,\ \forall t \in [0,1]\}$ is a path from the point $A_0 = \bigcap\{A \in \mathcal{F}_n : f(0) \in A\}$ to the point $A_1 = \bigcap\{A \in \mathcal{F}_n : f(1) \in A\}$ in $\mathcal{F}_n X$.

Corollary 3. If a topological space X is path-connected, then the space $\mathcal{F}_n X$ is also path-connected.

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DESCRIPTION OF LOCAL DERIVATIONS ON LOW-DIMENSIONAL JORDAN ALGEBRAS

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The present paper is devoted to local derivations on Jordan algebras. The Gleason-Kahane-Żelazko theorem is a basis of the concept of local derivations. This theorem is a fundamental contribution in the theory of Banach algebras.The concept of local derivations was introduced and studied by R. Kadison. He proved that each continuous local derivation from a von Neumann algebra into its dual Banach bemodule is a derivation. Later, B. Jonson extended the above result by proving that every local derivation from a C*-algebra into its Banach bimodule is a derivation. In particular, Johnson proved that local derivations of a C*-algebra A into a Banach A-bimodule X are continuous even if not assumed a priori to be so.

It is known that every local derivation on a JB-algebra is a derivation. In [2] Nuriddinov (the author of the present paper) gived the description of local derivations on five-dimensional nilpotent associative Jordan algebras. Also, in [3] Nuriddinov and Urinbayev gived the description of local derivations on four-dimensional nilpotent noncommutative Jordan algebras over $\mathbb{C}$. Arzikulov and Nuriddinov gived the description of local derivations on five-dimensional nilpotent nonassociative Jordan algebras in [4].

In the present paper I give the description of local derivations on Jordan algebras of dimension less than or equal to four.

Let $\mathcal{J}$ be a Jordan algebra of dimension five over an algebraically closed field $\mathbb{F}$ of characteristic $\neq 2$ with a basis $\{e_1, e_2, e_3, e_4\}$. Let x be an element in $\mathcal{J}$. Then $x = x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4$, for some elements x_1, x_2, x_3, x_4 in $\mathbb{F}$. Throughout of the paper let $\bar{x} = (x_1, x_2, x_3, x_4)^{Tr}$.

Definition. Recall that a linear mapping D on a Jordan algebra $\mathcal{J}$, satisfying, for each pair x, y of elements in $\mathcal{J}$, $D(xy) = D(x)y + xD(y)$, is called a derivation, and a linear mapping $\nabla : \mathcal{J} \to \mathcal{J}$ is called a local derivation if for every $x \in \mathcal{J}$ there exists a derivation $D : \mathcal{J} \to \mathcal{J}$ such that $\nabla(x) = D(x)$.

Throughout this paper, let $E_{i,j}$ be a linear mapping of the algebra $\mathcal{J}$ with basis $\{e_1, \ldots, e_4\}$, which maps the basis element e_j to e_i and the rest basis elements to zero. In the following tables a necessary and sufficient condition for a linear operator to be a derivation on all nilpotent noncommutative Jordan algebras of low-dimension is listed. The column of the tables indicates whether each local derivation of the corresponding Jordan algebra is a derivation or not, i.e., if yes, then sign "+" is put in the appropriate place of the column, if not, then sign "−" is put in this place. All notations of tables are taken from [1]

Theorem 1. Each local derivation on the algebras $\mathcal{T}_2$, $\mathcal{T}_5$, $\mathcal{T}_6$ $\mathcal{T}_7$, $\mathcal{T}_{10}$, $\mathcal{J}_2$, $\mathcal{J}_9$, $\mathcal{J}_{16}$, $\mathcal{J}_{17}$, $\mathcal{J}_{18}$, $\mathcal{J}_{25}$, $\mathcal{J}_{31}$, $\mathcal{J}_{32}$, $\mathcal{J}_{33}$,$\mathcal{J}_{36}$, $\mathcal{J}_{45}$, $\mathcal{J}_{48}$, $\mathcal{J}_{49}$,$\mathcal{J}_{52}$, $\mathcal{J}_{53}$, $\mathcal{J}_{55}$, $\mathcal{J}_{57}$, $\mathcal{J}_{58}$, $\mathcal{J}_{59}$, $\mathcal{J}_{62}$ is a derivation.

Table 1
Local derivations of three-dimensional Jordan algebras.

J	Common form of derivation	Is each local derivation a derivation?	J	Common form of derivation	Is each local derivation a derivation?
$\mathcal{T}_1$	$\{E_{2,2}+2E_{3,3}, E_{3,2}\}$	-	$\mathcal{T}_6$	$\{E_{2,1}, E_{2,2}, E_{3,3}\}$	+
$\mathcal{T}_2$	$\{E_{2,2}, E_{2,3}, E_{3,2}, E_{3,3}\}$	+	$\mathcal{T}_7$	$\{E_{2,1}, E_{2,2}, E_{2,3}, E_{3,1}, E_{3,2}, E_{3,3}\}$	+
$\mathcal{T}_3$	$\{E_{1,1}+2E_{2,2}+3E_{3,3}, E_{2,1}+2E_{3,2}, E_{3,1}\}$	-	$\mathcal{T}_8$	$\{E_{2,1}+2E_{3,2}, E_{2,2}+2E_{3,3}\}$	-
$\mathcal{T}_4$	$\{E_{1,1}+2E_{2,2}+E_{3,3}, E_{1,3}-E_{3,1}, E_{2,1}, E_{2,3}\}$	-	$\mathcal{T}_9$	$\{E_{2,1}-2E_{3,2}, E_{2,2}+2E_{3,3}\}$	-
$\mathcal{T}_5$	$\{E_{1,3}-E_{2,3}-\frac{1}{2}E_{3,1}+\frac{1}{2}E_{3,2}\}$	+	$\mathcal{T}_{10}$	$\{E_{3,1}-E_{3,2}, E_{3,3}\}$	+

Table 2
Local derivations of four-dimensional Jordan algebras.

J	Common form of derivation	Is each local derivation a derivation?	J	Common form of derivation	Is each local derivation a derivation?
$\mathcal{J}_2$	$\{E_{3,3}-E_{4,4}\}$	+	$\mathcal{J}_{52}$	$\{E_{2,2}+E_{4,4}, E_{3,3}+E_{4,4}, E_{4,1}\}$	+
$\mathcal{J}_9$	$\{E_{1,1}-E_{1,2}-E_{2,1}+E_{2,2}, E_{4,3}, E_{4,4}\}$	+	$\mathcal{J}_{53}$	$\{E_{2,2}+2E_{3,3}+ +2E_{4,4}, E_{4,1}\}$	+
$\mathcal{J}_{16}$	$\{E_{3,3}, E_{4,4}\}$	+	$\mathcal{J}_{55}$	$\{E_{2,2}+\frac{1}{2}E_{4,4}, E_{2,3}, E_{3,3}\}$	+
$\mathcal{J}_{17}$	$\{E_{3,3}, E_{4,4}\}$	+	$\mathcal{J}_{56}$	$\{E_{2,2}+2E_{3,3}, E_{3,2}, E_{4,4}\}$	-
$\mathcal{J}_{18}$	$\{E_{1,1}-E_{2,1}, E_{1,2}-E_{2,2}, E_{3,3}, E_{3,4}, E_{4,3}, E_{4,4}\}$	+	$\mathcal{J}_{57}$	$\{E_{2,2}+2E_{3,3}+E_{4,4}, E_{3,2}\}$	+
$\mathcal{J}_{25}$	$\{E_{3,3}+2E_{4,4}\}$	+	$\mathcal{J}_{58}$	$\{E_{2,2}+\frac{1}{2}E_{4,4}, , E_{3,3}E_{3,4}\}$	+
$\mathcal{J}_{31}$	$\{E_{2,2}, E_{2,3}, E_{3,2}, E_{3,3}, E_{4,4}\}$	+	$\mathcal{J}_{59}$	$\{E_{2,2}-\frac{1}{3}E_{3,4}+E_{4,4}, E_{3,3}+E_{3,4}-E_{3,4}\}$	+
$\mathcal{J}_{32}$	$\{E_{2,2}, E_{2,3}, E_{3,2}, E_{3,3}, E_{4,4}\}$	+	$\mathcal{J}_{60}$	$\{E_{2,2}+E_{4,4}, E_{3,3}+E_{4,4}, E_{4,3}\}$	-
$\mathcal{J}_{33}$	$\{E_{2,2}, E_{2,3}, E_{2,4}, E_{3,2}, E_{3,3}, E_{3,4}, E_{4,2}, E_{4,3}, E_{4,4}\}$	+	$\mathcal{J}_{61}$	$\{E_{1,1}+2E_{2,2}+3E_{3,3}+4E_{4,4}, E_{2,1}+2E_{3,2}+3E_{4,3}, E_{3,1}+2E_{4,2}, E_{4,1}\}$	-
$\mathcal{J}_{36}$	$\{E_{2,2}, E_{2,3}, E_{2,4}, E_{3,2}, E_{3,3}, E_{3,4}, E_{4,2}, E_{4,3}, E_{4,4}\}$	+	$\mathcal{J}_{62}$	$\{E_{1,1}+2E_{2,2}+3E_{3,3}+ +E_{4,4}, E_{3,1}, E_{3,4}\}$	+
$\mathcal{J}_{39}$	$\{E_{2,2}+2E_{3,3}+3E_{4,4}, E_{3,2}+2E_{4,3}, E_{4,2}\}$	-	$\mathcal{J}_{63}$	$\{E_{1,1}-\frac{1}{3}E_{1,4}+2E_{2,2}- -\frac{1}{3}E_{2,4}+\frac{8}{3}E_{3,3}-\frac{1}{3}E_{4,1}+E_{4,4}, E_{2,1}-E_{2,4}+2E_{3,2}, E_{3,1}, E_{3,4}\}$	-
$\mathcal{J}_{42}$	$\{E_{2,2}+2E_{3,3}+E_{4,4}, E_{3,2}, E_{3,4}, E_{4,2}\}$	-	$\mathcal{J}_{64}$	$\{E_{1,1}+2E_{2,2}+3E_{3,3}+ +E_{4,4}, E_{1,4}+E_{3,3}+E_{4,1}, E_{2,1}-E_{2,4}+2E_{3,2}, E_{3,1}, E_{3,4}\}$	-
$\mathcal{J}_{43}$	$\{E_{2,2}-E_{4,2}-E_{4,4}, E_{3,2}, E_{3,3}+\frac{1}{2}E_{4,2}+E_{4,4}, E_{3,4}\}$	-	$\mathcal{J}_{65}$	$\{E_{1,1}+2E_{2,2}-3E_{3,3}-2E_{4,4}, E_{2,1}-E_{2,4}+2E_{3,2}, E_{3,1}, E_{3,4}\}$	-
$\mathcal{J}_{45}$	$\{E_{2,2}+2E_{3,3}+E_{4,4}, E_{3,1}, E_{3,4}\}$	+	$\mathcal{J}_{66}$	$\{E_{1,1}+2E_{2,2}+3E_{3,3}+\frac{3}{2}E_{4,4}, E_{2,1}+2E_{3,2}, E_{2,4}-E_{4,1}, E_{3,1}, E_{3,4}\}$	-
$\mathcal{J}_{48}$	$\{E_{2,1}, E_{2,2}+\frac{1}{2}E_{4,4}, E_{3,3}, E_{3,4}\}$	+	$\mathcal{J}_{69}$	$\{E_{1,1}+2E_{2,2}+E_{4,4}, E_{2,1}, E_{2,3}, E_{3,1}+2E_{4,2}, E_{3,3}+E_{4,4}, E_{4,1}, E_{4,3}\}$	-
$\mathcal{J}_{49}$	$\{E_{2,1}, E_{2,2}-\frac{1}{2}E_{3,4}+E_{4,4}, E_{3,3}+E_{3,4}-E_{4,4}\}$	+	$\mathcal{J}_{70}$	$\{E_{1,1}+2E_{2,2}+2E_{4,4}, E_{1,3}-E_{4,1}, E_{1,4}-E_{3,1}, E_{2,1}, E_{2,3}, E_{2,4}, E_{3,3}-E_{4,4}\}$	-
$\mathcal{J}_{50}$	$\{E_{2,2}+E_{4,4}, E_{4,3}, E_{3,3}+E_{4,4}\}$	-			

Theorem 2. A linear operator on the Jordan algebras $\mathcal{T}_1$, $\mathcal{T}_3$, $\mathcal{T}_4$, $\mathcal{T}_8$, $\mathcal{T}_9$, $\mathcal{J}_{39}$, $\mathcal{J}_{42}$, $\mathcal{J}_{43}$, $\mathcal{J}_{50}$, $\mathcal{J}_{56}$, $\mathcal{J}_{60}$, $\mathcal{J}_{61}$, $\mathcal{J}_{63}$, $\mathcal{J}_{64}$, $\mathcal{J}_{65}$, $\mathcal{J}_{66}$, $\mathcal{J}_{69}$,$\mathcal{J}_{70}$ is a local derivation if and only if this linear operator belongs to the following subspace, respectively:

- $LocDer(\mathcal{T}_1) = \{E_{2,2}, E_{3,2}, E_{3,3}\}$;
- $LocDer(\mathcal{T}_3) = \{E_{1,1}, E_{2,1}, E_{2,2}, E_{3,1}, E_{3,2}, E_{3,3}\}$;
- $LocDer(\mathcal{T}_4) = \{E_{1,1}, E_{1,3}, E_{2,1}, E_{2,2}, E_{2,3}, E_{3,1}, E_{3,3}\}$;
- $LocDer(\mathcal{T}_8) = \{E_{2,1}, E_{2,2}, E_{3,2}, E_{3,3}\}$;
- $LocDer(\mathcal{T}_9) = \{E_{2,1}, E_{2,2}, E_{3,2}, E_{3,3}\}$;
- $LocDer(\mathcal{J}_{39}) = \{E_{2,2}, E_{3,2}, E_{3,3}, E_{4,2}, E_{4,3}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{42}) = \{E_{2,2}, E_{3,2}, E_{3,3}, E_{3,4}, E_{4,2}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{43}) = \{E_{2,2}, E_{3,2}, E_{3,3}, E_{3,4}, E_{4,2}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{50}) = \{E_{2,2}, E_{3,3}, E_{4,3}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{56}) = \{E_{2,2}, E_{3,2}, E_{4,3}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{61}) = \{E_{1,1}, E_{2,1}, E_{2,2}, E_{3,1}, E_{3,2}, E_{4,1}, E_{4,2}, E_{4,3}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{63}) = \{E_{1,1} - \frac{1}{3}E_{1,4} + 2E_{2,2} - \frac{1}{3}E_{2,4} - \frac{1}{3}E_{4,1} + E_{4,4}, E_{2,1} - E_{2,4}, E_{3,1}, E_{3,2}, E_{3,3}E_{3,4}\}$;
- $LocDer(\mathcal{J}_{64}) = \{E_{1,1}, E_{2,1} - E_{2,4}, E_{1,4}, E_{2,2}, E_{3,1}, E_{3,2}, E_{3,3}, E_{3,4}, E_{4,1}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{65}) = \{E_{1,1} + 2E_{2,2} - 3E_{4,1} - 2E_{4,4}, E_{3,1}, E_{3,3} + E_{4,4}, E_{4,1}\}$;
- $LocDer(\mathcal{J}_{66}) = \{E_{1,1}, E_{2,1}, E_{2,2}, E_{2,4}, E_{3,1}, E_{3,2}, E_{3,3}, E_{3,4}, E_{4,1}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{69}) = \{E_{1,1}, E_{2,1}, E_{2,2}, E_{2,3}, E_{3,1}, E_{3,2}, E_{4,1}, E_{4,2}, E_{4,3}, E_{4,4}\}$;
- $LocDer(\mathcal{J}_{70}) = \{E_{1,1}, E_{1,3}, E_{1,4}, E_{2,1}, E_{2,2}, E_{2,3}, E_{2,4}, E_{3,1}, E_{3,3}, E_{4,1}, E_{4,4}\}$.

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RELATION BETWEEN WEIGHTED GREEN FUNCTIONS AND EQUILIBRIUM STATES

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Let $f : \mathbb{C} \to \mathbb{C}$ be a polynomial of degree $d \geq 2$ i.e. $f(z) = z^d + a_{d-1}z^{d-1}... + a_0$, where $a_i \in \mathbb{C}$. Writing $f^n = f \circ ... \circ f$ for the $n-$ iterate of f, the *Julia set* is the set $J := \partial\{z \in \mathbb{C} : f^n(z) \to \infty \quad as \quad n \to \infty\}$ and the *Fatou set* F is the complement of J.

Definition 1. Let $\psi(z)$ be a bounded function on a compact set $K \subset \mathbb{C}$. Consider the class of functions

$$L(K, \psi) := \{u(z) \in psh(\mathbb{C}) : u(z)|_K \leq \psi(z), u(z) \leq c_u + ln^+|z|\}$$

and

$$V(z, K, \psi) := sup\{u(z) : u(z) \in L(K, \psi)\}$$

where $z \in \mathbb{C}$. Then $V^*(z, K, \psi) = \overline{\lim}_{\omega \to z} V(\omega, K, \psi)$ is said to be *a weighted Green function* on K with respect to $\psi(z)$.

Note that in the case $\psi(z) = 0$ the function $V^*(z, K, \psi)$ coincides with the *Green function* $V^*(z, K)$, i.e., $V^*(z, K, 0) = V^*(z, K, \psi)$. It is known that the Julia set of polynomial of degree at least 2 is compact. That's why we can take K as J. For $f-$invariant ergodic measure μ, we denote by $h_\mu(f)$ is the metric entropy of $f(z)$ and $h_t(f, J)$ is the topologic entropy of $f(z)$ on J and we know $h_t(f, J) = lnd$. If $sup\mu \subset J$ then $h_t(f, J) \geq h_\mu(f)$. We consider that the principle of variation s.t $h_t(f, J) = sup_\mu h_\mu(f)$.

Definition 2. If there exists ν such that $h_t(f, J) = h_\nu(f)$ then ν is called that *equilibrium measure.*

By [2] it is known that $dd^c V^*(z, J)\mu$ and $h_\mu(f) = lnd$. Now we put a pressure. Let ν be a measure invariant under f, $supp\nu \subset J$ and $\varphi : J \to \mathbb{R}^+$ be a continuous function then

$$P_\nu(\varphi) = \int_J \varphi d\nu + h_\nu(f)$$

is a pressure. If $\varphi \equiv 0$ then it's just entropy and $P(0) = \ln d$. Let $sup_\nu P_\nu(\varphi) = P(\varphi)$. We are interested in the case $\varphi \neq 0$. If $P(\varphi) = P_{\mu_\varphi}(\varphi)$ then μ_φ is called that *equilibrium state.*

Theorem. Let μ_φ be the equilibrium state of f with observable φ. Then there exists ψ such that $\mu_\varphi = dd^c V(z, J, \psi)$. Conversely, if $\psi(f) = d\psi + h$ for some harmonic function h then there exists φ such that $dd^c V(z, J, \psi) = \mu_\varphi$

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THE MODULAR AND CYCLIC MODULAR SPECTRUM OF PARTIAL INTEGRAL OPERATOR IN KAPLANSY-HILBERT MODULES

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Let (Ω, Σ, μ) be a measurable space having the direct sum property and let $L_0 = L_0(\Omega, \Sigma, \mu)$ be an algebra of all complex measurable functions on Ω (functions equal almost everywhere are identified). Consider a linear space X over the field of complex numbers $\mathbb{C}$. A mapping $\|\cdot\| : X \to L_0$ is called a L_0-valued norm on X, if for every $x, y \in X, \lambda \in \mathbb{C}$ the following relations hold:
1) $\|x\| \geq 0, \|x\| = 0 \Leftrightarrow x = 0$;
2) $\|\lambda x\| = |\lambda|\|x\|$;
3) $\|x + y\| \leq \|x\| + \|y\|$.

The pair $(X, \|\cdot\|)$ is called a lattice-normed space over L_0.

A lattice-normed space X is called d-decomposable, if for any $x \in X$ with $\|x\| = \lambda_1 + \lambda_2, 0 \leq \lambda_1, \lambda_2 \in L_0, \lambda_1\lambda_2 = 0$ there exist $x_1, x_2 \in X$ such that $x = x_1 + x_2$ and $\|x_1\| = \lambda_1, \|x_2\| = \lambda_2$. A net $\{x_\alpha\}_{\alpha\in A} \subset X$ (bo)-converges to an element $x \in X$, if the net $\{\|x_\alpha - x\|\}_{\alpha\in A}$ (o)-converges to zero in L_0 (note that the (o)-convergence in L_0 coincides with convergence almost everywhere). A (bo)-complete d-decomposable lattice-normed space over L_0 is called a Banach Kantorovich space over L_0. It is known that every Banach-Kantorovich space X over L_0 is a module over L_0 and $\|\lambda u\| = |\lambda|\|u\|$ for all $\lambda \in L_0, u \in X$ (see [1]).

The concept of the spectrum of an element of the Banach-Kantorovich algebra was investigated in [2], where the concept of the spectrum was introduced as a set of sp - all $\lambda \in L_0$ such that the element $\lambda e - x$ is not invertible in the Banach Kantorovich algebra, where e is the unit of the algebra. It was found that the sets $\mathrm{sp}(x)$ are not empty.

Now we consider a partial integral operators in a Kaplansky-Hilbert module $\left(L_2\left([a,b]^2\right), \langle\cdot,\cdot\rangle\right)$ over $L_2[a,b]$, and we will establish some properties of its modular spectrum.

Let $K(\omega, t, s)$ be a complex-valued measurable function on $[a,b]^3$ such that

$$\int_a^b \int_a^b |K(\omega, t, s)|^2 d\nu(t) d\nu(s)$$

exists for almost all $\omega \in [a,b]$ and this function satisfies the following conditions:
1) $K(\omega, t, s) = K_1(\omega)K_2(t, s), a \leq \omega \leq b$;
2) $K_1(\omega)$ is a continuous function on $[a,b]$;
3) $K_2(t, s)$- symmetric and positive defined function.

Assume that the partial integral operator T defined as

$$T(x)(\omega, t) = \int_a^b K(\omega, t, s)x(\omega, s)d\nu(s), \quad x \in L_2\left([a,b]^2\right) \tag{1}$$

maps the Kaplansky - Hilbert module $L_2\left([a,b]^2\right)$ into itself. For any $\omega \in \Omega$ we consider a integral operator T_ω with kernel $K_\omega(t,s)$, $T_\omega(x)(t) = \int\limits_a^b K_\omega(t,s)x(s)d\nu(s)$, $x(s) \in L_2\left([a,b]^2\right)$, where $K_\omega(t,s) = K(\omega,t,s)$. Also we consider an operator $T_2 : L_2[a,b] \to L_2[a,b]$ defined as follows:

$$T_2(x)(t) = \int\limits_a^b K_2(t,s)x(s)d\nu(s), \quad x(s) \in L_2\left([a,b]^2\right).$$

Let $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$ be eigenvalues of T_2. Set

$$\tilde{\lambda}_0(\omega) = 0, \qquad \tilde{\lambda}_n(\omega) = \frac{K_1(\omega)}{\lambda_n}, \quad n \geq 1,$$

$$m_n = \inf_\omega \tilde{\lambda}_n(\omega), \quad M_n = \sup_\omega \tilde{\lambda}_n(\omega), \quad n \geq 1.$$

Theorem. Let operator T defined as (1) has the kernel which satisfies the conditions 1)-3). Then the modular spectrum and cyclic modular spectrum of this operator has the following form respectively:

$$\mathrm{sp}(T) = \bigcup_{n\geq 1} [m_n, M_n] \cup \{0\},$$

and

$$\mathrm{spm}(T) = \mathrm{mix}\{\tilde{\lambda}_n : n \geq 0\}.$$

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ON DYNAMICS OF RATSIONAL MAPS OVER $\mathbb{Q}_2$

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We consider the following rational map the over field of p-adic numbers

$$f_{a,b,c,d}(x) = \left(\frac{ax+b}{cx+d}\right)^k, \tag{1}$$

where $a, b, c, d \in \mathbb{Z}_p$ and $k \geq 2$ integer.

One can see that (1) can be represented as follows

$$f_{\alpha,\beta}(x) = \left(\frac{\alpha x+1}{x+\beta}\right)^k, \quad \alpha, \beta \in \mathbb{Z}_p^* \tag{2}$$

if $|a|_p = |b|_p = |c|_p = |d|_p$.

For mapping $f : \mathbb{Q}_p \to \mathbb{Q}_p$ by $Fix(f)$ we denote a set of all fixed points of f i.e. $Fix(f) = \{x \in \mathbb{Q}_p : f(x) = x\}$. Let us try to describe the set of all fixed points of (2).

By strong triangle inequality we get

$$|f_{\alpha,\beta}(x)|_p = \frac{|\alpha x+1|_p^k}{|x+\beta|_p^k} = 1, \quad \forall x \in \mathbb{Q}_p \setminus \mathbb{Z}_p.$$

Again using strong triangle inequality we obtain

$$|f_{\alpha,\beta}(x)|_p = \frac{|\alpha x+1|_p^k}{|x+\beta|_p^k} = \frac{1}{|\beta|_p} = 1, \quad \forall x \in p\mathbb{Z}_p.$$

Thus we have shown that $Fix(f_{\alpha,\beta}) \subset \mathbb{Z}_p^*$.

The dynamics of $f_{\alpha,\beta}$ for $\alpha = \beta \in \mathcal{E}_p$ was studied in [1]. Recall that $\mathcal{E}_p = 1 + p\mathbb{Z}_p$. By this reason, we need to consider the cases $\alpha \neq \beta$ and $\alpha = \beta \notin \mathcal{E}_p$.

One can see that the equation $f_{\alpha,\beta}(x) = x$ for $k = 2$ equivalence to the following cubic equation

$$x^3 + (2\beta - \alpha^2)x^2 + (\beta^2 - 2\alpha)x - 1 = 0.$$

The last equation has a root $x_0 = 1$ if and only if $\alpha = \beta$. In this case other roots of cubic equations can be obtained from the following quadratic equation

$$x^2 + (2 - (\alpha-1)^2)x + 1 = 0 \tag{3}.$$

It is easy to check that equation (3) has roots if and only if $\sqrt{(\alpha+1)(\alpha-3)}$ exists in $\mathbb{Q}_p$.

Theorem 1. Let $f_{\alpha,\beta}$ be a rational map given by (2). Then $Fix(f_{\alpha,\beta}) \subset \mathbb{Z}_p^*$ for every prime p. Moreover, for $\alpha = \beta$ and $k = 2$ the following assertions hold:

1. if $\alpha \in \mathbb{Z}_3 \setminus \mathcal{E}_3$ then $f_{\alpha,\beta}$ has no fixed points.

2. if $\alpha \in \bigcup_{n=1}^{\infty}(3+5^{2n}+5^{2n+1}\mathbb{Z}_5)$ or $\alpha \in \bigcup_{n=1}^{\infty}(-1+5^{2n}+5^{2n+1}\mathbb{Z}_5)\cup(-1+4\cdot 5^{2n}+5^{2n+1}\mathbb{Z}_5)$ then $f_{\alpha,\beta}$ has one attracting fixed point and two repelling fixed points.

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ON DYNAMICS OF ISING-POTTS MAP OVER $\mathbb{Q}_2$

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By $\mathbb{Q}$, as usual, we denote the field of rational numbers. Let p be a fixed prime number, then every $x \neq 0$ can be represented as $x = p^r \frac{n}{m}$, where $r, n \in \mathbb{Z}$, m is a positive integer, and n and m are relatively prime with p: $(p, n) = 1$, $(p, m) = 1$. The p-adic norm of x is defined by

$$|x|_p = \begin{cases} p^{-r}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

We note that such a norm satisfies the so called *strong triangle inequality*

$$|x + y|_p \leq \max\{|x|_p, |y|_p\}.$$

The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$ (see [2]).

Let $x \neq 0$ be an arbitrary p-adic number, then it can be uniquely represented in the canonical form

$$x = p^{\gamma(x)}(x_0 + x_1 p + x_2 p^2 + \dots),$$

where $\gamma(x) \in \mathbb{Z}$ and the integers x_j satisfy: $x_0 > 0$, $0 \leq x_j \leq p - 1$. Moreover, one has $|x|_p = p^{-\gamma(x)}$.

We recall that $\mathbb{Z}_p = \{x \in \mathbb{Q}_p : |x|_p \leq 1\}$ and $\mathbb{Z}_p^* = \{x \in \mathbb{Q}_p : |x|_p = 1\}$ are the set of all *p-adic integers* and *p-adic units*, respectively.

Lemma 1. Let $k \geq 2$. Then for any $2 \leq n \leq k$ the following inequalities hold:

(1) if $|x|_2 = \frac{1}{2}$ then $|C_k^n x^{n-1}|_2 \leq |k|_2$;

(2) if $|x|_2 < \frac{1}{2}$ then $|C_k^n x^{n-1}|_2 < |k|_2$.

Lemma 2. Let $k \in \mathbb{N}$ and $\alpha, \beta \in \mathbb{Z}_2^*$. Then the following holds

$$|\alpha^k - \beta^k|_2 \leq |k(\alpha - \beta)|_2.$$

Corollary 1. Let $\alpha, \beta \in \mathbb{Z}_2^*$ and $k \in \mathbb{N}$. Then the following assertions hold:

(1) If k is odd then
$$|\alpha^k - \beta^k - k(\alpha - \beta)|_2 < |k(\alpha - \beta)|_2.$$

(2) If k is even then
$$\alpha^k - \beta^k = o[2k].$$

For convenience, we denote $\mathcal{E}_2 = 1 + 4\mathbb{Z}_2$ and consider the following function on $\mathbb{Q}_2$

$$f(x) = \left(\frac{ax + 1}{x + a}\right)^k, \qquad a \in \mathcal{E}_2 \setminus \{1\}. \tag{1}$$

We denote

$$\mathcal{P}(f) = \bigcup_{n\geq 1} f^{-n}(-\theta),$$

which is called *a set of all bad points of* f.

Proposition 2. Let f be a function defined by (1). Then the following assertions hold:

(i) if k is even then $f^2(x) \in \mathcal{E}_2$ for any $x \neq -\theta$;

(ii) if k is odd then $f^2(y) \in \mathcal{E}_2$ for any $y \notin \mathcal{K} \cup \{-\theta\}$, where $\mathcal{K} = \{z \in \mathbb{Z}_2^* : z+1 = o[\theta-1]\}$.

Corollary 3. Let f be a function defined by (1). Then

$$\mathcal{P}(f) \subset \begin{cases} \varnothing, & \text{if } k \text{ is even;} \\ \mathcal{K}, & \text{if } k \text{ is odd.} \end{cases}$$

Theorem 1. Let f be a function defined by (1). Then the following assertions hold:

(i) if k is even then $\lim\limits_{n\to\infty} f^n(x) = 1$ for any $x \neq -a$.

(ii) if k is odd then $\lim\limits_{n\to\infty} f^n(x) = 1$ for any $x \notin \mathcal{K} \cup \{-a\}$.

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SUBADDITIVE MEASURE ON PROJECTORS OF A REAL W*-ALGEBRA

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Let $A \subset B(H)$ be a real (or complex) W*-algebra and let $P(A)$ be the set of all projections of A. By subadditive measure in A we mean a mapping $m : P(A) \to [0, \infty]$ with the following properties:

(i) $m(0) = 0$, and $m(p) = 0$ implies $p = 0$ (faithfulness);

(ii) $p \leq q$ implies $m(p) \leq m(q)$ (monotonicity);

(iii) $p \sim q$ implies $m(p) = m(q)$,

(iv) $m(p \vee q) \leq m(p) + m(q)$ (subadditivity);

(v) $p_n \uparrow p$ implies $m(p_n) \uparrow m(p)$.

The measure m is called finite if $m(1) < \infty$. Making use of the relation $p \vee q - q \sim p - p \wedge q$, it is not hard to notice that instead of (iv) one may assume

(iV^*) $m(p \vee q) = m(p + q) \leq m(p) + m(q)$ for $p \perp q$.

The following proposition establishes the basic properties of the measure m.

Proposition. (i') $p \prec q$ implies $m(p) \leq m(q)$,

(ii') $m(\vee_{i=1}^{\infty} p_i) \leq \sum\limits_{i=1}^{\infty} m(p_i)$,

(iii') if $p(H) \subset D(a)$, $\|ap\| \leq \lambda$ then $m(e_\lambda{}^\perp) = m(q^\perp)$, where $|a| = \int_0^\infty \lambda de_\lambda$, $a \eta A$ - densely defined,

(iv') $m(p) < m(q)$ implies $p^\perp \wedge q \neq 0$;

(v') fix $q \in P(A)$; if for any $\varepsilon > 0$, there exists some $p \in P(A)$, $m(p^\perp) \leq \varepsilon$ and $p \wedge q = 0$, then $q = 0$.

(vi') fix $q_1, q_2 \in P(A)$; if for any $\varepsilon > 0$, there exists some $p \in P(A)$, $m(p^\perp) \leq \varepsilon$ and $q_1 \wedge p = q_2 \wedge p$, then $q_1 = q_2$.

Definition. A linear space $D \eta A$ is said to be m-dense in H if, for any $\varepsilon > 0$ there exists a projection $p \in P(A)$ such that $p(H) \subset D$ and $m(p^\perp) \leq \varepsilon$.

An operator a is called m-measurable if (i) $a \eta A$; (ii) $D(a)$ is m-dense; (iii) a is closed.

The main result of the paper is the following theorem.

Theorem. Let an operator $a \eta A$ be densely defined.

(i) a is m-measurable if and only if $|a|$ is m-measurable;

(ii) Let $a \geq 0$ and let $a = \int_0^\infty \lambda de_\lambda$ be the spectral decomposition of a. Then a is m-measurable if and only if $m(e_\lambda{}^\perp) \to 0$ for $\lambda \to \infty$.

(iii) a is m-measurable if and only if a^* is m-measurable.

UNIQUENESS OF SOLUTION OF THE CAUCHY PROBLEM FOR AIRY-TYPE EQUATIONS WITH VARIOUS FRACTIONAL DERIVATIVES GIVEN ON THE STAR GRAPH

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In recent years, there has been a notable rise in interest regarding the examination of initial and initial-boundary value problems associated with fractional-order equations. In this work, we delve into the initial boundary value problem concerning the time-fractional Airy-type equation on a graph with infinite bonds. Our investigation focuses on exploring the characteristics of potentials related to this equation. By leveraging these characteristics, we were able to derive solutions for the problem under consideration. Furthermore, we established the uniqueness theorem by employing an analogue of the Grönwall-Bellman inequality and an a-priory estimate.

Let us consider the graph with k incoming and m outgoing bonds. In the incoming k bonds coordinate is set from $-\infty$ to 0, and on the outgoing k bonds the coordinates are set from 0 to ∞. The bonds of the graph are denoted by $b_j, j = \overline{1, k+m}$.

We explore the equation

$$ {}_C D_{0,t}^{\alpha_j} u_j(x,t) - \frac{\partial^3}{\partial x^3} u_j(x,t) = f_j(x,t), 0 < t \le T \tag{1}$$

on the each bond b_j ($j = \overline{1, k+m}$) of the above graph. Let $t \in 0 \le t \le T$, and $x \in \overline{b_j}, j = \overline{1, k+m}$. We need to impose following initial conditions

$$ u(x,0) = u_0(x), \tag{2}$$

vertex conditions

$$ a_j D_{0t}^{\alpha_j - 1} u_j(0,t) = D_{0t}^{\alpha_1 - 1} u_1(0,t), \;\; 0 < t \le T, \;\; j = \overline{2, n}, \tag{3}$$

where $u^- = (u_1, u_2, ..., u_k)^T, u^+ = (u_{k+1}, u_{k+2}, ..., u_{k+m})^T$,

$$ \sum_{j=1}^{n} D_{0t}^{\alpha_j - 1} \tilde{u}_{j,x}^2(0,t) = 0, \;\; 0 < t \le T, \tag{4}$$

And one more vertex condition

$$ \sum_{j=1}^{n} \frac{1}{a_j} \frac{\partial^2}{\partial x^2} D_{0,t}^{\alpha_j - 1} u_j(x,t)|_{x=0} = \lambda D_{0,t}^{\alpha_1 - 1} u_1(0,t), \;\; 0 < t \le T, \;\; \lambda \ge 0. \tag{5}$$

where $a_j \neq 0$ for $j = \overline{2, k+m}$.

We put

$$V_\sigma(\Gamma) = \{(u_1(x,t), ..., u_j(x,t)) : \lim_{x \to +\infty} D_{0,t}^{\alpha_j} u_j(x,t) \exp\{-\sigma x\} = 0, \, 0 < t \le T, \, j = \overline{1, k+m}\}.$$

Theorem. The Cauchy problem (1) – (5) has at most one regular solution in $V_{\sigma_0}(\Gamma)$ for any $\sigma_0 > 0$.

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DYNAMICS OF HENON-LIKE MAPS

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Henon maps are among the most studied dynamical systems. Henon-like maps are invertible holomorphic maps, defined on some convex bounded domain of $\mathbb{C}^k$, that have an expanding behaviour in p directions and contracting behaviour in the remaining $k-p$ directions. They form a large class of dynamical systems in any dimension, that contain Henon maps in dimension 2. In this talk, we show that the sequence of their dynamical degrees is non-decreasing until the main dynamical degree, and non-increasing after that. As an application, we also show that their Green currents are woven. This is a joint work with Fabrizio Bianchi and Tien-Cuong Dinh.

FIXED POINTS OF A MAPPING RELATED TO THE ISING MODEL ON $\mathbb{Q}_p^2$

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p-adic numbers. Let $\mathbb{Q}$ be the field of rational numbers. For a fixed prime number p, every rational number $x \neq 0$ can be represented in the form $x = p^r \frac{n}{m}$, where, $r, n \in \mathbb{Z}$, m is a positive integer, and n and m are relatively prime with p. The p-adic norm of x is given by

$$|x|_p = \begin{cases} p^{-r}, \ x \neq 0, \\ 0, \quad x = 0. \end{cases}$$

This norm is non-Archimedean, i.e. it satisfies the strong triangle inequality $|x+y|_p \leq \max\{|x|_p, |y|_p\} (\leq |x|_p + |y|_p)$ for all $x, y \in \mathbb{Q}$.
The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$.

Any p-adic number $x \neq 0$ can be uniquely represented in the canonical form

$$x = p^{\gamma(x)}(x_0 + x_1 p + x_2 p^2 + ...)$$

where $\gamma(x) \in \mathbb{Z}$ and the integers x_j satisfy that $x_0 \neq 0, \ x_j \in \{0, 1, ..., p-1\}$, $j \in \mathbb{N}$. In this case $|x|_p = p^{-\gamma(x)}$.

For $a \in \mathbb{Q}_p$ and $r > 0$ we denote

$$B(a, \ r) = \{x \in \mathbb{Q}_p : \ |x - a|_p < r\}.$$

p-adic *exponential* is defined by

$$\exp_p(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

which converges for $x \in B\left(0, p^{-1/(p-1)}\right)$. Denote

$$\mathcal{E}_p = \left\{x \in \mathbb{Q}_p : |x-1|_p < p^{-1/(p-1)}\right\} = B(1, p^{-1/(p-1)}).$$

The dynamics on $\mathbb{Q}_p^2$. Consider a non-linear mapping

$$F : x = (x_1, x_2, ..., x_m) \in \mathbb{Q}_p^m \to x' = F(x) = (x_1', x_2', ..., x_m') \in \mathbb{Q}_p^m$$

given by $x_k' = F_k(x_1, x_2, ..., x_m), \ \ k = 1, 2, ..., m$, where $F_i : \mathbb{Q}_p^m \to \mathbb{Q}_p^m, \ \ i = 1, 2, ..., m$ is a continuously differentiable single-valued function. If $F(x^\star) = x^\star$ then $x^\star \in \mathbb{Q}_p^m$ is called a *fixed point* of the mapping F. The eigenvalues of the Jacobian matrix J_G determine the local behavior of the non-linear dynamical system (see [2]).

Definition.[2] A fixed point $x^\star$ of a mapping F is called hyperbolic point of its Jacobian J_G at $x^\star$ has no eigenvalues on the unit circle. The hyperbolic point is called

- attracting point if all the eigenvalues of the Jacobi matrix $J_G(x^\star)$ are less than 1 in absolute value;

- repelling point if all the eigenvalues of the Jacobi matrix $J_G(x^\star)$ are greater than 1 in absolute value;
- a saddle point otherwise.

The problem of finding H_A-weakly periodic p-adic generalized Gibbs for the Ising model on the binary tree brings to consider the following mapping:

$$W_\theta : \mathbf{h} = (x, y) \in \mathbb{Q}_p^2 \to \mathbf{h}' = \left(\frac{\theta + y}{\theta y + 1} \cdot \frac{\theta x + 1}{x + \theta}; \left(\frac{\theta x + 1}{x + \theta} \right)^2 \right) \in \mathbb{Q}_p^2, \tag{1}$$

We study the dynamics of W_θ. Firstly, we find fixed points of (1)

$$(x_1, y_1) = (1, 1), \ \ (x_2, y_2) = (-1, 1),$$

$$(x_3, y_3) = \left(\frac{\theta^2 - 2\theta - 1 + (\theta - 1)\sqrt{(\theta - 3)(\theta + 1)}}{2}, \frac{\theta^2 - 2\theta - 1 + (\theta - 1)\sqrt{(\theta - 3)(\theta + 1)}}{2} \right),$$

$$(x_4, y_4) = \left(\frac{\theta^2 - 2\theta - 1 - (\theta - 1)\sqrt{(\theta - 3)(\theta + 1)}}{2}, \frac{\theta^2 - 2\theta - 1 - (\theta - 1)\sqrt{(\theta - 3)(\theta + 1)}}{2} \right).$$

Stability of these fixed points are as follows:

Theorem. For the fixed points of W_θ the following statements are true:
1) The fixed point (x_1, y_1) is attracting fixed point;
2) The fixed point (x_2, y_2) is saddle;
3) If $p \equiv 1 (\mathrm{mod}\, 4)$ then fixed points (x_3, y_3) and (x_4, y_4) are repelling fixed point.

Remark. If $p \not\equiv 1 (\mathrm{mod}\, 4)$ then $(x_3, y_3) \notin \mathbb{Q}_p^2$ and $(x_4, y_4) \notin \mathbb{Q}_p^2$.

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GROUND STATES AND GIBBS MEASURES FOR THE SOS MODEL WITH AN EXTERNAL FIELD AND COUNTABLE SET OF SPIN VALUES ON A CAYLEY TREE OF ORDER TWO

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At low temperatures, a periodic ground state corresponds to a periodic Gibbs measure. Therefore, the problem of description of periodic ground states naturally arises (see [1]).

Let $\Gamma^k = (V, L)$ be the Cayley tree, where each vertex has $k+1$ neighbors with V being the set of vertices and L the set of edges (see [1]). For an arbitrary vertex $x_0 \in V$, we put $W_n = \{x \in V| \ \ d(x, x^0) = n\}, V_n = \{x \in V| \ d(x, x^0) \leq n\}$, where $d(x, y)$ is the distance between x and y in the Cayley tree, i.e. the number of edges of the path between x and y. Two vertices x and y are called *nearest neighbors* if there exists an edge $l \in L$ connecting them and we denote $l = \langle x, y\rangle$. Let $S(x) = \{y \in W_{n+1} : d(y, x) = 1\}$ be the set of direct successors of x.

It is well-known that there exists a one-to-one correspondence between the set V of vertices of the Cayley tree of order $k \geq 1$ and the group G_k of the free products of $k+1$ cyclic groups of second order with generators $a_1, a_2, a_3, ..., a_{k+1}$ (see [1]). Assume that spin takes values in the set of integers $\mathbb{Z}$. By a configuration σ on V we mean a function taking $\sigma : V \to \mathbb{Z}$. The set of all configurations coincides with the set $\Omega = \mathbb{Z}^V$.

Denote

$$\Omega' = \{\sigma : \ \langle x, y\rangle \in L, \ \ | \ \sigma(x) - \sigma(y) \ | \leq P\},$$

where P is a finite natural number. In this work we study translation-invariant ground states and Gibbs measures of the model with spin values in the set $\mathbb{Z}$ on the Cayley tree of order two.

Definition 1. A configuration $\sigma(x)$ is said to be G_k^*-periodic if $\sigma(x) = \sigma_i$ for all $x \in G_k$ with $x \in H_i$. A G_k-periodic configuration is said to be translation-invariant.

The SOS model with an external field is given by Hamiltonian:

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} |\sigma(x) - \sigma(y)| + \alpha \sum_{x \in V} \sigma(x) \bmod P, \tag{1}$$

where $J \in \mathbb{R}$ is coupling constant, $\alpha \in \mathbb{R}$ is external field and $\sigma \in \Omega$.

By the *restricted configuration* σ_b we mean the restriction of a configuration σ to a unit ball $b \in M$. Let c_b denote the center of a unit ball b. The energy of a configuration σ_b on b is defined by the formula

$$U(\sigma_b) = -\frac{1}{2} J \sum_{\substack{\langle x,y\rangle: \\ x,y \in b}} |\sigma(x) - \sigma(y)| + \alpha\sigma(c_b) \bmod P. \tag{2}$$

Therefore, we consider $\Omega' = \{\sigma(x) : \ \langle x, y\rangle \in L, \ | \ \sigma(x) - \sigma(y) \ | \leq P\}$, where P is a finite natural number and on the set Ω' we study the translation-invariant ground states

of model (1) with spin values in the set $\mathbb{Z}$ on the Cayley tree of order two. It is easy to see that $U(\sigma_b) \in \{U_{i,j} : \ i = 0, 1, 2, \dots, 3P,\ j = 0, 1, 2, \dots, P-1\}$ for all σ_b, where

$$U_{0,0} = 0,\ U_{0,1} = \alpha,\ U_{0,2} = 2\alpha, \dots, U_{0,P-1} = (P-1)\alpha,$$

$$U_{1,0} = -\frac{1}{2}J,\ U_{1,1} = -\frac{1}{2}J + \alpha,\ U_{1,2} = -\frac{1}{2}J + 2\alpha, \dots, U_{1,P-1} = -\frac{1}{2}J + (P-1)\alpha,$$

$$\dots,$$

$$U_{3P-1,0} = -\frac{3P-1}{2}J,\ U_{3P-1,1} = -\frac{3P-1}{2}J + \alpha, \dots, U_{3P-1,P-1} = -\frac{3P-1}{2}J + (P-1)\alpha.$$

$$U_{3P,0} = -\frac{3P}{2}J,\ U_{3P,1} = -\frac{3P}{2}J + \alpha,\ U_{3P,2} = -\frac{3P}{2}J + 2\alpha, \dots, U_{3P,P-1} = -\frac{3P}{2}J + (P-1)\alpha.$$

Definition 2. A configuration $\varphi \in \Omega'$ is called a *ground state* for the Hamiltonian H, if $U(\varphi_b) = \min\{U_{i,j} : \ i = 0, 1, 2, \dots, 3P,\ j = 0, 1, 2 \dots, P-1\}$, for any $b \in M$.

For $i = 0, 1, 2, \dots, 3P,\ j = 0, 1, 2, \dots, P-1$. We put

$$A_{i,j} = \{(J, \alpha) \in \mathbb{R}^2 : U_{i,j} = \min\{U_{0,0},\ U_{0,1},\ U_{0,2}, ..., U_{3P,P-1}\}.$$

Quite cumbersome, but not difficult calculations show that:

$$A_{0,0} = \{(J, \alpha) \in \mathbb{R}^2 : J \le 0, \alpha \ge 0\},\ A_{i,0} = \{(J, \alpha) \in \mathbb{R}^2 : J = 0, \alpha \ge 0\},\ i = 1, 2, \dots, 3P-1,$$

$$A_{3P,0} = \{(J, \alpha) \in \mathbb{R}^2 : J \ge 0, \alpha \ge 0\},\ A_{0,j} = \{(J, \alpha) \in \mathbb{R}^2 : J \le 0, \alpha = 0\},\ j = 1, 2, \dots, P-2,$$

$$A_{i,j} = \{(J, \alpha) \in \mathbb{R}^2 : J = 0, \alpha = 0\},\ i = 1, 2, \dots, 3P-1,\ j = 1, 2, \dots, P-2,$$

$$A_{3P,j} = \{(J, \alpha) \in \mathbb{R}^2 : J \ge 0, \alpha = 0\},\ j = 1, 2, \dots, P-2,\ A_{0,P-1} = \{(J, \alpha) \in \mathbb{R}^2 : J \le 0, \alpha \le 0\},$$

$$A_{i,P-1} = \{(J, \alpha) \in \mathbb{R}^2 : J = 0, \alpha \le 0\},\ i = 1, 2, \dots, 3P-1,$$

$$A_{3P,P-1} = \{(J, \alpha) \in \mathbb{R}^2 : J \ge 0, \alpha \le 0\}, \quad \text{and} \quad \bigcup_{i,j} A_{i,j} = \mathbb{R}^2.$$

Theorem 1. Let $\alpha \neq 0$. For the Hamiltonian (1) the following assertions hold:

i) If $(J, \alpha) \in A_{0,0}$, then there exists at least a countable set of ground states, which is given by $\{\sigma(x) \equiv 0 \pmod P),\ \forall x \in V\}$.

ii) If $(J, \alpha) \in A_{0,P-1}$, then there exists at least a countable set of ground states, which is given by $\{\sigma(x) \equiv P-1 \pmod P),\ \forall x \in V\}$.

For simplicity, we consider only case $(J, \alpha) \in \widetilde{A}_{0,0}$, where

$$\widetilde{A}_{0,0} = \{(J, \alpha) \in \mathbb{R}^2 : J < 0, \quad \alpha > 0\}.$$

In this case by Theorem 1 we have $GS(H) = \{\sigma \equiv zP,\ z \in \mathbb{Z}\}$. Using Lemmas 13 and 15 by very similar argument of [2] one can prove

Theorem 2. If $(J, \alpha) \in \widetilde{A}_{0,0}$ then for all sufficiently large β ($\beta = \frac{1}{T}$, $T > 0$ – temperature) there exists a countable set of Gibbs measures for the model (1) on Cayley tree of order two.

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CONSTRUCTIVE GIBBS MEASURES FOR THE ISING MODEL ON THE CAYLEY TREE

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Let $\tau^k = (V, L), k \geq 1$, be the Cayley tree of order k, i.e. an infinite graph such that every vertex of which is incident to exactly $k+1$ edges. Here V is the set of all vertices, L is the set of all edges of the tree τ^k. It is known that τ^k can be represented as a group G_k, which is the free product of $k+1$ cyclic groups of the second order [1], [3].

For an arbitrary point $x^0 \in V$ we set

$$W_n = \{x \in V | d(x^0, x) = n\}, V_n = \bigcup_{m=0}^{n} W_m,$$

$$L_n = \{\langle x, y\rangle \in L | x, y \in V_n\},$$

$$S(x) = \{y \in W_{n+1} : d(x, y) = 1\}, \quad x \in W_n$$

where $d(x, y)$ is the distance between the vertices x and y on the Cayley tree, i.e. the number of edges in the shortest path joining the vertices x and y.

Let $\Phi = \{-1, 1\}$ and let $\sigma \in \Omega = \Phi^V$ be a configuration, i.e. $\sigma = \{\sigma(x) \in \Phi : x \in V\}$. Let $A \subset V$. We let Ω_A denote the space of configurations defined on the set A and taking values in Φ.

We consider the Hamiltonian of the Ising model:

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} \sigma(x)\sigma(y),$$

where $J \in R$, $\sigma(x) \in \Phi$ and $\langle x, y\rangle$ are nearest neighbors.

We are going to describe new Gibbs measures of the Ising model on the semi-infinite Cayley tree of order $k = 2$. We define the set of quantities $h = \{h_x, x \in V\}$ as follows **(A)** If on the vertex of the x, we have $h_x = h_1$, then at each vertex of $S(x)$ we put the value h_2. If on the vertex of the x, we have $h_x = h_2$, then at one vertex of $S(x)$ we put the value h_1, and on another vertex of $S(x)$ we put one of the values of 0. If on the vertex of the x, we have $h_x = 0$, then at each vertex from $S(x)$ we put the value 0.

It is easy to see that the boundary conditions defined by the **(A)** are compatible iff the following system of equations is satisfied:

$$\begin{cases} h_1 = 2f(h_2, \theta), \\ h_2 = f(h_1, \theta), \end{cases} \tag{1}$$

where $f(h, \theta) = \operatorname{arctanh}(\theta \tanh h) = \frac{1}{2} \ln \frac{(1+\theta)e^{2h}+(1-\theta)}{(1-\theta)e^{2h}+(1+\theta)}$, $\alpha = \frac{1-\theta}{1+\theta}$, $z_i = e^{2h_i}$, $i = 1, 2$. It is easy to check that $\alpha \in (0, 1) \cup (1, +\infty)$. It is clear that $h_1 = h_2 = 0$ is a solution of (1).

By the last notations, (1) yields the following system of equations:

$$\begin{cases} z_1 = \left(\frac{z_2+\alpha}{\alpha z_2+1}\right)^2, \\ z_2 = \frac{z_1+\alpha}{\alpha z_1+1}. \end{cases}$$

Theorem. For the Gibbs measures which are defined by rule **(A)** for the Ising model on the Cayley tree of order two the following statements hold:

(i) $\alpha \in \left(0, 3-2\sqrt{2}\right) \cup \left(3+2\sqrt{2}, +\infty\right)$, then there are three Gibbs measures, which two of them are non-translation-invariant Gibbs measures;

(ii) if $\alpha \in (3-2\sqrt{2}, 1) \cup (1, 3+2\sqrt{2}]$, then there exists a unique translation-invariant Gibbs measure.

Remark. Let $\alpha \in (3-2\sqrt{2}, 1) \cup (1, 3+2\sqrt{2}]$. Then the translation-invariant measure corresponding to $(z_1, z_2) = (1, 1)$ is studied in [3–6].

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TRANSLATION-INVARIANT GIBBS MEASURES FOR THE POTTS-SOS MODEL WITH AN EXTERNAL FIELD

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The Cayley tree Γ^k (see, e.g., [1,2]) of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, from each vertex of which exactly $k+1$ edges issue. Let $\Gamma^k = (V, L, i)$, where V is the set of vertices of Γ^k, L is the set of edges of Γ^k and i is the incidence function associating each edge $l \in L$ with its endpoints $x, y \in V$. It is known (see [1]) that there exists a one-to-one correspondence between the set V of vertices of the Cayley tree of order $k \geq 1$ and the group G_k of the free products of $k+1$ cyclic groups $\{e, a_i\}$, $i = 1, ..., k+1$ of the second order (i.e. $a_i^2 = e$, $a_i^{-1} = a_i$) with generators $a_1, a_2, ..., a_{k+1}$.

We consider model, where the spin takes values in the set $\Phi = \{0, 1, 2, ..., m\}$, $m \geq 1$.

The Hamiltonian of the Potts-SOS model with an external field has the form

$$H(\sigma) = -J_p \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)} - J \sum_{\langle x,y\rangle \in L} \mid \sigma(x) - \sigma(y) \mid + \alpha \sum_{x \in V} \sigma(x) \tag{1}$$

where $J_p, J \in \mathbb{R}$ are nonzero coupling constants, $\alpha \in \mathbb{R}$ is an external field, $\sigma \in \Phi^V$.

Remark. Note that model (1) coincides with the Potts-SOS model, if $\alpha = 0$. Translation-invariant Gibbs measures for the Potts-SOS model were studied in [3,4].

Let $h : x \mapsto h_x = (h_{0,x}, h_{1,x}, ..., h_{m,x}) \in \mathbb{R}^{m+1}$ be a real vector-valued function of $x \in V \setminus \{x^0\}$. We consider the probability distributions $\mu^{(n)}$ on Φ^{V_n} for given $n = 1, 2, ...$ defined by

$$\mu^{(n)}(\sigma_n) = Z_n^{-1} \exp\big(- \beta H(\sigma_n) + \sum_{x \in W_n} h_{\sigma(x),x}\big), \tag{2}$$

where $\sigma_n \in \Phi^{V_n}$ and the partition function Z_n can be expressed as follows:

$$Z_n(\widetilde{\sigma}_n) = \sum_{\widetilde{\sigma}_n \in \Phi^{V_n}} \exp\big(- \beta H(\widetilde{\sigma}_n) + \sum_{x \in W_n} h_{\widetilde{\sigma}(x),x}\big). \tag{3}$$

We say that the probability distributions $\mu^{(n)}$ are compatible if $\forall n \geq 2$ and $\sigma_{n-1} \in \Phi^{V_{n-1}}$:

$$\sum_{\omega_n \in \Phi^{W_n}} \mu^{(n)}\big(\sigma_{n-1} \vee \omega_n\big) = \mu^{(n-1)}\big(\sigma_{n-1}\big), \tag{4}$$

here $\big(\sigma_{n-1} \vee \omega_n\big) \in \Phi^{V_n}$ is the concatenation of σ_{n-1} and ω_n.

If for probability distributions $\mu^{(n)}$ on Φ^{V_n} the equality (4) holds, then then by the Kolmogorov extension theorem there is a unique measure μ on Φ^V such that, $\forall n$ and $\sigma_n \in \Phi^{V_n}$,

$$\mu\big(\{\sigma \mid \sigma_{V_n} = \sigma_n\}\big) = \mu^{(n)}(\sigma_n).$$

The measure μ is called splitting Gibbs measure (SGM) corresponding to Hamiltonian H and function $h : x \mapsto h_x, x \neq x^0$.

The next theorem expresses the requirement on function h that the distributions $\mu^{(n)}(\sigma_n)$ hold the compatibility conditions.

Theorem 1. Defined as in (2) the probability distributions $\mu^{(n)}(\sigma_n), n = 1, 2, ...,$ satisfy the compatibility condition if and only if

$$h_x^* = \sum_{y \in S(x)} F\big(h_y^*, m, \rho, \theta, \gamma\big) \tag{5}$$

is satisfied for any $x \in V \setminus \{x^0\}$, where

$$\rho = \exp(J_p\beta), \quad \theta = \exp(J\beta), \quad \gamma = \exp(-\alpha\beta)$$

and also $\beta = 1/T$ is the inverse temperature. Here h_x^* represents the vector

$$(h_{0,x} - h_{m,x}, h_{1,x} - h_{m,x}, ..., h_{m-1,x} - h_{m,x})$$

and the vector function $F(., m, \rho, \theta, \gamma) : \mathbb{R}^m \to \mathbb{R}^m$ is defined as follows

$$F(h, m, \rho, \theta, \gamma) = \big(F_0(h, m, \rho, \theta, \gamma), F_1(h, m, \rho, \theta, \gamma), ..., F_{m-1}(h, m, \rho, \theta, \gamma)\big)$$

where

$$F_i(h, m, \rho, \theta, \gamma) = \ln \frac{\sum_{j=0}^{m-1} \rho^{\delta_{ij}} \theta^{|i-j|} \gamma^j \exp(h_j) + \rho^{\delta_{mi}} \theta^{m-i} \gamma^m}{\sum_{j=0}^{m-1} \rho^{\delta_{mj}} \theta^{m-j} \gamma^j \exp(h_j) + \rho \gamma^m},$$

$h = (h_0, h_1, ..., h_{m-1}), i = 0, 1, 2, ..., m-1.$

Namely, for any collection of functions satisfying the functional equation (5) there exists a unique splitting Gibbs measure, the correspondence being one-to-one.

Theorem 2. Let $m = 2$, $\rho = \theta^2$. Let N_{TISGM} be the number of translation-invariant SGMs for the Potts-SOS model with an external field on the Cayley tree with order two. In this case, the following statement is appropriate:

$$N_{TISGM} = \begin{cases} 3, & \text{if } Q(\theta, \gamma) < 0, \\ 2, & \text{if } Q(\theta, \gamma) = 0, \\ 1, & \text{if } Q(\theta, \gamma) > 0, \end{cases}$$

where

$$Q(\theta, \gamma) = \frac{1}{27}\Big(-\frac{1}{3}\gamma\theta^2 + \gamma^2\theta + \theta\Big)^3 + \frac{1}{4}\Big(-\frac{2}{27}\gamma\theta^3 + \frac{1}{3}\gamma^2\theta^2 + \frac{1}{3}\theta^2 - \gamma^2 - 1\Big)^2.$$

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p-ADIC QUASI-GIBBS MEASURES FOR THE POTTS MODEL WITH AN EXTERNAL FIELD ON A CAYLEY TREE OF ORDER TWO

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We study translation-invariant p-adic quasi Gibbs measures for Potts model with external field on a Cayley tree of order two.

The Cayley tree Γ^k (see, e.g., [1,2]) of order $k \geq 1$ is an infinite tree, i.e. a graph without cycles, from each vertex of which exactly $k+1$ edges issue. Let $\Gamma^k = (V, L, i)$, where V is the set of vertices of Γ^k, L is the set of edges of Γ^k and i is the incidence function associating each edge $l \in L$ with its endpoints x, y originate from V. It is known (see [1]) that there exists a one-to-one correspondence between the set V of vertices of the Cayley tree of order $k \geq 1$ and the group G_k of the free products of $k+1$ cyclic groups $\{e, a_i\}$, $i = 1, ..., k+1$ of the second order (i.e. $a_i^2 = e$, $a_i^{-1} = a_i$) with generators $a_1, a_2, ..., a_{k+1}$.

Let us set

$$W_n = \{x \in V : \ d(x, x^0) = n\}, \quad V_n = \bigcup_{m=0}^{n} W_m,$$

$$L_n = \{\langle x, y\rangle \in L : \ x, y \in V_n\}.$$

We consider p-adic Potts model with external field on a Cayley tree, where the spin takes values in the set $\Phi := \{1, 2, ..., q\}$.

The (formal) Hamiltonian of p-adic Potts model with a nonzero external field is defined by

$$H(\sigma) = J \sum_{\langle x, y\rangle \in L} \delta_{\sigma(x)\sigma(y)} + \alpha \sum_{x \in V} \delta_{q\sigma(x)}, \tag{1}$$

where $J, \alpha \in B(0, p^{-1/(p-1)})$ are parameters, $\langle x, y\rangle$ stands for nearest neighbor vertices and δ_{ij} is the Kronecker symbol, i.e.

$$\delta_{ij} = \begin{cases} 0, & \text{if } i \neq j, \\ 1, & \text{if } i = j. \end{cases}$$

We remark that in this case $\alpha = 0$, the translation-invariant p-adic Gibbs measures for the Potts model was studied in [3]. Assume that $h : V\backslash\{x^{(0)}\} \to \mathbb{Q}_p^{\Phi}$ is a mapping, i.e. $h_x = (h_{1,x}, h_{2,x}, ..., h_{q,x})$, where $h_{i,x} \in \mathbb{Q}_p$ $(i \in \Phi)$ and $x \in V\backslash\{x^{(0)}\}$. Given $n \in \mathbb{N}$, we consider a p-adic probability measure $\mu_{h,\sigma}^{(n)}$ on Ω_{V_n} defined by

$$\mu_h^{(n)}(\sigma) = \frac{1}{Z_n^{(h)}} \exp_p\{H_n(\sigma)\} \prod_{x \in W_n} h_{\sigma(x),x}, \tag{2}$$

here, $\sigma \in \Omega_{V_n}$, and $Z_n^{(h)}$ is the corresponding normalizing factor

$$Z_n^{(h)} = \sum_{\sigma \in \Omega_{V_n}} \exp_p\{H_n(\sigma)\} \prod_{x \in W_n} h_{\sigma(x),x}. \tag{3}$$

Finding translation-invariant p-adic quasi Gibbs measure for the Potts model with external field is equivalent to find a solution of the following system of equations [1]

$$\widehat{h}_i = \left(\frac{(\theta-1)\widehat{h}_i + \sum\limits_{j=1}^{q-1} \widehat{h}_j + \eta}{\sum\limits_{j=1}^{q-1} \widehat{h}_j + \theta\eta} \right)^k, \quad i = 1, 2, ..., q-1. \tag{4}$$

Where $\theta = \exp_p(J)$, $\eta = \exp_p(\alpha)$ and below a vector $\widehat{\mathbf{h}} = \left(\widehat{h}_1, \widehat{h}_2, ..., \widehat{h}_{q-1}\right) \in \mathbb{Q}_p^{q-1}$ is defined by a vector $\mathbf{h} = (h_1, h_1, ..., h_q) \in \mathbb{Q}_p^q$ as follows

$$\widehat{h}_i = \frac{h_i}{h_q}, \ i = 1, 2, ..., q-1.$$

We introduce the notation

$$I = \{\mathbf{h} = \{h_1, h_2, ..., h_{q-1}\} \in \mathbb{Q}_p^{q-1} | h_1 = h_2 = ... = h_{q-1}\}.$$

Let $\widehat{\mathbf{h}} = (\widehat{h}_1, \widehat{h}_2 ..., \widehat{h}_{q-1}) \in I$, we set $\widehat{h} = \widehat{h}_1 = \widehat{h}_2 = ... = \widehat{h}_{q-1}$. Then the system of equations (4) reduce to following

$$\widehat{h} = \left(\frac{(\theta+q-2)\widehat{h} + \eta}{(q-1)\widehat{h} + \theta\eta} \right)^k. \tag{5}$$

Theorem 1. *Let $q \notin \mathcal{E}_p$ and $\mathcal{N}_{TP}$ be the number of translation-invariant p-adic quasi-Gibbs measures for the Potts model with external field on the Cayley tree of order two. Then following statements are true:*

$$\mathcal{N}_{TP} = \begin{cases} 3, & \text{if } \ p > 3, p \nmid q, (1-q)^{\frac{p-1}{2}} \equiv 1 (\mathrm{mod}\, p), \\ 1, & \text{if } \ p = 3, \ \ 3 \nmid q, \\ 1, & \text{if } \ p > 3, p \nmid q, \ \ (1-q)^{\frac{p-1}{2}} \not\equiv 1 (\mathrm{mod}\, p), \\ 0, & \text{if } \ p \geq 3, p \mid q. \end{cases}$$

where $\mathcal{E}_p = \{x \in \mathbb{Q}_p : |x-1|_p < p^{\frac{-1}{p-1}}\}$.

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H_A-PERIODIC GROUND STATES FOR THE VANNIMENUS MODEL ON THE CAYLEY TREE OF ORDER TWO

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The Cayley tree $\Gamma^k = (V, L)$ (see, e.g., [1,2]) of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, from each vertex of which exactly $k+1$ edges issue. It is known (see [1]) that there exists a one-to-one correspondence between the set V of vertices of the Cayley tree of order $k \geq 1$ and the group G_k of the free products of $k+1$ cyclic groups $\{e, a_i\}$, $i = 1, ..., k+1$ of the second order (i.e. $a_i^2 = e$, $a_i^{-1} = a_i$) with generators $a_1, a_2, ..., a_{k+1}$. Denote $S(x)$ the set of direct successors of $x \in G_k$. Let $S_1(x)$ be the set of all nearest neighboring vertices of $x \in G_k$. Consider model where the spin takes values in the set $\Phi = \{-1, +1\}$. For $A \subseteq V$ a spin *configuration* σ_A on A is defined as a function $x \in A \mapsto \sigma_A(x) \in \Phi$; the set of all configurations coincides with $\Omega_A = \Phi^A$. Denote $\Omega = \Omega_V$ and $\sigma = \sigma_V$.

Definition 1. A configuration $\sigma \in \Omega$ is called G_k^* -periodic, if $\sigma(yx) = \sigma(x)$ for any $x \in G_k$ and $y \in G_k^* \subset G_k$.

For a given periodic configuration the index of the subgroup is called the *period of the configuration.*

Definition 2. A configuration that is invariant with respect to all shifts is called *translation-invariant.*

The Vannimenus model (see [2]) is defined by the following Hamiltonian

$$H(\sigma) = -J_p \sum_{\rangle x,y \langle} \sigma(x)\sigma(y) - J \sum_{\langle x,y\rangle \in L} \sigma(x)\sigma(y), \tag{1}$$

where $J_p, J \in \mathbb{R}$, and $\sigma \in \Omega$.

Remark 1. Recall that model (1) coincides with the Ising model under the condition $J_p = 0$ (see, e.g., [2]).

Let M be the set of all unit balls with vertices in V and $S_1(x)$ be the set of all nearest neighboring vertices of $x \in V$.

We call the restriction of a configuration σ to the ball $b \in M$ a *bounded configuration* σ_b. The energy of configuration σ_b on b is defined by the formula

$$U(\sigma_b) = -\frac{J}{2} \sum_{x \in S_1(c_b)} \sigma(x)\sigma(c_b) - J_p \sum_{\langle x,y\rangle : x,y \in b} \sigma(x)\sigma(y),$$

where $(J, J_p) \in \mathbb{R}^2$ and c_b is the center of the unit ball b.

The Hamiltonian (1) can be written as

$$H(\sigma) = \sum_{b \in M} U(\sigma_b).$$

Let $A \subset \{1, 2, 3\}$, $H_A = \{x \in G_k : \sum_{j \in A} w_j(x) - \text{even}\}$, where $w_j(x)$ is the number of letters a_j in the word x. Note that H_A is a normal subgroup of index two (see [1]). Let

$G_k/H_A = \{H_A, G_k \setminus H_A\}$ be the quotient group. In this work we study all H_A-periodic ground states for the Vannimenus model on the Cayley tree of order two.

Lemma. *Let $k = 2$. Then for each configuration φ_b, we have the following*

$$U(\varphi_b) \in \{U_i : i = 1, 2, ..., 6\},$$

where

$$U_1 = -2J_p - \frac{3J}{2}; U_2 = -2J_p + \frac{3J}{2}; U_3 = -\frac{J}{2};$$
$$U_4 = \frac{J}{2}; U_5 = 2J_p - \frac{J}{2}; U_6 = 2J_p + \frac{J}{2}.$$

Definition 3. A configuration φ is called a ground state for the Hamiltonian (1), if

$$U(\varphi_b) = \min\{U_i : i = 1, 2, ..., 6\}$$

for any $b \in M$.

We denote $A_\xi = \{(J, \alpha) \in \mathbb{R}^2 : U_\xi = \min\{U_i : i = 1, 2, ..., 6\}\}$.

Calculations show that:

$$A_1 = \{(J, J_p) \in \mathbb{R}^2 : J \geq 0, J \geq -4J_p\};$$
$$A_2 = \{(J, J_p) \in \mathbb{R}^2 : J \leq 0, J \leq 4J_p\};$$
$$A_3 = A_4 = \{(J, \alpha) \in \mathbb{R}^2 : J = 0, J_p = 0\};$$
$$A_5 = \{(J, J_p) \in \mathbb{R}^2 : 0 \leq J \leq -4J_p, J_p \leq 0\};$$
$$A_6 = \{(J, J_p) \in \mathbb{R}^2 : 4J_p \leq J \leq 0, J_p \leq 0\}.$$

Theorem 1. *Let $k = 2$. If $(J, \alpha) \in A_1$, then $\sigma(x) = -1, \forall x \in V$ and $\sigma(x) = +1, \forall x \in V$ are translation-invariant ground states.*

Theorem 2. *Let $k = 2$ and $\mid A \mid \in \{1, 2, 3\}$. If $(J, J_p) \neq (0, 0)$, then all H_A-periodic ground states are translation-invariant.*

Remark 2. The results obtained in Theorems 1 and 2 differ from those obtained for the Ising model with competing interactions (see [3]).

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PERIODIC GROUND STATES FOR THE CHUI-WEEKS'S MODEL ON THE CAYLEY TREE OF ORDER TWO

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The Cayley tree $\Gamma^k = (V, L)$ (see, e.g., [1,2]) of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, from each vertex of which exactly $k+1$ edges issue. It is known (see [1]) that there exists a one-to-one correspondence between the set V of vertices of the Cayley tree of order $k \geq 1$ and the group G_k of the free products of $k+1$ cyclic groups $\{e, a_i\}$, $i = 1, ..., k+1$ of the second order (i.e. $a_i^2 = e$, $a_i^{-1} = a_i$) with generators $a_1, a_2, ..., a_{k+1}$. Consider model where the spin takes values in the set $\Phi = \{0, 1, 2\}$. For $A \subseteq V$ a spin *configuration* σ_A on A is defined as a function $x \in A \mapsto \sigma_A(x) \in \Phi$; the set of all configurations coincides with $\Omega_A = \Phi^A$. Denote $\Omega = \Omega_V$ and $\sigma = \sigma_V$.

Definition 1. A configuration $\sigma \in \Omega$ is called G_k^* -periodic, if $\sigma(yx) = \sigma(x)$ for any $x \in G_k$ and $y \in G_k^* \subset G_k$. For a given periodic configuration the index of the subgroup is called the *period of the configuration.*

For convenience, let us denote the following $G_k^{(2)}$-periodic configuration as (l, m):

$$\sigma(x) = \begin{cases} l, & \text{if } x \in G_k^{(2)}, \\ m, & \text{if } x \in G_k \setminus G_k^{(2)}, \end{cases}$$

where $l, m \in \Phi$.

Definition 2. A configuration that is invariant with respect to all shifts is called *translation-invariant.*

The Chui-Weeks's model (see [3]) is defined by the following Hamiltonian

$$H(\sigma) = J \sum_{\langle x,y \rangle \in L} \mid \sigma(x) - \sigma(y) \mid + \alpha \sum_{x \in V} \delta_{\sigma(x),0}, \tag{1}$$

where $J, \alpha \in \mathbb{R}$, α is an external field and $\sigma \in \Omega$.

Remark. Recall that model (1) coincides with the SOS model under the condition $\alpha = 0$ (see, e.g., [2]).

Let M be the set of all unit balls with vertices in V and $S_1(x)$ be the set of all nearest neighboring vertices of $x \in V$.

We call the restriction of a configuration σ to the ball $b \in M$ a *bounded configuration* σ_b. The energy of configuration σ_b on b is defined by the formula

$$U(\sigma_b) = \frac{J}{2} \sum_{x \in S_1(c_b)} \mid \sigma(x) - \sigma(c_b) \mid + \frac{\alpha}{k+2} \sum_{x \in b} \delta_{\sigma(x),0}.$$

where $J = (J, \alpha) \in \mathbb{R}^2$ and c_b is the center of the unit ball b.

The Hamiltonian (1) can be written as

$$H(\sigma) = \sum_{b \in M} U(\sigma_b).$$

In this work we study all $G_2^{(2)}$-periodic ground states for the Chui-Weeks's model on the Cayley tree of order two.

We have the following lemma.

Lemma. *Let $k = 2$. Then for each configuration φ_b, we have the following*

$$U(\varphi_b) \in \{U_i : i = 1, 2, ..., 19\},$$

where

$$U_1 = 0; U_2 = \alpha; U_3 = \frac{J}{2}; U_4 = \frac{J}{2} + \frac{\alpha}{4}; U_5 = \frac{J}{2} + \frac{3\alpha}{4};$$

$$U_6 = J; U_7 = J + \frac{\alpha}{4}; U_8 = J + \frac{\alpha}{2}; U_9 = J + \frac{3\alpha}{4}; U_{10} = \frac{3J}{2};$$

$$U_{11} = \frac{3J}{2} + \frac{\alpha}{4}; U_{12} = \frac{3J}{2} + \frac{\alpha}{2}; U_{13} = \frac{3J}{2} + \frac{3\alpha}{4}; U_{14} = 2J + \frac{\alpha}{4};$$

$$U_{15} = 2J + \frac{\alpha}{2}; U_{16} = \frac{5J}{2} + \frac{\alpha}{4}; U_{17} = \frac{5J}{2} + \frac{\alpha}{2}; U_{18} = 3J + \frac{\alpha}{4}; U_{19} = 3J + \frac{3\alpha}{4}.$$

Definition 3. A configuration φ is called a ground state for the Hamiltonian (1), if

$$U(\varphi_b) = \min\{U_i : i = 1, 2, ..., 19\}$$

for any $b \in M$.

We denote $A_\xi = \{(J, \alpha) \in \mathbb{R}^2 : U_\xi = \min\{U_i : i = 1, 2, ..., 19\}\}$.

Calculations show that:

$$A_1 = \{(J, \alpha) \in \mathbb{R}^2 : J \geq 0, \alpha \geq 0\};$$

$$A_2 = \{(J, \alpha) \in \mathbb{R}^2 : 12J \geq \alpha, \alpha \leq 0\};$$

$$A_3 = A_6 = \{(J, \alpha) \in \mathbb{R}^2 : J = 0, \alpha \geq 0\};$$

$$A_4 = A_5 = A_7 = A_8 = A_9 = A_{11} = A_{12} = ... = A_{17} = \{(J, \alpha) \in \mathbb{R}^2 : J = 0, \alpha = 0\};$$

$$A_{10} = \{(J, \alpha) \in \mathbb{R}^2 : -\alpha \leq 6J \leq 0, \alpha \geq 0\};$$

$$A_{18} = \{(J, \alpha) \in \mathbb{R}^2 : J \leq 0, 0 \leq \alpha \leq -6J\};$$

$$A_{19} = \{(J, \alpha) \in \mathbb{R}^2 : J \leq 0, 12J \leq \alpha \leq 0\}.$$

Theorem. *Let $k = 2$. Then for the Chui-Weeks's model the following assertions hold*
i) if $(J, \alpha) \in A_1$, then $\sigma(x) = 1, \forall x \in V$ and $\sigma(x) = 2, \forall x \in V$ are translation-invariant ground states;
ii) if $(J, \alpha) \in A_2$, then $\sigma(x) = 0, \forall x \in V$ is the translation-invariant ground state;
iii) if $(J, \alpha) \in A_{10}$, then $(1, 2)$ and $(2, 1)$ are $G_2^{(2)}$-periodic ground states;
iv) if $(J, \alpha) \in A_{18} \cap A_{19}$, then $(0, 2)$ and $(2, 0)$ are $G_2^{(2)}$-periodic ground states.

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ON GIBBS MEASURES OF P-SOS MODEL ON CAYLEY TREE OF ORDER THREE

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Let Γ^k be a Cayley three that is an infinite graph without cycles such that exactly $k+1$ edges originate from each vertex. The distance $d(x,y)$ on the Cayley tree is the number of edges of the shortest path from x to y. For a fixed $x^0 \in V$, called the *root*, we set

$$W_n = \{x \in V : d(x,x^0) = n\}, \qquad V_n = \bigcup_{m=0}^{n} W_m$$

and denote by

$$S(x) = \{y \in W_{n+1} : d(x,y) = 1\}, \quad x \in W_n,$$

the set of *direct successors* of x.

We consider spin-configurations σ which are functions from the vertices of a Cayley tree of order $k \geq 1$ to the local configuration space $\Phi = \{0,1,\dots,m\}$, $m \geq 1$.

By $\langle x,y\rangle$ we denote a pair of nearest-neighbor vertices. A two-parametric solid-on-solid model (called p-SOS) is a spin system with spins taking values in Φ, and with formal Hamiltonian

$$H(\sigma) = -J \sum_{\langle x,y\rangle} |\sigma(x) - \sigma(y)|^p, \tag{1}$$

where $p > 0$ and $J \in \mathbb{R}$ the coupling constant. Next, we consider real vector-valued function from $V \setminus \{x^0\}$ to $\mathbb{R}^{m+1}$ given as

$$h\colon x \mapsto h_x = (h_{0,x}, h_{1,x}, \dots, h_{m,x}),$$

and the corresponding probability distributions $\mu^{(n)}$ on Φ^{V_n}, the set of all configuration given on V_n, $n \in \mathbb{N}$, defined as

$$\mu^{(n)}(\sigma_n) = Z_n^{-1} \exp\Big(- H(\sigma_n) + \sum_{x \in W_n} h_{\sigma(x),x}\Big). \tag{2}$$

Here, $\sigma_n\colon x \in V_n \mapsto \sigma(x) \in \{0,\dots,m\}$ and Z_n is the corresponding partition function

$$Z_n = \sum_{\widetilde{\sigma}_n \in \Phi^{V_n}} \exp\Big(- H(\widetilde{\sigma}_n) + \sum_{x \in W_n} h_{\widetilde{\sigma}(x),x}\Big). \tag{3}$$

We say that the sequence of probability distributions $(\mu^{(n)})_{n\geq 1}$ are *compatible* if for all $n \geq 1$ and $\sigma_{n-1} \in \Phi^{V_{n-1}}$ we have that

$$\sum_{\omega_n \in \Phi^{W_n}} \mu^{(n)}(\sigma_{n-1} \vee \omega_n) = \mu^{(n-1)}(\sigma_{n-1}). \tag{4}$$

Here $\sigma_{n-1} \vee \omega_n \in \Phi^{V_n}$ is the concatenation of σ_{n-1} and ω_n. If this is the case, then, by the Kolmogorov extension theorem, there exists a unique measure μ on Φ^V such that, for all n and $\sigma_n \in \Phi^{V_n}$,

$$\mu(\{\sigma|_{V_n} = \sigma_n\}) = \mu^{(n)}(\sigma_n).$$

Let us now turn our attention to the p-SOS model with Hamiltonian defined in (1). The following statement presents a system of functional equations whose solutions correspond to (infinite-volume) Gibbs measures of the p-SOS model on Cayley trees. Let us note that every extremal Gibbs measure also arises in this way, but not necessarily every measure which arises in this way is extremal, see Chapter 12 of [1].

In other words, the following statement describes conditions on h_x guaranteeing compatibility of distributions $\mu^{(n)}(\sigma_n)$.

Theorem 1. [2,3] Probability distributions $\mu^{(n)}$, $n = 1, 2, \ldots$, in (2) are compatible iff, for any $x \in V \setminus \{x^0\}$, the following equation holds,

$$h_x^* = \sum_{y \in S(x)} F(h_y^*, m, \theta). \tag{5}$$

Here $\theta := \exp(J)$ and h_x^* stands for the vector $(h_{0,x} - h_{m,x}, h_{1,x} - h_{m,x}, \ldots, h_{m-1,x} - h_{m,x})$ and the vector-valued function $F(\cdot, m, \theta)\colon \mathbb{R}^m \to \mathbb{R}^m$ is

$$F(h, m, \theta) := (F_0(h, m, \theta), \ldots, F_{m-1}(h, m, \theta)),$$

with

$$F_i(h, m, \theta) := \ln \frac{\sum_{j=0}^{m-1} \theta^{|i-j|^p} \exp(h_j) + \theta^{(m-i)^p}}{\sum_{j=0}^{m-1} \theta^{(m-j)^p} \exp(h_j) + 1}, \tag{6}$$

where $h := (h_0, h_1, \ldots, h_{m-1}), i = 0, \ldots, m-1$.

To each solution of (6) corresponds a Gibbs measure. One has to begin with translation-invariant solutions: $h_x = h \in \mathbb{R}^m$ for all x. In this case (6) becomes

$$z_i = \left(\frac{\sum_{j=0}^{m-1} \theta^{|i-j|^p} z_j + \theta^{(m-i)^p}}{\sum_{j=0}^{m-1} \theta^{(m-j)^p} z_j + 1} \right)^k, \qquad i = 0, \ldots, m-1, \tag{7}$$

where $z_i = \exp(h_i)$. In [3] the case $k = m = 2$ this equation have been studied. We considered the case $k = 3$. Our main result is

Theorem 2. The equation (7) has up to seven solution for $k = 3$, $m = 2$ and any $p > 0$, $\theta > 0$, i.e., there are up to seven translation invariant Gibbs measures.

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ON AN HAMMERSTEIN'S INTEGRAL EQUATION RELATED TO GIBBS MEASURES

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A Cayley tree $\Gamma^k = (V, L)$ (see [1]) of order $k \geq 1$ is an infinite regular tree, i.e., a graph without cycles, with exactly $k+1$ edges incident to each vertices. Here V is the set of vertices and L that of edges.

Consider spins $\sigma(x)$, $x \in V$ with values from $[0, 1]$. A configuration σ on V is then defined as a function $x \in V \mapsto \sigma(x) \in [0, 1]$; the set of all configurations is $\Omega = [0, 1]^V$.

Define the (formal) Hamiltonian on Ω as

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} \xi_{\sigma(x)\sigma(y)}, \tag{1}$$

where $J \in R \setminus \{0\}$ and $\xi : (u, v) \in [0, 1]^2 \to \xi_{uv} \in R$ is a given bounded, measurable function. As usually, $\langle x, y\rangle$ stands for nearest neighbor vertices.

It is known (see [1], [2]) that to each translation-invariant Gibbs measure of Hamiltonian (1) corresponds a solution $f(t)$, $t \in [0, 1]$ of the following functional equation

$$f(t) = \left(\frac{\int_0^1 K(t, u) f(u) du}{\int_0^1 K(0, u) f(u) du} \right)^k, \tag{2}$$

where $K(t, u) = \exp(J\xi_{tu}) > 0, f(t) > 0, t, u \in [0, 1]$.

Note that this equation can be rewritten as a Hammerstein's integral equation. In general, it is very difficult to solve functional equation (2), but in [1]-[4] (see also references therein), under some conditions on the kernel $K(t, u)$, some solutions are studied.

Here we consider separable kernel, i.e.

$$K(t, u) = \sum_{i=1}^{m} p_i(t) q_i(u), \tag{3}$$

where $m \in \mathbb{N}$, p_i and q_i are some given positive functions.

Consider the following system of non-linear equations

$$\alpha_i \left(\sum_{j=1}^{m} p_i(0) \alpha_j \right)^k - \int_0^1 q_i(u) \left(\sum_{j=1}^{m} p_j(u) \alpha_j \right)^k du = 0, \quad i = 1, \ldots, m. \tag{4}$$

Theorem 1. For each solution $\alpha = (\alpha_1, \ldots, \alpha_m)$, $\alpha_i > 0$ of system (4) corresponds a solution

$$f(t) = \left(\frac{\sum_{i=1}^m p_i(t) \alpha_i}{\sum_{i=1}^m p_i(0) \alpha_i} \right)^k$$

of equation (2) in the case of kernel (3).

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THE PATH IN THE SPACE $SP^n_G X$

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Let X^n be the n-th power of a compact X. The permutation group S_n of all permutations, acts on the n-th power X^n as permutation of coordinates. The set of all orbits of this action with quotient topology we denote by $SP^n X$. Thus, points of the space $SP^n X$ are finite subsets (equivalence classes) of the product X^n. Thus, two points $(x_1, x_2, ..., x_n)$, $(y_1, y_2, ..., y_n) \in X^n$ are considered to be equivalent, if there is a permutation $\sigma \in S_n$ such that $y_i = x_{\sigma(i)}$. The space $SP^n X$ is called the n-permutation degree of a space X.

Let G be any subgroup of the group S_n. Then it also acts on X^n as group of permutations of coordinates. Consequently, it generates a G-symmetric equivalence relation on X^n. This quotient space of the product of X^n under the G-symmetric equivalence relation is called G-permutation degree of the space X and it is denoted by $SP^n_G X$ [1].

An operation SP^n_G is the covariant functor in the category of compacts and it is said to be a functor of G-permutation degree. For every map $f : X \to Y$ a mapping $SP^n_G f : SP^n_G X \to SP^n_G Y$ is defined [1] by the formula

$$SP^n_G f[(x_1, x_2, ..., x_n)]_G = [(f(x_1), f(x_2), ..., f(x_n))]_G.$$

A continuous mapping $f : [0, 1] \to X$ is called a *path* in X. The point $f(0)$ is called the *initial point* and $f(1)$ is called the *final* or *terminal point* of this path [2].

If f and g are two paths in X with $f(1) = g(0)$ then by the product of f and g we mean the path $f * g$, which is defined [2] by

$$(f * g)(t) = \begin{cases} f(2t), & \text{if } 0 \le t \le 1/2, \\ g(2t-1), & \text{if } 1/2 \le t \le 1. \end{cases}$$

Theorem 1. If the maps $f_i : I \to X$ are paths in X from the points x^i_0 to the points x^i_1, $i = 1, 2, ..., n$ respectively, then a map $SP^n_G f^n : I \to SP^n_G X$ which is defined by that $SP^n_G f^n(t) = [(f_1(t), f_2(t), ..., f_n(t))]_G$ is also path from a point $[(x^1_0, x^2_0, ..., x^n_0)]_G$ to a point $[(x^1_1, x^2_1, ..., x^n_1)]_G$ in $SP^n_G X$.

Theorem 2. Let $SP^n_G f^n$ and $SP^n_G g^n$ be paths from $[(x^1_0, x^2_0, ..., x^n_0)]_G$ to $[(x^1_1, x^2_1, ..., x^n_1)]_G$ and from $[(x^1_2, x^2_2, ..., x^n_2)]_G$ to $[(x^1_3, x^2_3, ..., x^n_3)]_G$ (as Corollary 1), respectively , with
$SP^n_G f^n(1) = SP^n_G g^n(0)$. Then we can define the multiplication of the paths in $SP^n_G X$ as follows:

$$(SP^n_G f^n * SP^n_G g^n)(t) = [((f_1 * g_1)(t), (f_2 * g_2)(t), ..., (f_n * g_n)(t))]_G.$$

This path sharing the points $[(x^1_0, x^2_0, ..., x^n_0)]_G$ and $[(x^1_3, x^2_3, ..., x^n_3)]_G$.

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DIFFERENTIAL GAME WITH THE ℓ-CAPTURE PROBLEM FOR PONTRYAGIN'S CONTROL EXAMPLE

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Differential game problems began to be examined systematically by Isaacs [1] in the 1950's. Fundamental results of differential games were obtained by Krasovsky [2], Pontryagin [3] and others. The problem for the case of ℓ-approach [4] was first studied by Indian mathematician Ramchundra. Similar effects in the cases of geometric and integral constraint were considered in the works of Chikrii, Rappoport, Pshenichnyi, Ostapenko, Petrosjan, Satimov, Grigorenko, Azamov, Ibragimov, Samatov and etc.

We investigate the ℓ-capture problem for Pontryagin's control example in a differential game with two conflicting controlled players, where the controls of both players are subject to geometric constraints. The goal of the first player (the pursuer) is to ℓ-capture the second player (the evader) by closing the distance between them to a value of ℓ. Conversely, the goal of the second player is to prevent the pursuer from approaching it at a distance of ℓ. The problem is divided into two parts. In the first part, we assume that both players have the same movement dynamics, and we construct an approach strategy for the pursuer, thereby identifying sufficient conditions for ℓ-capture. In the second part, we examine the ℓ-capture problem in the differential game when the players have different movement dynamics. We directly solve the problem by applying the Π-strategy.

Assume that a player $\mathbb{P}$ (the pursuer) chases another player $\mathbb{E}$ (the evader) in space $\mathbf{R}^n$. Represent by x a state of the pursuer and by y that of the evader in $\mathbf{R}^n$. Let us consider the ℓ-capture problem when the players' movements are based on the differential equations with initial conditions

$$\mathbb{P}: \quad \ddot{x} + k\dot{x} = u, \quad x(0) = x_{10}, \quad \dot{x}(0) = x_{11}, \tag{1}$$

$$\mathbb{E}: \quad \ddot{y} + k\dot{x} = v, \quad y(0) = y_{10}, \quad \dot{y}(0) = y_{11}, \tag{2}$$

where $x, y, u, v \in \mathbf{R}^n$, $n \geq 2$, $k > 0$; x_{10}, y_{10} are the initial states of the players accordingly, and it is presumed that $|x_{10} - y_{10}| > \ell$, $\ell > 0$; x_{11}, y_{11} are the playersŸ initial velocity vectors correspondingly; the acceleration vectors u, v act as control parameters of the players respectively, and they depend on the time t, $t \geq 0$.

The temporal variations of u and v must be Lebesgue measurable functions $u(\cdot) : [0, +\infty) \to \mathbf{R}^n$, $u(\cdot) : [0, +\infty) \to \mathbf{R}^n$, and these functions are involved fulfilling the geometric constraints (shortly, G-constraints)

$$\operatorname{ess}\sup_{0 \leq t \leq t^*} |u(t)| \leq \alpha, \tag{3}$$

$$\operatorname{ess}\sup_{0 \leq t \leq t^*} |v(t)| \leq \beta, \tag{4}$$

where α, β are the given positive parametric numbers which represent the maximal acceleration values of the players.

Introduce the denotation: $z = x - y$, $z_{10} = x_{10} - y_{10}$, $z_{11} = x_{11} - y_{11}$.

We will construct a solution to the ℓ-capture problem using the provided initial states z_{10} and z_{11}. It is important to note that there are only two possible cases in this situation:

Case 1: Players have the same movement dynamics, which is supposed to be $x_{11} = y_{11}$ or the vectors z_{10} and z_{11} are collinear, meaning that there exists a finite number r such that $z_{11} = rz_{10}$.

Case 2: The vectors z_{10} and z_{11} are non-collinear, meaning that they are linearly independent.

Definition 1. [5] Let $\alpha > \beta$. Then in the differential game (1)–(4), we term the function
$\mathbf{u}(z_{10}, v) = v + \lambda_{\Pi_\ell}(z_{10}, v)\,(m(z_{10}, v) - z_{10})$ the Π_ℓ-strategy of the pursuer, where

$$\lambda_{\Pi_\ell}(z_{10}, v) = \left[\langle v, z_{10}\rangle + \alpha\ell + \sqrt{(\langle v, z_{10}\rangle + \alpha\ell) + (\alpha^2 - |v|^2)(|z_{10}|^2 - \ell^2)}\right] /(|z_{10}|^2 - \ell^2),$$

$$m(z_{10}, v) = -(v - \lambda_{\Pi_\ell}(z_{10}, v)z_{10})\ell/(|v - \lambda_{\Pi_\ell}(z_{10}, v)z_{10}|).$$

Definition 2. [6] Let $\alpha > \beta$. Then in the differential game (1)–(4), we term the function
$\mathbf{u}(v) = v + \lambda_{\Pi}(v)\xi_{11}$, the Π-strategy of the pursuer, where

$$\lambda_{\Pi}(v) = \langle v, z_{11}\rangle + \sqrt{\langle v, z_{11}\rangle^2 + \alpha^2 - |v|^2}, \quad \xi_{11} = z_{11}/|z_{11}|.$$

Proposition 1. If $\alpha > \beta$ and $r \leq 0$, then the equation

$$e^{-kt} = -At + B, \quad A = \frac{(\alpha - \beta)a}{\alpha - \beta + kr|z_{10}|}, \quad B = 1 + \frac{k^2(|z_{10}| - \ell)}{\alpha - \beta + kr|z_{10}|}$$

has a positive root with respect to t, and we denote it by t^*.

Theorem 1. Let $\alpha > \beta$ and let the vectors z_{10} and z_{11} be collinear in the differential game (1)–(4). Then the Π_ℓ-strategy is winning on the interval $[0, t^*]$.

Theorem 2. Let $\alpha > \beta$ and let the vectors z_{10} and z_{11} be non-collinear in the differential game (1)–(4). Then using the Π and Π_ℓ-strategies can complete the ℓ-capture.

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CRITERION FOR UNIQUENESS AND NON-UNIQUENESS OF THE p-ADIC MARKOV CHAIN WITH A COUNTABLE STATE

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p-Adic numbers. Let $\mathbb{Q}$ be the field of rational numbers. For a fixed prime number p, every rational number $x \neq 0$ can be represented in the form $x = p^r \frac{n}{m}$, where $r, n \in \mathbb{Z}$, m is a positive integer, and n and m are relatively prime with p: $(p, n) = 1$, $(p, m) = 1$. The p-adic norm of x is given [1] by $|x|_p = p^{-r}$, for $x \neq 0$, $|x|_p = 0$, for $x = 0$. The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$. Denote

$$\mathcal{E}_p = \left\{x \in \mathbb{Q}_p : |x-1|_p < p^{-1/(p-1)}\right\}.$$

Tree. A tree is a connected graph without cycles (see [2] for more details). Let $\mathcal{T} = (V, L)$ be a tree, where V is the set of vertices and L is the set of edges. Two vertices x and y are called *nearest neighbors* if there exists an edge $b \in L$ connecting them. We will use the notation $b = \langle x, y\rangle$ for the edge connecting the vertices x and y. A collection of nearest neighbor pairs $\langle x, x_1\rangle, \langle x_1, x_2\rangle, ..., \langle x_{d-1}, y\rangle$ is called a *path* from x to y. The distance $d(x, y)$ on the tree is the number of edges of the shortest path from x to y. For each edge $\langle x, y\rangle \in L$ we let xy denote the associated oriented edge which points from x to y. We denote the set of all oriented edges by $\overrightarrow{L}$. For $z \in V$,

$$^zL := \left\{xy \in \overrightarrow{L} : d(z, y) = d(z, x) + 1\right\}.$$

p-adic Markov chain and boundary laws We consider a system with nearest neighbor interactions on a tree where the spins assigned to the vertices of the tree take values in the set of natural numbers $\mathbb{N}$. A configuration σ_A on $A \subset V$ is defined as a function $x \in A \mapsto \sigma_A(x) \in \mathbb{N}$. The set of all configurations is $\mathbb{N}^A$.

By p-adic probability vector we mean a vector with p-adic valued coordinates summing to 1. A p-adic stochastic matrix is a matrix with each row being a p-adic probability vector.

For each edge $b = \langle x, y\rangle \in L$ we consider a stochastic matrix $\mathbb{P}_b = (P_b(i, j))_{i,j\in\mathbb{N}}$. For each $x \in V$ consider a probability vector $\alpha_x = (\alpha_{1,x}, \alpha_{2,x}, \alpha_{3,x}, \dots)$

Definition 1. [3] A p-adic probability distribution (measure) μ is called a p-adic Markov chain with transition matrices $(P_b)_{b\in L}$ and marginal distribution α_x at $x \in V$ if for all finite, connected set $\Lambda \subset V$, and all $\zeta \in \mathbb{N}^\Lambda$ and $z \in \Lambda$ the following holds

$$\mu(\sigma_\Lambda = \zeta) = \alpha_{\zeta_z,z} \prod_{\substack{\langle x,y\rangle \in {}^zL: \\ x,y\in\Lambda}} P_{\langle x,y\rangle}(\zeta_x, \zeta_y).$$

Consider for each edge $b = \langle x, y\rangle$ a matrix $\mathcal{Q}_b = (Q_b(i, j))_{i,j\in\mathbb{N}}$. We always assume

$$Q_{\langle x,y\rangle}(i, j) = Q_{\langle y,x\rangle}(j, i), \quad \sum_{j\in\mathbb{N}} Q_{\langle x,y\rangle}(j, i) = 1. \tag{1}$$

Let $\boldsymbol{z}(x, y) \in m_p := \{z = \{z_i\}_{i\in\mathbb{N}} : \quad z_i \in \mathbb{Q}_p,\ \sup_{i\in\mathbb{N}} |z_i|_p < +\infty\}$.

Definition 2. [3] For $(\mathcal{Q}_b)_{b\in L}$ satisfying (1), a *p-adic boundary law* $\{\boldsymbol{z}(x,y)\}_{\langle x,y\rangle\in L}$ is such that for any $\langle x,y\rangle \in L$, and for all $i \in \mathbb{N}$, it holds

$$z_i(x,y) = c(x,y) \prod_{v\in\partial\{x\}\setminus\{y\}} \sum_{j\in\mathbb{N}} z_j(v,x) Q_{\langle v,x\rangle}(j,i), \tag{2}$$

where $c(x,y)$ is an arbitrary constant (not depending on $i \in \mathbb{N}$).

For the uniqueness, we assume that the matrix $\mathcal{Q}_b = (Q_b(i,j))_{i,j\in\mathbb{N}}$ satisfies the following conditions

$$|Q_{\langle x,y\rangle}(j,i)|_p \le 1, \quad \left|Q_{\langle x,y\rangle}(j,i) - Q_{\langle x,y\rangle}(j,1)\right|_p \le \frac{1}{p}, \quad \forall\langle x,y\rangle, \quad \forall i,j. \tag{3}$$

Theorem 1. Assume each vertex of the tree has degree at least two and that the matrix $\mathcal{Q}_b = (Q_b(i,j))_{i,j\in\mathbb{N}}$ satisfies (1) and (3). Then the equation (2) has a unique solution $\boldsymbol{z}(x,y) = \mathbf{1} = (1,1,1,\dots) \in \mathcal{E}_p^{\mathbb{N}}$, $\langle x,y\rangle \in L$.

Corollary. Under the conditions of Theorem 1, there exists a unique p-adic Markov chain.

We consider non-uniqueness of the p-adic Markov chains on the Cayley tree of order $k \ge 1$. Suppose the matrix $\mathcal{Q}_b$ in the system of equations (2) satisfies the condition

$$Q_{\langle v,x\rangle}(1,2) = Q_{\langle v,x\rangle}(1,i), \quad \forall\langle v,x\rangle, \quad i > 2. \tag{4}$$

We assume further that $Q_{\langle v,x\rangle}(1,1)$ and $Q_{\langle v,x\rangle}(1,2)$ are independent on $\langle v,x\rangle$, that is

$$1-\alpha \equiv Q_{\langle v,x\rangle}(1,2), \quad 1-\beta \equiv Q_{\langle v,x\rangle}(1,1), \forall\langle v,x\rangle \in L. \tag{5}$$

Theorem 2. If (4), (5) are satisfied, α,β are p-adic integers, and there exists $\gamma \in \{0,1,2,\dots\}$ such that

$$k(\beta-\alpha)+1 \ne 0, \quad k(\beta-\alpha)+1 \equiv 0 \,(\mathrm{mod}\, p^{2\gamma+1}),$$

$$k\beta + \frac{k(k-1)}{2}(\beta^2-\alpha^2) \equiv 0 \,(\mathrm{mod}\, p^{\gamma}),$$

$$k\beta + \frac{k(k-1)}{2}(\beta^2-\alpha^2) \ne 0 \,(\mathrm{mod}\, p^{\gamma+1}),$$

then the equation (2) has at least two solutions.

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p-ADIC DYNAMICAL SYSTEMS OF THE FUNCTION ax^{-k}

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In this work we consider the discrete dynamical system associated with the function $f\colon\ \mathbb{Q}_p \to \mathbb{Q}_p$ defined by

$$f(x) = \frac{a}{x^k}, \quad a \neq 0, \quad a \in \mathbb{Q}_p, \quad k \in \mathbb{N}, \tag{1}$$

where $x \neq \hat{x} = 0$.

The fixed points of this function are solutions of the equation $x^{k+1} = a$ in $\mathbb{Q}_p$. The work [1] provides detailed information about the solutions of the equation $x^k = a$ in the field $\mathbb{Q}_p$. Using these data, we obtain the following results. Define the set

$$Sol_p(x^{k+1} - a) = \{\xi \in \mathbb{F}_p : \xi^k \equiv a(\mathrm{mod}\, p)\}, \ \ a \in \mathbb{Z}_p^*,$$

where $\mathbb{F}_p := \{0, 1, 2, ..., p-1\}$ and $\mathbb{Z}_p^* := \{x \in \mathbb{Q}_p : \ \ |x|_p = 1\}$. We also denote the number of elements of the set $Sol_p(x^{k+1} - a)$ by κ_p. Note that for any number $k \in \mathbb{N}$, there are numbers $m \in \mathbb{N}$ and $s \in \mathbb{Z}_+$ such that $k = mp^s$, $(m, p) = 1$ is valid. Let

$$a_0 + a_1 \cdot p + a_2 \cdot p^2 + a_3 \cdot p^3 + \dots \tag{2}.$$

Theorem 1. Let $p \geq 3$ and $k + 1 = mp^s$. If the canonical expansion of the number $a \in \mathbb{Z}_p^*$ is (2), then the following assertions are valid:

(i) If $Sol_p(x^{k+1} - a) = \emptyset$, then the equation $x^{k+1} = a$ hasn't solution;

(ii) Let $Sol_p(x^{k+1} - a) \neq \emptyset$. Then

(ii_1) if $|a - a_0^{p^s}|_p \geq p^{-s}$, then the equation $x^{k+1} = a$ hasn't solution;

(ii_2) if $|a - a_0^{p^s}|_p < p^{-s}$, then the number of solutions of the equation $x^{k+1} = a$ is equal to κ_p.

It can be seen from this theorem that the number of fixed points of the function (1) may be equal to κ_p.

Theorem 2. The fixed point of the function (1) is an attractive for $p \mid k$ and an indifferent for $p \nmid k$.

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REGULARITY OF POSITIVE RIESZ TYPE STOCHASTIC OPERATORS

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It is known [4] that nonlinear dynamical systems appear in some problems of physics, biology, chemistry, economics, etc. However, unlike linear dynamical systems, there is still no general approach to the study of nonlinear dynamical systems. For example, there are quadratic stochastic operators on a finite-dimensional simplex that have applications in population genetics and economy, which have been studied for almost a century (see [1], [4]). But, even in this class, we have yet to study a full description of the dynamics. This is because the dynamics of such operators involve several polynomials with degrees greater than or equal to two at each step. On the other hand, there are other nonlinear stochastic operators that lie between linear and quadratic stochastic operators, which have several applications in many areas of natural sciences. This paper focuses on studying such types of operators. We introduce nonlinear operators, which are generated by a linear one. Initially, it might seem that studying such kinds of operators is straightforward. However, they are much more difficult in reality. Our motivation for studying such kinds of operators came from [2], [6]. In [6], several non-stochastic dynamical systems were presented, which naturally arise from biological systems. Moreover, these types of operator normalizations give a class of nonlinear stochastic dynamical systems. Another class of nonlinear stochastic operators, called Lotka-Volterra type operators, has been recently investigated in [5].

Let us denote by $\mathbb{R}$ and $\mathbb{R}_+$ the set of all real and non-negative real numbers, respectively. Then for a given $m \geq 2$ we denote by S^{m-1} the set of all probability vectors (or distributions), i.e.

$$S^{m-1} = \left\{ \mathbf{x} = (x_1, x_2, \ldots, x_m) \in \mathbb{R}_+^m : \sum_{k=1}^{m} x_k = 1 \right\}.$$

One can see that S^{m-1} is a norm closed subset of $\mathbb{R}^m$. Moreover, S^{m-1} is a simplex, i.e. every $\mathbf{x} \in S^{m-1}$ is written as follows $\mathbf{x} = \sum_{k=1}^{m} x_k \mathbf{e}_k$, where $\{\mathbf{e}_k\}_{k=1}^{m}$ is the standard basis of $\mathbb{R}^m$.

Every mapping that maps the set S^{m-1} into itself is known as a *stochastic operator*. In this paper, we consider a nonlinear stochastic operator T_A obtained by renormalizing the linear operator A at each point for a given linear operator A on $\mathbb{R}^m$.

For a given matrix

$$A = \begin{pmatrix} a_{11} & 0 & 0 & \ldots & 0 \\ a_{21} & a_{22} & 0 & \ldots & 0 \\ a_{31} & a_{32} & a_{33} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ a_{m1} & a_{m2} & a_{m3} & \ldots & a_{mm} \end{pmatrix}, \tag{1}$$

we consider the following nonlinear operator on $\mathbb{R}^m$

$$(T_A(\mathbf{x}))_i = \frac{(A\mathbf{x})_i}{\sum\limits_{k=1}^{m}(A\mathbf{x})_k}, \quad i = \overline{1,m}.$$

Definition 1. [3] Let a matrix $A = (a_{ij})_{i,j=1}^m$ have the following properties: $(A1)$ $a_{ij} \geq 0$ for every $i, j \in \{1, 2 \ldots, m\}$; $(A2)$ $\sum\limits_{j=1}^{i} a_{ij} = 1$ for all $i \in \{1, 2, \ldots, m\}$. Then the associated operator T_A is called a Riesz-type stochastic operator (in short, RSO). Moreover, if $a_{ij} > 0$ for any $i \geq j$ then T_A is called a positive Riesz-type stochastic operator.

Theorem 1. Let $A = (a_{ij})_{i,j=1}^m$ be a positive upper triangular matrix with (1). If $a_{22} > a_{33} > \cdots > a_{mm}$ then the positive RSO T_A has exactly m fixed points: $\mathbf{q}_k = (q_{k1}, q_{k2}, \ldots, q_{km})$, $k \in \{1, 2, \ldots, m\}$ where

$$q_{kj} = \begin{cases} 0, & \text{if } j < k; \\ \frac{c_{jk}}{c_{kk}+\cdots+c_{mk}}, & \text{if } k \leq j \leq m; \end{cases}$$

and

$$c_{jk} = \lim_{n\to\infty} \frac{a_{jk}^{(n)}}{a_{kk}^{(n)}}, \qquad \forall j \geq k.$$

Moreover, for any $\mathbf{x} \in \Gamma_k$, $k \in \{1, 2, \ldots, m\}$ it holds

$$\lim_{n\to\infty} T_A^n(\mathbf{x}) = \mathbf{q}_k.$$

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TRANSPOSED POISSON STRUCTURES ON INFINITE DIMENSIONAL LIE ALGEBRAS

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Recently, Bai, Bai, Guo, and Wu have introduced a dual notion of the Poisson algebra [1], called a transposed Poisson algebra, by exchanging the roles of the two multiplications in the Leibniz rule defining a Poisson algebra. In 2021 a relation between $\frac{1}{2}$-derivations of Lie algebras and transposed Poisson algebras has been established by Ferreira, Kaygorodov [3]. These ideas were used to describe all transposed Poisson structures on Witt and Virasoro algebras in [3]. The present work is a continuation of this research. Namely, we describe transposed Poisson structures on algebras $W(G,1)$ defined in [2].

Definition 1. The triple $(\mathfrak{L},\cdot,[\cdot,\cdot])$ is called a transposed Poisson algebra if $(\mathfrak{L},\cdot)$ is a commutative associative algebra and $(\mathfrak{L},[\cdot,\cdot])$ is a Lie algebra that satisfies the following compatibility condition

$$2z\cdot[x,y]=[z\cdot x,y]+[x,z\cdot y].$$

Definition 2. Let $(\mathfrak{L},[\cdot,\cdot])$ be a Lie algebra, $\varphi:\mathfrak{L}\to\mathfrak{L}$ be a linear map. Then φ is a $\frac{1}{2}$-derivation if it satisfies $\varphi([x,y])=\frac{1}{2}\big([\varphi(x),y]+[x,\varphi(y)]\big)$.

Let G be any nontrivial additive subgroup of $\mathbb{C}$, and $\mathbb{C}[G\times\mathbb{Z}_+]$ the semigroup algebra of $G\times\mathbb{Z}_+$ with basis $\{x^\alpha t^i \mid \alpha\in G, i\in\mathbb{Z}_+\}$ and product $(x^\alpha t^i)\cdot(x^\beta t^j)=x^{\alpha+\beta}t^{i+j}$.

Denote $\partial=\partial_x+t^2\partial_t$. Then the algebra $W(G,1)$ is $\mathbb{C}[G\times\mathbb{Z}_+]\partial$ with basis $\{L_{\alpha,i}=x^\alpha t^i\partial \mid \alpha\in G, i\in\mathbb{Z}_+\}$ and relation

$$[L_{\alpha,i},L_{\beta,j}]=(\beta-\alpha)L_{\alpha+\beta,i+j}+(j-i)L_{\alpha+\beta,i+j+1},\quad \alpha,\beta\in G,\quad i,j\in\mathbb{Z}_+.$$

First, we compute $\frac{1}{2}$-derivations of $W(G,1)$.

Theorem 1. Let φ be a $\frac{1}{2}$-derivation of the algebra $W(G,1)$, then

$$\varphi(L_{\alpha,i})=\sum_{d\in G}\sum_{k\in\mathbb{Z}_+,\ k\geq i}a^{d,k-i}L_{\alpha+d,k}.$$

Now we aim to classify all transposed Poisson structures on $W(G,1)$. Let $\mathcal{H}$ be an associative-commutative algebra with basis $\{L_{\alpha,i} \mid \alpha\in G, i\in\mathbb{Z}_+\}$ and multiplication

$$L_{\alpha,i}L_{\beta,j}=L_{\alpha+\beta,i+j}.$$

Theorem 2. Let $(\mathfrak{L},\cdot,[\cdot,\cdot])$ be a transposed Poisson structure defined on the Lie algebra $W(G,1)$. Then $(\mathfrak{L},\cdot,[\cdot,\cdot])$ is not Poisson algebra and $(\mathfrak{L},\cdot)$ is a mutation of the algebra $\mathcal{H}$. On the other hand, every mutation of the algebra $\mathcal{H}$ gives a transposed Poisson structure with the Lie part isomorphic to the algebra $W(G,1)$.

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FIXED POINTS AND STABILITY OF THE PHYTOPLANKTON-ZOOPLANKTON SYSTEM

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In [1] the following continuous-time phytoplankton-zooplankton model is considered:

$$\begin{cases} \dfrac{dP}{dt} = bP(1-\dfrac{P}{k}) - \alpha f(P)Z, \\ \dfrac{dZ}{dt} = \beta f(P)Z - rZ - \theta g(P)Z, \end{cases} \tag{1}$$

where P is the density of phytoplankton and Z is the density of the zooplankton population; $\alpha > 0$ and $\beta > 0$ are predation and conversion rates of the zooplankton on the phytoplankton population, respectively; $b > 0$ is the growth rate, $k > 0$ is carrying capacity of the phytoplankton; $r > 0$ is the death rate of the zooplankton; $f(P)$ represents the predator functional response; $g(P)$ represents the distribution of the toxin substances; $\theta > 0$ denotes the rate of toxin liberation by the phytoplankton population. Many authors have studied the model (1) with different types of $f(x)$ and $g(x)$.

Let us consider the model (1) by choosing $f(u) = \frac{u^h}{1+cu^h}$, $g(u) = u^h$. In the case of $h = 1$, the model is famous as a phytoplankton-zooplankton system with Holling type II predator functional response, and for the discrete analogue of the model - the existence and local stability of positive fixed points, the occurrence of the Neimark-Sacker bifurcation of positive fixed points was shown in [2].

Let's denote

$$\overline{t} = bt,\ \overline{u} = \frac{P}{k},\ \overline{v} = \frac{\alpha k^{h-1} Z}{b},\ \overline{c} = ck^h,\ \overline{\beta} = \frac{\beta k^h}{b},\ \overline{r} = \frac{r}{b},\ \overline{\theta} = \frac{\theta k^h}{b}.$$

Then by dropping the overline sign at time $t \geq 0$ we get:

$$\begin{cases} \dfrac{du}{dt} = u(1-u) - \dfrac{u^h v}{1+cu^h} \\ \dfrac{dv}{dt} = \dfrac{\beta u^h v}{1+cu^h} - rv - \theta u^h v. \end{cases} \tag{2}$$

where β, r, θ, c are positive parameters.

The discrete-time version of the model (2) has the following form:

$$V : \begin{cases} u^{(1)} = u(2-u) - \dfrac{u^h v}{1+cu^h} \\ v^{(1)} = \dfrac{\beta u^h v}{1+cu^h} + (1-r)v - \theta u^h v, \end{cases} \tag{3}$$

where $u \geq 0, v \geq 0$ and $h > 0$. It is easy to see that, $E_0 = (0;0)$ and $E_1 = (1,0)$ are fixed points of V. From a biological point of view, the existence and stability of positive fixed points are essential.

Proposition 1. The following statements hold true:

(i) If $\beta > r+\theta$ and $0 < c < \frac{\beta-r-\theta}{r+\theta}$ then there exists unique positive fixed point $E_- = (u_-, v_-)$;

(ii) If $\beta > \frac{(r+\theta)^2}{\theta}$ and $\frac{\beta-r-\theta}{r+\theta} < c < \frac{\beta+\theta-2\sqrt{\beta\theta}}{r}$ then there exist two positive fixed points $E_- = (u_-, v_-)$ and $E_+ = (u_+, v_+)$;

(iii) If $\beta > \frac{(r+\theta)^2}{\theta}$ and $c = \frac{\beta+\theta-2\sqrt{\beta\theta}}{r}$ then there exists unique positive fixed point $\overline{E} = (u_0, v_0)$,

where

$$u_\mp = \sqrt[h]{\frac{\beta - rc - \theta \mp \sqrt{(\beta - rc - \theta)^2 - 4cr\theta}}{2c\theta}}, \quad v_\mp = (1-u_\mp)(1+cu_\mp^h)u_\mp^{1-h},$$

$$u_0 = \sqrt[h]{\frac{r}{\sqrt{\theta}(\sqrt{\beta}-\sqrt{\theta})}}, \quad v_0 = (1-u_0)(1+cu_0^h)u_0^{1-h}.$$

Note that, for the remaining cases of Proposition 1, there is no positive fixed point.

Proposition 2. For fixed points of V, the following statements are true:

(i) the fixed point $\overline{E} = (u_0, v_0)$ is non-hyperbolic;

(ii)

$$E_0 = \begin{cases} \text{saddle}, & \text{if } 0 < r < 2 \\ \text{nonhyperbolic}, & \text{if } r = 2 \\ \text{repelling}, & \text{if } r > 2, \end{cases}$$

(iii)

$$E_1 = \begin{cases} \text{attractive}, & \text{if } \frac{\beta}{1+c} < r+\theta < 2+\frac{\beta}{1+c} \\ \text{nonhyperbolic}, & \text{if } r+\theta = \frac{\beta}{1+c} \text{ or } r+\theta = 2+\frac{\beta}{1+c} \\ \text{saddle}, & \text{if otherwise.} \end{cases}$$

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MHD STEFAN FLOW OF CASSON NON-FLUID THROUGH A POROUS MEDIUM IN THE PRESENCE OF CHEMICAL REACTION WITH THE EFFECT OF THOMPSON AND TROIAN SLIP OVER A PLATE IN THE COMPANY OF RADIATION

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In this study, we present the impact of Stefan blowing, Thompson and Troian slip on the behavior of magnetohydrodynamic (MHD) Casson nanofluid past a porous medium in the presence of a chemical reaction. Using a two-phase model for nanofluids, we also investigate the influence of velocity distribution, Joel heat, and radiation parameters. The main Partial Differential Equations (PDEs) can be transformed into Ordinary Differential Equations (ODEs) using similarity transformations. To solve nonlinear equations,MATLAB Shooting and Runge-Kutta techniques can be used. The changes in non-dimensional perimeters reveal how fluid flow, heat, and mass transfer characteristics are affected. It is observed that with the expansion of Stefan blowing parameter S, the skin friction coefficient decreases. The fluid concentration reduces with the increasing values of Thompson and Troian slip parameters. The heat of the fluid increases with the increase of , and k, but the concentration decreases. The outcomes of this examination provide many appealing features that is going to give merits for further study of the problems..

Let us think about a forced convective Casson nano liquid flow over a plate of enormously small width and greatly bigger span, fixed in the medium (see, Fig. 1). The axis is normal to the upward direction plate of axis. We assume that the applied magnetic field is perpendicular to the plate. The magnetic field is applied to the strength of the transverse magnetic field. Assuming as the velocity apparatuses of parallel and normal to the plate respectively. The appropriate governing equations of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_f\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho_f}(u-u_\infty)-\frac{\mu}{k}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{(\rho\, c)_f}\frac{\partial q_r}{\partial y}+\frac{\nu}{(\rho\, c)_f}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial c}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}+k_1\,(C-C_\infty) \tag{4}$$

Here, u and v are the velocity components in the x and y directions respectively,

The appropriate boundary conditions for this problem are given below as,

$$u = \gamma\left(1 - \zeta\frac{\partial u}{\partial y}\right)^{-1/2}\frac{\partial u}{\partial y}, v = \frac{-D_B}{(1-C_w)}\frac{\partial C}{\partial y}, T = T_\infty, C = C_w \; at \; y = 0 \tag{5a}$$

$$u = u_\infty(y) = \beta_1 y, T = T_\infty, C = C_\infty \; at \; y \to \infty \tag{5b}$$

We now put the following similarity transformation relations for u, v as

$$u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x} \tag{6}$$

where, ψ is the stream function. Again, let us introduce the following dimensionless variables,

$$\eta = \frac{y}{L}\left(\frac{x}{L}\right)^{-1/2}, \;\; \psi = v\left(\frac{x}{L}\right)^{2/3} f(\eta) \text{ and } \theta = \frac{T - T_\infty}{T_w - T_\infty}, \; \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

Using the relations (6)–(7) in the boundary layer Eq. (2), energy Eq. (3) and concentration Eq. (4) the following equations are obtained.

$$\left(1 + \frac{1}{\beta}\right) f''' + \frac{2}{3} f\, f'' - \frac{1}{3}(f')^2 - \left(M + \frac{1}{K}\right) f' = 0 \tag{8}$$

$$\frac{1}{\Pr}\left(1 + \frac{4R}{3}\right)\theta'' + N_b\,\theta'\,\phi' + N_t(\theta')^2 + \frac{2}{3} f\,\theta' + Ec\left[(f')^2 + (f'')^2\right] = 0 \tag{9}$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + Le\left[\frac{2}{3} f\,\phi' - k\phi\right] = 0 \tag{10}$$

the boundary conditions finally become

$$f = \frac{3S}{2\,Le}\phi', f' = \delta\left(1 - \beta_1 f''\right), \theta = 1, \phi = 1 \text{ at } \eta = 0 \tag{11}$$

$$f'' = 1, \; \theta = 0, \; \phi = 0 \text{ at } \eta \to \infty \tag{12}$$

Stefan flow of Casson nanofluid over a plate in the presence of shear flow, porous, MHD, and Radiation has been investigated. The effects of the Thompson-Troian slip at the boundary have also been inspected. Numerical solutions have been obtained and a compa- rison has been made with the available data and found excellent agreement. The following observations are made.

(i)Temperature increases with the augmented values of the thermophoresis parameter N_1 and reduction in concentration is noted for mounting values of the Brownian motion parameter N_2

(ii) Temperature decreases whereas the concentration oscillates with the rising values of chemical reaction parameter k.

(iii) Heat is transported from the plate to the liquid

(iv) Skin friction coefficient reduces with the growing values of Stefan blowing parameter S

(v) Mass transport rate diminishes with the rise in slip parameter δ and critical shear rate β.

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FOKAS METHOD FOR THE HEAT EQUATION ON METRIC GRAPHS

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Let $\Gamma = E \bigcup V$ be a connected metric graph, where $E = \{b_j\}_{j=1}^{n}$ is the set of its edges, and $V = \{v_j\}_{j=1}^{m}$ is the set of vertices [2]. Let us define the coordinates x_j on the edges of the graphs via the isometric mapping of these edges onto the intervals $(0,\, L_j)$, $j = 1, 2, ..., n$.

We say that the vertex ν *touches* the edge b_j, if it is the end of this edge, and denote it by $b_j \sim v$. The number of the elements of the set $\{b : b \sim v,\, b \in E\}$ is called the *valence* of the vertex v. If the valence of the vertex is equal to one, then it is called *boundary*. Let $\{\gamma_1, \gamma_2, ..., \gamma_{m_1}\} = \partial\Gamma \subset V$ be the boundary vertices of the graph. Without loss of generality we will further use x instead of x_j.

Consider the heat equation

$$u_t^{(j)}(x,t) = u_{xx}^{(j)}(x,t),\ x \in b_j,\ t > 0,\ j = 1, 2, ..., n. \tag{1}$$

on each edge of the graph.

We claim that the solution of Eq. (1) satisfies the initial condition

$$u^{(j)}(x,0) = u_0^{(j)}(x), j = 1, 2, ..., n. \tag{2}$$

In the branching points (i.e., in the internal vertices) of the graph the solution must satisfy the following gluing conditions:

a) the values at the vertex v of all function's derivatives $u^{(j)}$, for which $b_j \sim v$, are the same;

b) the sum of functions at each vertex ν of all functions $u^{(j)}$, for which $b_j \sim v$, is equal to zero:

$$\sum_{b_j \sim v} \sigma_{b_j,v} u^{(j)}(x,t)\Big|_v = 0,\ v \in V \backslash \partial\Gamma,\ t \in [0,T], \tag{3}$$

where $\sigma_{b_j,v} = -1$ if v is left end of the bond b_j and $\sigma_{b_j,v} = -1$ if v is right end of the bond b_j.

The first of these equations is called the *condition of the continuity* of the solution at the vertex, and the second – the *condition of flow preservation*. These conditions are also called δ' *type* vertex conditions.

We claim that the following boundary conditions hold at the boundary vertices of the graph:

$$u^{(j)}(x,t)|_\nu = g_0^{(j)}(t), x_j = 0\, at\, the\, vertex\ \nu,$$

$$u^{(j)}(x,t)|_\nu = h_0^{(j)}(t), x_j = L_j\, at\, the\, vertex\ \nu, b_j \sim \nu. \tag{4}$$

The initial data $u_0^{(j)}$, $j = \overline{1,n}$, satisfy the gluing conditions at the nonboundary vertices of the graph and the matching conditions at the boundary vertices.

We find the solution of the problem (1)–(4) using Fokas method. This method uses special generalization of the Fourier transformation, called the unified transformation method [1], [3]–[4]. To satisfy boundary and vertex conditions we obtained the system of linear algebraic equations, using so called global relations.

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FOUR ELECTRON SYSTEMS IN THE IMPURITY HUBBARD MODEL. FIRST SINGLET STATE. ONE-DIMENSIONAL CASE

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We consider of the energy operator of four-electron systems in the impurity Hubbard model [1] and investigated the structure of essential spectra and discrete spectrum of the system in the one-dimensional lattice for the first singlet state of the system. The Hamiltonian of the considering system has the form

$$H = A\sum_{m,\gamma} a^+_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^+_{m,\gamma}a_{m+\tau,\gamma} + U\sum_{m} a^+_{m,\uparrow}a_{m,\uparrow}a^+_{m,\downarrow}a_{m,\downarrow} +$$

$$+(A_0 - A)\sum_{\gamma} a^+_{0,\gamma}a_{0,\gamma} + (B_0 - B)\sum_{\tau,\gamma}(a^+_{0,\gamma}a_{\tau,\gamma} + a^+_{\tau,\gamma}a_{0,\gamma}) + (U_0 - U)a^+_{0,\uparrow}a_{0,\uparrow}a^+_{0,\downarrow}a_{0,\downarrow}.$$

Here, A (A_0) is the electron energy at a regular (impurity) lattice site; $B > 0$ ($B_0 > 0$) is the transfer integral between electrons (between electron and impurity) in a neighboring sites, and the summation over τ ranges the nearest neighbors, U (U_0) is the parameter of the on-site Coulomb interaction of two electrons, correspondingly in the regular (impurity) lattice site; γ is the spin index,$\gamma =\uparrow$ or $\gamma =\downarrow$, and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m \in Z^\nu$.

The first singlet state corresponds four electron bound states (or antibound states) to the basis functions: ${}^1s^0_{p,q,r,t} = a^+_{p,\uparrow}a^+_{q,\uparrow}a^+_{r,\downarrow}a^+_{t,\downarrow}\varphi_0$. The subspace ${}^1\widetilde{\mathcal{H}}^0_s$, corresponding to the first singlet state is the set of all vector's of the form: ${}^1\psi^0_s = \sum_{p,q,r,t\in Z^\nu} f(p,q,r,t){}^1s^0_{p,q,r,t}, f \in l^{as}_2$, where l^{as}_2 is the subspace of antisymmetric functions in $l_2((Z^\nu)^4)$.

In the four-electron systems exists quintet state, two type singlet states, and three type triplet states. The Hamiltonian H acts in the antisymmetric Fock space $\widetilde{\mathcal{H}}_{as}$. Let φ_0 be the vacuum vector in the space $\widetilde{\mathcal{H}}_{as}$. We denote by ${}^1H^0_s$ the restriction of the operator H to the space ${}^1\widetilde{\mathcal{H}}^0_s$. We call the operator ${}^1H^0_s$ the four-electron first singlet state operator.

We let $\mathcal{F}$ denote the Fourier transform: $\mathcal{F} : l_2((Z^\nu)^4) \to L_2((T^\nu)^4) \equiv {}^1\widetilde{\mathcal{H}}^0_s$, where T^ν is the $\nu-$ dimensional torus endowed with the normalized Lebesgue measure $d\lambda$, $\lambda(T^\nu) = 1$. We set ${}^1\widetilde{H}^0_s = \mathcal{F}\,{}^1\overline{H}^0_s\mathcal{F}^{-1}$. In the quasimomentum representation, the operator ${}^1\overline{H}^0_s$ acts in the Hilbert space $L^{as}_2((T^\nu)^4)$, where L^{as}_2 is the subspace of antisymmetric functions in $L_2((T^\nu)^4)$.

Theorem. Let $\nu = 1$. Then

A). If $\varepsilon_2 = -B$ and $\varepsilon_1 < -2B$ (respectively, $\varepsilon_2 = -B$ and $\varepsilon_1 > 2B$), then the essential spectrum of the operator ${}^1\widetilde{H}^0_s$ is consists of the union of ten segments: $\sigma_{ess}({}^1\widetilde{H}^0_s) = [4A-8B, 4A+8B] \cup [3A-6B+z, 3A+6B+z] \cup [2A-4B+2z, 2A+4B+2z] \cup [A-2B+3z, A+2B+3z] \cup [2A-4B+z_3, 2A+4B+z_3] \cup [A-2B+z+z_3, A+2B+z+z_3] \cup [2A-4B+z_4, 2A+4B+z_4] \cup [A-2B+z+z_4, A+2B+z+z_4] \cup [2A-4B+z_5, 2A+4B+z_5] \cup [A-2B+z+z_5, A+2B+z+z_5]$ and the discrete spectrum of the operator ${}^1\widetilde{H}^0_s$ is consists of six eigenvalues: $\sigma_{disc}({}^1\widetilde{H}^0_s) = \{4z, 2z+z_3, 2z+z_4, 2z+z_5, z_3+z_4, z_3+z_5\}$,

where $z = A + \varepsilon_1$ and z_3 and z_4 and z_5 are the additional eigenvalues of the operator $\widetilde{H}_2$ and $\widetilde{H}_3$, respectively.

B). If $\varepsilon_2 = -2B$ or $\varepsilon_2 = 0$ and $\varepsilon_1 < 0$ (respectively, $\varepsilon_2 = -2B$ or $\varepsilon_2 = 0$ and $\varepsilon_1 > 0$), then the essential spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of the union of ten segments, and discrete spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of six eigenvalues.

C). If $\varepsilon_1 = 0$ and $\varepsilon_2 > 0$ or $\varepsilon_1 = 0$ and $\varepsilon_2 < -2B$, then the essential spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of the union of sixteen segments, $\sigma_{ess}({}^1\widetilde{H}_s^0) = [4A-8B, 4A+8B] \cup [3A-6B+z_1, 3A+6B+z_1] \cup [3A-6B+z_2, 3A+6B+z_2] \cup [2A-4B+2z_1, 2A+4B+z_1] \cup [2A-4B+2z_2, 2A+4B+z_2] \cup [A-2B+3z_1, A+2B+3z_1] \cup [A-2B+3z_2, A+2B+3z_2] \cup [2A-4B+z_4, 2A+4B+z_4] \cup [2A-4B+z_5, 2A+4B+z_5] \cup [A-2B+z_1+z_4, A+2B+z_1+z_4] \cup [A-2B+z_1+z_5, A+2B+z_1+z_5] \cup [A-2B+z_2+z_4, A+2B+z_2+z_4] \cup [A-2B+z_2+z_5, A+2B+z_2+z_5] \cup [2A-4B+z_3, 2A+4B+z_3] \cup [A-2B+z_1+z_3, A+2B+z_1+z_3] \cup [A-2B+z_2+z_3, A+2B+z_2+z_3]$, and discrete spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of ten eigenvalues: $\sigma_{disc}({}^1\widetilde{H}_s^0) = \{4z_1, 2z_1+z_4, 2z_1+z_5, 4z_2, 2z_2+z_4, 2z_2+z_5, 2z_1+z_3, 2z_2+z_3, z_3+z_4, z_3+z_5\}$, where $z_1 = A - \frac{2BE}{\sqrt{E^2-1}}$ and $z_2 = A + \frac{2BE}{\sqrt{E^2-1}}$ and $E = \frac{(B+\varepsilon_2)^2}{\varepsilon_2^2+2B\varepsilon_2}$.

D). If $\varepsilon_1 = \frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$ (respectively, $\varepsilon_1 = -\frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$), or If $\varepsilon_2 > 0$ and $\varepsilon_1 > \frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$ (respectively, $\varepsilon_2 < -2B$ and $\varepsilon_1 > \frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$), or If $\varepsilon_2 > 0$ and $\varepsilon_1 < -\frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$ (respectively, $\varepsilon_2 < -2B$ and $\varepsilon_1 < -\frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$), then the essential spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of the union of ten segments, and discrete spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of six eigenvalues.

E). If $\varepsilon_2 > 0$ and $0 < \varepsilon_1 < \frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$ (respectively, $\varepsilon_2 < -2B$ and $0 < \varepsilon_1 < \frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B}$), then the essential spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of the union of nineteen segments: $\sigma_{ess}({}^1\widetilde{H}_s^0) = [4A-8B, 4A+8B] \cup [3A-6B+z_1, 3A+6B+z_1] \cup [3A-6B+z_2, 3A+6B+z_2] \cup [2A-4B+2z_1, 2A+4B+2z_1] \cup [2A-4B+2z_2, 2A+4B+2z_2] \cup [2A-4B+z_1+z_2, 2A+4B+z_1+z_2] \cup [A-2B+3z_1, A+2B+3z_1] \cup [A-2B+3z_2, A+2B+3z_2] \cup [A-2B+2z_1+z_2, A+2B+2z_1+z_2] \cup [A-2B+z_1+2z_2, A+2B+z_1+2z_2] \cup [2A-4B+z_3, 2A+4B+z_3] \cup [2A-4B+z_4, 2A+4B+z_4] \cup [A-2B+z_1+z_3, A+2B+z_1+z_3] \cup [A-2B+z_1+z_4, A+2B+z_1+z_4] \cup [A-2B+z_2+z_3, A+2B+z_2+z_3] \cup [A-2B+z_2+z_4, A+2B+z_2+z_4] \cup [2A-4B+z_5, 2A+4B+z_5] \cup [A-2B+z_1+z_5, A+2B+z_1+z_5] \cup [A-2B+z_2+z_5, A+2B+z_2+z_5]$, and discrete spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of sixteen eigenvalues: $\sigma_{disc}({}^1\widetilde{H}_s^0) = \{4z_1, 3z_1+z_2, 2z_1+2z_2, z_1+3z_2, 4z_2, 2z_1+z_3, z_1+z_2+z_3, 2z_2+z_3, 2z_1+z_4, 2z_2+z_4, z_1+z_2+z_4, 2z_1+z_5, z_1+z_2+z_5, 2z_2+z_5, z_3+z_4, z_3+z_5\}$, where $z_1 = A + \frac{2B(\alpha+E\sqrt{E^2-1+\alpha^2})}{E^2-1}$ and $z_2 = A + \frac{2B(\alpha-E\sqrt{E^2-1+\alpha^2})}{E^2-1}$ and $E = \frac{(B+\varepsilon_2)^2}{\varepsilon_2^2+2B\varepsilon_2}$ and the real number $0 < \alpha < 1$.

F). If $\varepsilon_2 > 0$ and $-\frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B} < \varepsilon_1 < 0$ (respectively, $\varepsilon_2 < -2B$ and $-\frac{2(\varepsilon_2^2+2B\varepsilon_2)}{B} < \varepsilon_1 < 0$), then the essential spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of the union of nineteen segments, and discrete spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of sixteen eigenvalues.

K). If $-2B < \varepsilon_2 < 0$, then the essential spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of the union of four segments: $\sigma_{ess}({}^1\widetilde{H}_s^0) = [4A-8B, 4A+8B] \cup [2A-4B+z_3, 2A+4B+z_3] \cup [2A-4B+z_4, 2A+4B+z_4] \cup [2A-4B+z_5, 2A+4B+z_5]$, and discrete spectrum of the operator ${}^1\widetilde{H}_s^0$ is consists of two eigenvalues: $\sigma_{disc}({}^1\widetilde{H}_s^0) = \{z_3+z_4, z_3+z_5\}$.

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WEAKLY PERIODIC P-ADIC QUASI GIBBS MEASURES FOR THE POTTS MODEL ON A CAYLEY TREE

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In the present paper we study weakly periodic p-adic quasi Gibbs measures for the three-state Potts model on a Cayley tree of order two. We find the number of the weakly periodic (non-periodic) p-adic quasi Gibbs measures.

Let $\mathbb{Q}_p$ be the field of p-adic numbers (see e.g.[1]) and $\Phi = \{1, 2, ..., q\}$ be a finite set. A configuration σ on $A \subset V$ is defined by the function $x \in A \to \sigma(x) \in \Phi$. The set of all configurations on A is denoted by Ω_A and $\Omega = \Omega_V$ (see e.g.[2]). The (formal) Hamiltonian of the p-adic Potts model is defined by

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)}, \tag{1}$$

where $J \in B(0, p^{-1/(p-1)})$ is a coupling constant, $\langle x,\ y\rangle$ stands for nearest neighbor vertices and δ_{ij} is the Kronecker's symbol. Assume that $\mathbf{h} : V\backslash\{x^{(0)}\} \to \mathbb{Q}_p^q$ is a mapping, i.e. $\mathbf{h}_x = (h_{1,x}, h_{2,x}, ..., h_{q,x})$, where $h_{i,x} \in \mathbb{Q}_p$, $i \in \Phi$ and $x \in V\backslash\{x^{(0)}\}$. Given $n \in \mathbb{N}$, we consider a p-adic probability distribution $\mu_{\mathbf{h}}^{(n)}$ on Ω_{V_n} defined by

$$\mu_{\mathbf{h}}^{(n)}(\sigma) = \frac{1}{Z_n^{(\mathbf{h})}} \exp_p\{H_n(\sigma)\} \prod_{x \in W_n} h_x^{\sigma(x)}, \tag{2}$$

where $\sigma \in \Omega_{V_n}$, and $Z_n^{(\mathbf{h})}$ is the corresponding normalizing factor. If p-adic probability distributions (2) are compatibly (see [2]) then a non-Archimedean analogue of the Kolmogorov's extension theorem [3], there exists a unique p-adic quasi *Gibbs measure* corresponding to the given hamiltonian (1).

Let G_k be a group of the free product of $k+1$ cyclic groups of the second order with generators $a_1, a_2, ..., a_{k+1}$, respectively. There exists a one-to-one correspondence between the set of vertices V of the Cayley tree Γ^k and the group G_k (see [4]). Let G_k^* be a normal subgroup of the group G_k with index r, i.e. $G_k/G_k^* = \{H_1, H_2, ..., H_r\},\ r \geq 1$ is a quotient group of G_k by G_k^*. A set $h = \{h_x, x \in G_k\}$ of quantities is called G_k^*-*periodic* if $h_x = h_i$, for all $x \in H_i$. A G_k-periodic quantities are called *translation-invariant*. A set of quantities $h = \{h_x, x \in G_k\}$ is called G_k^*-*weakly periodic* if $h_x = h_{ij}$, for any $x \in H_i$ and $y_\downarrow \in H_j$. A p-adic quasi Gibbs measure μ is said to be G_k^*-(weakly) periodic if it corresponds to a G_k^*-(weakly) periodic h. A G_k-periodic measure is called a translation-invariant measure [2].

Let

$$H_{a_i} = \{x \in G_k : \sum_{i \in A} \omega_x(a_i) \text{ is an even number}\}, \tag{3}$$

where $\emptyset \neq A \subseteq N_k = \{1, 2, 3, ..., k+1\}$, and $\omega_x(a_i)$ is the number of letters a_i in a word $x \in G_k$. Note that H_{a_i} which is defined by (3) is a normal divisor of index 2 in G_k (see

[2]). It can be check that weakly periodic Gibbs measure depends on choosing the normal subgroup of G_k. In real case, different weakly periodic Gibbs measures for the Potts model are found in [5]. Note that, the set of weakly periodic Gibbs measures includes the set of periodic (in particular translation-invariant) Gibbs measures. The translation-invariant p-adic Gibbs measures for the Potts model were studied in [6]. G_2-periodic p-adic quasi Gibbs measures for the Potts model on the Cayley tree of order two were considered in [7].

Theorem. Let $p > 3$, $\mathcal{N}$ be a number of H_{a_i}-weakly periodic (non-periodic) p-adic quasi Gibbs measures for the three-state Potts model on a Cayley tree of order two. Then

$$\mathcal{N} = \begin{cases} 5, \text{if } \ p \equiv 5(\mathrm{mod}\, 8) \ \text{ or } \ p \equiv 7(\mathrm{mod}\, 8), \\ 7, \text{if } \ p \equiv 1(\mathrm{mod}\, 8) \ \text{ or } \ p \equiv 3(\mathrm{mod}\, 8). \end{cases}$$

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DYNAMICAL SYSTEM OF A QUADRATIC FUNCTION

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Let X be a topological space. In order to define (see [1], [2]) a discrete-time dynamical system consider a function $f : X \to X$. For $x \in X$ denote by $f^n(x)$ the n-fold composition of f with itself:

$$f^n(x) = \underbrace{f(f(...f(x)...))}_{n}$$

For arbitrary given $x^{(0)} \in X$ and $f : X \to X$ the discrete-time dynamical system is the sequence of points

$$x^{(0)}, x^{(1)} = f(x^{(0)}), x^{(2)} = f^2(x^{(0)}), x^{(3)} = f^3(x^{(0)}), \dots \tag{1}$$

The main problem is to know behavior of the sequence (1) for each initial point $x^{(0)} \in X$.

A point $x \in X$ is called a fixed point for $f : X \to X$ if $f(x) = x$.

Denote the set of all fixed points by $Fix(f)$.

In this work we consider $X = \mathbb{R}$ and f is the following special function:

$$f(x) = ax^2 + bx + \frac{b^2 - 2b}{4a}, \tag{1}$$

where $a, b \in \mathbb{R}$, $a \neq 0$.

By mathematical induction one can prove the following

Lemma 1. For function (1) the following equality holds

$$f^n(x) = \frac{2(\alpha(x))^{2^n} - b}{2a}, \quad \text{with} \quad \alpha(x) = \frac{2ax + b}{2}.$$

This lemma is used to prove

Theorem 1. For any initial point $x^{(0)} \in \mathbb{R}$ the following limit holds

$$\lim_{n \to +\infty} f^n(x^{(0)}) = \frac{1}{2a} \begin{cases} -b, & \text{if } |\alpha(x^{(0)})| < 1 \\ 2 - b & \text{if } |\alpha(x^{(0)})| = 1 \\ +\infty, & \text{if } |\alpha(x^{(0)})| > 1 \end{cases}$$

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ON REPRESENTATION FORMULAS OF FRACTIONAL ORDER LINEAR SYSTEMS

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The talk is devoted to the representation formulas of time-fractional order linear systems of pseudo-differential and differential-operator equations. Such formulas were known in particular cases of scalar order (commensurate case) or vector-order with rational components (incommensurate case). In the talk we provide representation formulas for arbitrary vector-order systems with real components of the order.

ON MAXIMAL SOLVABLE EXTENSION OF GIVEN NILRADICAL CONTAINING CHARACTERISTICALLY NILPOTENT IDEALS

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Lie algebras plays an essential role in mathematics and physics, especially in quantum field theory, quantum mechanics and string theory. An extensive study of Lie algebras gave many beautiful results and generalizations. From the classical theory of finite-dimensional Lie algebras, it is known that an arbitrary Lie algebra over a field of zero characteristic is decomposed into a semidirect sum of the solvable radical and its semisimple subalgebra (Levi's theorem). Thanks to Malcev's and Mubarakzjanov's results the study of non-nilpotent solvable Lie algebras is reduced to the study of nilpotent ones and their derivations. Therefore, the study of finite-dimensional Lie algebras is focused on nilpotent algebras and representations of semisimple algebras.

One of the most important classes of Lie algebras is Characteristically nilpotent Lie algebras. The first example of CNLA was found by Dixmier and Lister 1957. In 1972 French matematician G.Favre discovered the lowest dimensional CNLA. Actually such algebras exist in any dimension starting from seven.

In this thesis we consider two non-isomorph maximal solvable Lie algebras whose nilradical containing characteristically nilpotent ideals .

Definition 1. A vector space with a bilinear bracket $(L, [-,-])$ over a field of $\mathbb{F}$ is called a Lie algebra if for any $x, y, z \in L$ the following identities hold:

$$[x, y] = -[y, x]$$

$$[[x, y], z] + [[y, z], x] + [[z, x], y] = 0$$

For a given Lie algebra $(L, [-,-])$ the lower central and the derived series are defined recursively as follows:

$$L^1 = L, L^{k+1} = [L^k, L], k \geq 1; \quad L^{[1]} = L, L^{[k+1]} = [L^{[k]}, L^{[k]}], k \geq 1.$$

Definition 2. An algebra $(L, [-,-])$ is said to be solvable (respectively, nilpotent) if there exists $m \in \mathbb{N}$ (respectively, $n \in \mathbb{N}$) such that $L^{[m]} = 0$ ($L^n = 0$).

Definition 3. A linear map $d : L \to L$ of a Lie algebra $(L, [-,-])$ is said to be a derivation if for all $x, y \in L$ the following condition holds:

$$d([x, y]) = [d(x), y] + [x, d(y)]$$

The set of all derivations of L is denoted by $Der(L)$.

If all derivations of a given algebra are nilpotent, then this algebra is called **characteristically nilpotent**.

Let consider the direct sum of 8-dimensional characteristically nilpotent Lie algebra and one dimensional abelian algebra:

$$N_8 : \begin{cases} [e_1, e_2] = e_5, & [e_1, e_3] = e_6, & [e_1, e_4] = e_7, & [e_1, e_5] = -e_8, & [e_2, e_3] = e_8 \\ [e_2, e_4] = e_6, & [e_2, e_6] = -e_7, & [e_3, e_4] = -e_5, & [e_3, e_5] = -e_7, & [e_6, e_4] = e_8. \end{cases}$$

$$N = N_8 \oplus < e_9 >$$

Proposition . Any derivations of the algebra N have the following matrix form:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & d_{16} & d_{17} & d_{18} & d_{19} \\ 0 & 0 & 0 & 0 & d_{25} & d_{26} & d_{27} & d_{28} & d_{29} \\ 0 & 0 & 0 & 0 & -d_{26} & d_{25} & d_{37} & d_{38} & d_{39} \\ 0 & 0 & 0 & 0 & d_{16} & -d_{15} & d_{47} & d_{48} & d_{49} \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{16} & -d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{15} & -d_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{97} & d_{98} & d_{99} \end{pmatrix}$$

Lemma. One has $CodimN \leq 1$.

Theorem. R_1 and R_2 are maximal solvable extensions of N, and they are non-isomorphic to each other:

$$R_1 \begin{cases} [e_1, e_2] = e_5 & [e_2, e_3] = e_8 \\ [e_1, e_3] = e_6 & [e_2, e_4] = e_6 \\ [e_1, e_4] = e_7 & [e_2, e_6] = -e_7 \\ [e_1, e_5] = -e_8 & [e_3, e_4] = -e_5 \\ [e_6, e_4] = e_8 & [e_3, e_5] = -e_7 \\ [e_7, x] = e_7 & \end{cases}$$

$$R_2 \begin{cases} [e_1, e_2] = e_5 & [e_2, e_3] = e_8 \\ [e_1, e_3] = e_6 & [e_2, e_4] = e_6 \\ [e_1, e_4] = e_7 & [e_2, e_6] = -e_7 \\ [e_1, e_5] = -e_8 & [e_3, e_4] = -e_5 \\ [e_6, e_4] = e_8 & [e_3, e_5] = -e_7 \\ [e_7, x] = e_7 & [e_1, x] = e_8 \end{cases}$$

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ON DYNAMICS OF LINEAR OPERATORS OVER NON-ARCHIMEDEAN SEQUENCE SPACES

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Let X and Y be topological vector spaces over non-Archimedean valued field $\mathbb{K}$. By $L(X,Y)$ we denote the set of all continuous linear operators from X to Y. If $X=Y$ then $L(X,Y)$ is denoted by $L(X)$. In what follows, we use the following terminology: T is a linear continuous operator on X means that $T\in L(X)$. The T-*orbit* of a vector $\mathbf{x}\in X$, for some operator $T\in L(X)$, is the set

$$O(\mathbf{x},T):=\{T^n(\mathbf{x}):n\in\mathbb{Z}_+\}.$$

An operator $T\in L(X)$ is called *supercyclic* if there exists some vector $\mathbf{x}\in X$ such that whose projective orbit

$$\mathbb{K}\cdot O(\mathbf{x},T):=\{\lambda T^n(\mathbf{x}):n\in\mathbb{Z}_+,\ \lambda\in\mathbb{K}\}$$

is dense in X. The corresponding vector $\mathbf{x}$ is called T-*hypercyclic*, and the set of all T-supercyclic vectors is denoted by $SC(T)$. we recall that T is called *cyclic* if there exists $\mathbf{x}\in X$ such that

$$\mathbb{K}[T]\mathbf{x}:=\text{span}O(\mathbf{x},T)=\{P(T)\mathbf{x};P\text{ polynomial}\}$$

is dense in X. The set of all T-cyclic vectors is denoted by $CC(T)$.

Some basic properties of supercyclic operator over non-Archimedean vector spaces were studied in [2].

Definition 1. [1] Let X be a topological vector space, and let $T\in L(X)$. It is said that T satisfies the **Supercyclicity Criterion** if there exist an increasing sequence of integers (n_k), two dense sets $\mathcal{D}_1,\mathcal{D}_2\subset X$ and a sequence of maps $S_{n_k}:\mathcal{D}_2\to X$ such that:

(1) $\|T^{n_k}(\mathbf{x})\|\cdot\|S_{n_k}(\mathbf{y})\|\to\mathbf{0}$ for any $\mathbf{x}\in\mathcal{D}_1$ and for every $\mathbf{y}\in\mathcal{D}_2$;

(2) $T^{n_k}S_{n_k}(\mathbf{y})\to\mathbf{y}$ for any $\mathbf{y}\in\mathcal{D}_2$.

Theorem 1. [2] Let $T\in L(X)$, where X is a separable Banach space. Assume that T satisfies the Supercyclic Criterion. Then T is supercyclic.

Let us consider an infinite dimensional upper-triangular matrix $\mathcal{B}=(b_{i,j})_{i,j=1}^{\infty}$ over a non-Archimedean field $\mathbb{K}$, such that

$$\sup_{i,j}\{|b_{i,j}|\}<\infty,\qquad b_{k,l}=0,\qquad \forall k\geq l. \tag{1}$$

For a given $\mathcal{B}$ with (1), we define the following linear operator on c_0 (here c_0 be a space of null sequences over non-Archimedean valued field $\mathbb{K}$) by

$$B_{\mathcal{B}}(\mathbf{e}_n)=\begin{cases}0, & \text{if } n=1;\\ \sum_{j=1}^{n-1}b_{j,n}\mathbf{e}_j, & \text{if } n\geq 2.\end{cases} \tag{2}$$

The linear operator (2) is called *the generalized weighted backward shift* operator. Recall that if matrix $\mathcal{B}$ has the following extra condition $b_{k,l} = 0$ for all $k \neq l+1$, then the corresponding linear operator $B_{\mathcal{B}}$ is reduced to *weighted backward shift*. In this setting, the operator acts as follows: $B_{\mathbf{a}}(\mathbf{e}_1) = 0$ and $B_{\mathbf{a}}(\mathbf{e}_n) = a_{n-1}\mathbf{e}_{n-1}$ if $n \geq 2$, where $a_{n-1} := b_{n-1,n-1}$ is called a *weighted backward shift*. Here, $\mathbf{a} = (a_n)_{n\in\mathbb{N}}$. The operator $B_{\mathbf{a}}$ is called a *backward shift* if $b_{n-1,n-1} = 1$ for all $n \geq 1$, and such a shift is denoted by B.

We notice that the supercyclicity and hypercyclicity of weighted backward shift operators have been studied in [2].

Theorem 2. Let $\mathcal{B}$ be a matrix given by (1). Assume that $b_{k,k+1} \neq 0$ for all $k \geq 1$. Then the generalized weighted backward shift operator $B_{\mathcal{B}}$ is supercyclic on c_0.

Corollary 1. Let $\mathcal{B}$ be a matrix given by (1). Assume that $b_{k,k+1} \neq 0$ for all $k \geq 1$. Then the generalized weighted backward shift $B_{\mathcal{B}}$ is cyclic on c_0.

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LOCAL AUTOMORPHISMS OF P-FILIFORM LEIBNIZ ALGEBRAS

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In last decades a series of papers have been devoted to study of mappings which are close to automorphism and derivation of associative algebras (especially of operator algebras and C*-algebras). Namely, the problems of describing so-called local automorphisms (respectively, local derivations) and 2-local automorphisms (respectively, 2-local derivations) have been considered. Later similar problems were extended for non associative algebras, in particular, for the case of Lie algebras.

Let $\mathcal{A}$ be an algebra. Recall that a linear mapping Φ of $\mathcal{A}$ into itself is called a local automorphism (respectively, a local derivation) if for every $x \in \mathcal{A}$ there exists an automorphism (respectively, a derivation) Φ_x of $\mathcal{A}$, depending on x, such that $\Phi_x(x) = \Phi(x)$.

Local automorphisms of certain finite-dimensional simple Lie and Leibniz algebras are investigated in [1]. Concerning local automorphism, T.Becker, J.Escobar, C.Salas and R.Turdibaev in [2] established that the set of local automorphisms $LAut(sl_2)$ coincides with the group $Aut^{\pm}(sl_2)$ of all automorphisms and anti-automorphisms. Later in [4] M.Costantini proved that a linear map on a simple Lie algebra is a local automorphism if and only if it is either an automorphism or an anti-automorphism.

An algebra $(\mathcal{L}, [\cdot,\cdot])$ over a field $\mathbb{F}$ is called a (right) Leibniz algebra if it satisfies the property

$$[x,[y,z]] = [[x,y],z] - [[x,z],y],$$

which is called Leibniz identity.

Definition 1. A vector space with bilinear bracket $(\mathcal{L}, [\cdot,\cdot])$ is called a Leibniz algebra if for any $x, y, z \in \mathcal{L}$ the so-called Leibniz identity

$$\big[x,[y,z]\big] = \big[[x,y],z\big] - \big[[x,z],y\big],$$

holds.

Let $\mathcal{L}$ be a Leibniz algebra. For a Leibniz algebra $\mathcal{L}$ consider the following central lower and derived sequences:

$$\mathcal{L}^1 = \mathcal{L}, \quad \mathcal{L}^{k+1} = [\mathcal{L}^k, \mathcal{L}^1], \quad k \geq 1,$$

$$\mathcal{L}^{[1]} = \mathcal{L}, \quad \mathcal{L}^{[s+1]} = [\mathcal{L}^{[s]}, \mathcal{L}^{[s]}], \quad s \geq 1.$$

Definition 2. A Leibniz algebra $\mathcal{L}$ is called nilpotent (respectively, solvable), if there exists $p \in \mathbb{N}$ ($q \in \mathbb{N}$) such that $\mathcal{L}^p = 0$ (respectively, $\mathcal{L}^{[q]} = 0$).The minimal number p (respectively, q) with such property is said to be the index of nilpotency (respectively, of solvability) of the algebra $\mathcal{L}$.

Now let us define a naturally graduation for a nilpotent Leibniz algebra.

Definition 3. Given a nilpotent Leibniz algebra $\mathcal{L}$, put $\mathcal{L}_i = \mathcal{L}^i/\mathcal{L}^{i+1}$, $1 \leq i \leq n-1$, and $gr(\mathcal{L}) = \mathcal{L}_1 \oplus \mathcal{L}_2 \oplus \cdots \oplus \mathcal{L}_{n-1}$. Then $[\mathcal{L}_i, \mathcal{L}_j] \subseteq \mathcal{L}_{i+j}$ and we obtain the graded algebra $gr(\mathcal{L})$. If $gr(\mathcal{L})$ and L are isomorphic, then we say that an algebra $\mathcal{L}$ is naturally graded.

Now we define the notion of characteristic sequence, which is one of the important invariants. For a finite-dimensional nilpotent Leibniz algebra N and for the matrix of

the linear operator R_x denote by $C(x)$ the descending sequence of its Jordan blocks' dimensions. Consider the lexicographical order on the set $C(N) = \{C(x) \mid x \in N\}$.

Definition 4. The sequence

$$\left(\max_{x\in N\setminus N^2} C(x)\right)$$

is said to be the characteristic sequence of the nilpotent Leibniz algebra N.

Definition 5. A Leibniz algebra $\mathcal{L}$ is called p-filiform, if the characteristic sequence is $C(\mathcal{L}) = (n-p, \underbrace{1, \dots, 1}_{p})$.

Now we give the definitions of automorphisms and local automorphisms.

Definition 6. A linear bijective map $\varphi : \mathcal{L} \to \mathcal{L}$ is called an automorphism, if it satisfies $\varphi([x,y]) = [\varphi(x), \varphi(y)]$ for all $x, y \in \mathcal{L}$.

Definition 7. Let $\mathcal{L}$ be an algebra. A linear map $\Delta : \mathcal{L} \to \mathcal{L}$ is called a local automorphism, if for any element $x \in \mathcal{L}$ there exists an automorphism $\varphi_x : \mathcal{L} \to \mathcal{L}$ such that $\Delta(x) = \varphi_x(x)$.

Since we shall consider naturally graded non-Lie p-filiform nilradical we give its classication [3].

Theorem 1. An arbitrary n-dimensional naturally graded non-split p-filiform Leibniz algebra $(n-p \geq 4)$ is isomorphic to one of the following non-isomorphic algebras:

if $p = 2k$, then

$$\mu_1 : \begin{cases} [e_i, e_1] = e_{i+1}, & 1 \leq i \leq n-2k-1, \\ [e_1, f_j] = f_{k+j}, & 1 \leq j \leq k, \end{cases}$$

$$\mu_2 : \begin{cases} [e_i, e_1] = e_{i+1}, & 1 \leq i \leq n-2k-1, \\ [e_1, f_1] = e_2 + f_{k+1}, & \\ [e_i, f_1] = e_{i+1}, & 2 \leq i \leq n-2k-1, \\ [e_1, f_j] = f_{k+j}, & 2 \leq j \leq k, \end{cases}$$

if $p = 2k+1$, then

$$\mu_3 : \begin{cases} [e_i, e_1] = e_{i+1}, & 1 \leq i \leq n-2k-2, \\ [e_1, f_j] = f_{k+1+j}, & 1 \leq j \leq k, \\ [e_i, f_{k+1}] = e_{i+1}, & 1 \leq i \leq n-2k-2, \end{cases}$$

where $\{e_1, e_2, \dots, e_{n-p}, f_1, f_2, \dots, f_p\}$ is the basis of the algebra and the omitted products are equal to zero.

Now we give the main theorem concerning local automorphism on p-filiform Leibniz algebras.

Theorem 2. The algebras μ_1, μ_2 and μ_3 admit local automorphism which are not automorphism.

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ON σ-COMPACT OF MODIFICATION NIEMYTZKI PLANE

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We denote an order pair by $\langle x, y\rangle$ the set of real numbers by $\mathbb{R}$, the natural numbers by $\mathbb{N}$, and the rationals by $\mathbb{Q}$. Let $X = \{\langle x, y\rangle \in \mathbb{R}^2 : y \geq 0\}\}$ and $P = \{\langle x, y\rangle \in \mathbb{R}^2 : y > 0\}$, so the x-axis is $L = X\backslash P$. Denote the usual metric topology on X by $\mathcal{U}$ and the Niemytzki topology on X by $\mathcal{N}$. For every $\langle x, 0\rangle \in L$ and $r \in \mathbb{R}, r > 0$, let $D(\langle x, 0\rangle, r)$ be the set of all points of P inside the circle of radius r tangent to $x-$axis at $\langle x, 0\rangle$ and let $D_r(\langle x, 0\rangle) = D(\langle x, 0\rangle, r) \bigcup \{\langle x, 0\rangle\}$. For every $\langle x, 0\rangle \in X$ and $r > 0$, let $U_r(\langle x, y\rangle)$ be the set of all points of X inside the circle of radius r and centered at $\langle x, y\rangle$. Recall that the Niemytzki topology $\mathcal{N}$ on X is generated by the following neighborhood system: For every $\langle x, 0\rangle \in L$, let $\mathcal{B}(\langle x, 0\rangle) = \{D_r(\langle x, 0\rangle) : r > 0\}$. For every $\langle x, y\rangle \in P$, let $\mathcal{B}(\langle x, y\rangle) = \{U_r(\langle x, y\rangle) : r > 0\}$. Observe that P as a subspace of X with the usual metric topology coincides with P as a subspace of X with the Niemytzki topology.

Definition 1. Let A be a non-empty proper subset of the $x-$axis L. For each $\langle a, 0\rangle \in A$, $\mathcal{B}(\langle a, 0\rangle) = \{U_r(\langle a, 0\rangle) : r > 0\}$ let, where $U_r(\langle a, 0\rangle)$ is the set of all points of X inside the circle of radius r and centered at $\langle a, 0\rangle$. For each $\langle a, b\rangle \in P$, let $\mathcal{B}(\langle a, b\rangle) = \{U_r(\langle a, b\rangle) : r > 0\}$. So, the points in $A \bigcup P$ will have the same local base as in $(X, \mathcal{U})$. For each $\langle c, 0\rangle \in L\backslash A$, let $\mathcal{B}(\langle c, 0\rangle) = \{D_r(\langle c, 0\rangle) : r > 0\}$. So, the points in $L\backslash A$ will have the same local base as in $(X, \mathcal{N})$. We call the topology on X generated by the neighborhood system $\{\mathcal{B}(\langle x, y\rangle) : \langle x, y\rangle \in X\}$ the H-generated topology on X from $\mathcal{U}$ and $\mathcal{N}$ and denote it by $\mathcal{U}_A\mathcal{N}$. We call X with this *H-generated topology* an *H-space* and denote it by $(X, \mathcal{U}_A\mathcal{N})$ [1].

Definition 2. A topological space is called *σ-compact* if it is the union of countably many compact sets. A subset of a topological space is called a *σ-compact* if it is a σ-compact space with the subspace topology [2].

Clearly every compact spase is σ-compact.

Proposition 1. For an arbitrary subset A of L, if $(X, \mathcal{U}_A\mathcal{N})$ is $\sigma-$compact, then $L\backslash A$ is countable and nowhere dense in $(X, \mathcal{U}_A\mathcal{N})$.

Definition 3. A topological space T is called *scattered* if it contains no nonempty dense-in-itself subset. Equivalently, every nonempty subset A of T, contains a point isolated in A. A subset of a topological space is called a *scattered set* if it is a scattered space with the subspace topology [2].

Proposition 2. For an arbitrary subset A of L, if $L\backslash A$ is countable and scattered in $(X, \mathcal{U}_A\mathcal{N})$, then $(X, \mathcal{U}_A\mathcal{N})$ is σ-compact.

Proposition 3. For an arbitrary subset A of, if $L\backslash A$ is countable and it is a G_δ-subset of L, then $(X, \mathcal{U}_A\mathcal{N})$ is σ-compact

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A NOTE MODIFICATION OF NIEMYTZKI PLANE

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Definition 2. A topological space is called *σ-compact* if it is the union of countably many compact sets. A subset of a topological space is called a *σ-compact* if it is a σ-compact space with the subspace topology [2].

Lemma 1. Let $A \subseteq L$ and $B \subseteq A$ and $C \subseteq L\backslash A$. Then
(i) $\mathcal{U}_A\mathcal{N}|_B = \mathcal{U}|_B$ and
(ii) $\mathcal{U}_A\mathcal{N}|_B = \mathcal{N}|_B$.

Theorem 1. Let $A \subseteq L$ and $B = L\backslash A$. Then the following conditions are equivalent.
(a) There exist a σ-compact subset D and a closed subset C of the space $(X, \mathcal{U}_A\mathcal{N})$ such that $B \subseteq C \subseteq D$.
(b) There exists a closed σ-compact subset C of the space $(X, \mathcal{U}_A\mathcal{N})$such that $B \subseteq C$.
(c) The closure $Cl_{(X,\mathcal{U})}(B)$ of B in L is σ-compact in $(X, \mathcal{U}_A\mathcal{N})$.
(d) The closure $Cl_{(X,\mathcal{U}_A\mathcal{N})}(B)$ of B in the space $(X, \mathcal{U}_A\mathcal{N})$ is σ-compact in $(X, \mathcal{U}_A\mathcal{N})$.
(e) the space $(X, \mathcal{U}_A\mathcal{N})$ is σ-compact.

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KAPUTO KASR TARTIBLI OPERATORNI O‘Z ICHIGA OLUVCHI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMA UCHUN CHEGARAVIY MASALA

Abdulboqiyev X.

Farg‘ona davlat universiteti, Farg‘ona, O‘zbekiston;

$\Omega = \{(x,y) : 0 < x < p;\ \ 0 < y < q\}$ sohada

$$\left(\partial^2/\partial x^2\right)\ \left({}_C D^{\alpha}_{0y}\, u\,(x,y)\right) + bu\,(x,y) = f\,(x,y) \tag{1}$$

xususiy hosilali differensial tenglamani qaraylik, bu yerda $u\,(x,y)$- noma’lum funksiya, b-o‘zgarmas haqiqiy son, $0 < \alpha < 1$, $f\,(x,y)$-berilgan funksiya va ${}_C D^{\alpha}_{0y} u\,(x,y)$ - $u\,(x,y)$ funksiyaga y argumenti bo‘yicha ta’sir etuvchi Kaputo ma’nosidagi kasr tartibli operator [1]

$${}_C D^{\alpha}_{0y}\, u\,(x,y) = \frac{1}{\Gamma\,(1-\alpha)} \int\limits_0^y \frac{u_z\,(x,z)}{(y-z)^{\alpha}} dz$$

A **masala.** Shunday $u\,(x,y)$ funksiya topilsinki, u quyidagi xossalarga ega bo‘lsin:
1) $u\,(x,y)\,,\ u_x\,(x,y)\ \in C\left(\overline{\Omega}\right)$, $(\partial^2/\partial x^2)\,{}_C D^{\alpha}_{0y} u\,(x,y) \in C\,(\Omega)$;
2) Ω sohada (1) tenglamani qanoatlantiradi;
3) Ω soha chegarasida esa

$$u\,(0,y) = \varphi_1\,(y)\,,\ u_x\,(0,y) = \varphi_2\,(y)\,,\ y \in [0,q]\,; \tag{2}$$

$$u\,(x,0) = \tau\,(x)\,,\ x \in [0,p] \tag{3}$$

shartlarni qanoatlantirsin, bu yerda $\varphi_1\,(y)$, $\varphi_2\,(y)$ va $\tau\,(x)$ - berilgan funksiyalar.

Teorema. Agar $\tau\,(x) \in C^1\,[0,p]$, $\varphi_1\,(y)\,,\ \varphi_2\,(y) \in C\,[0,q]$, $f\,(x,y) \in C\left(\overline{\Omega}\right)$, $\tau\,(0) = \varphi_1\,(0)\,,\ \tau'\,(0) = \varphi_2\,(0)$ bo‘lsa, u holda A masalaning yechim

$$u\,(x,y) = +\tau\,(x) + [\varphi_1\,(y) - \varphi_1\,(0)] + x\,[\varphi_2\,(y) - \varphi_2\,(0)] -$$

$$-by^{\alpha} \int\limits_0^x (x-\xi)\, E_{(2,2),\,(\alpha,\alpha+)} \left[-b(x-\xi)^2 y^{\alpha}\right] \tau\,(\xi)\, d\xi -$$

$$-bx^3 \int\limits_0^y (y-\eta)^{\alpha-1}\, [\varphi_2\,(\eta) - \varphi_2\,(0)]\, E_{(2,4),(\alpha,\alpha)} \left[-b(y-\eta)^{\alpha} x^2\right] d\eta -$$

$$-bx^2 \int\limits_0^y (y-\eta)^{\alpha-1}\, [\varphi_1\,(\eta) - \varphi_1\,(0)]\, E_{(2,3),(\alpha,\alpha)} \left[-b(y-\eta)^{\alpha} x^2\right] d\eta +$$

$$+ \int\limits_0^x \int\limits_0^y f\,(\xi,\eta)\,(x-\xi)\,(y-\eta)^{\alpha-1} E_{(2,2),\,(\alpha,\alpha)} \left[-b(x-\xi)^2 (y-\eta)^{\alpha}\right]\ d\xi\, d\eta$$

ko‘rinishda bo‘ladi, bu yerda

$$E_{(\alpha_1,\beta_1),(\alpha_2,\beta_2)}\,(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma\,(\alpha_1 k + \beta_1)\,\Gamma\,(\alpha_2 k + \beta_2)}$$

- to‘rt parametrlik Mittag -Leffler funksiyasi [2].

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PANJARADAGI IKKI FERMIONLI SISTEMAGA MOS SHRODINGER OPERATORINING XOS QIYMATLARI SONI VA UNING JOYLASHGAN O‘RNI

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Biz ikki o'lchamli panjaradagi birinchi va ikkinchi eng yaqin tugunlarda o'zaro ta'sirlashuvchi ikkita bir xil fermionlar sistemasini qaraymiz.

Bu tezisning asosiy maqsadi ikki o'lchamli panjaradagi ikkita bir xil fermionlar sistemaga mos Schrödinger operatori $H_{\lambda\mu}(K)$, $K \in \mathbb{T}^2$ ning muhim spektridan chapdagi xos qiymatlari soni va joylashgan o'rnini aniqlashdan iborat.

Faraz qilaylik, $\mathbb{Z}^2$-ikki o'lchamli panjara va $\mathbb{T}^2 = (-\pi, \pi]^2$ esa ikki o'lchamli tor bo'lib, $\ell^{2,a}(\mathbb{Z}^2 \times \mathbb{Z}^2)$ va $L^{2,o}(\mathbb{T}^2)$ lar orqali mos ravishda $\mathbb{Z}^2 \times \mathbb{Z}^2$ va $\mathbb{T}^2$ da aniqlangan kvadrati bilan jamlanuvchi antisimmetrik va kvadrati bilan integrallanuvchi toq funksiyalarning Hilbert fazolari belgilangan bo'lsin.

Panjaradagi birinchi va ikkinchi eng yaqin tugunlarda $\widehat{\mathbb{V}}_{\lambda\mu}$ potinsial orqali ta'sirlashuvchi ikki fermionli sistemaga mos $\widehat{\mathbb{H}}_{\lambda\mu}$ hamiltonian koordinata tasviri quydagicha aniqlanadi:

$$\widehat{\mathbb{H}}_{\lambda\mu} = \widehat{\mathbb{H}}_0 - \widehat{\mathbb{V}}_{\lambda\mu}, \ \lambda, \mu > 0.$$

bunda $\widehat{\mathbb{H}}_0$ sistemaning kinetik energiya operatori bo'lib, $\ell^{2,a}(\mathbb{Z}^2 \times \mathbb{Z}^2)$ da quyidagicha aniqlanadi:

$$[\widehat{\mathbb{H}}_0 \hat{f}](x,y) = \sum_{m \in \mathbb{Z}^2} \hat{\varepsilon}(x-m)\hat{f}(m,y) + \sum_{n \in \mathbb{Z}^2} \hat{\varepsilon}(y-n)\hat{f}(x,n), \ \hat{f} \in \ell^{2,a}(\mathbb{Z}^2 \times \mathbb{Z}^2),$$

bu yerda

$$\hat{\varepsilon}(x) = \begin{cases} 2, & |x| = 0, \\ -\frac{1}{2}, & |x| = 1, \\ 0, & |x| > 1, \end{cases}$$

va $|x| = |x_1| + |x_2|$, $x = (x_1, x_2) \in \mathbb{Z}^2$. Birinchi va ikkinchi eng yaqin qo'shni tugunning o'zaro ta'sir potentsiali $\widehat{\mathbb{V}}_{\lambda\mu}$, $\hat{v}_{\lambda\mu}$ funksiyaga ko'paytirish operatori:

$$[\widehat{\mathbb{V}}_{\lambda\mu}\hat{f}](x,y) = \hat{v}_{\lambda\mu}(x-y)\hat{f}(x,y), \ \hat{f} \in \ell^{2,a}(\mathbb{Z}^2 \times \mathbb{Z}^2),$$

bu yerda

$$\hat{v}_{\lambda\mu}(x) = \begin{cases} \frac{\lambda}{2}, & x = \{(\pm 1, 0)(0, \pm 1)\}, \\ \frac{\mu}{2}, & x = \{(\pm 2, 0)(0, \pm 2)\}, \\ 0, & \text{qolgan hollarda.} \end{cases}$$

Schrödinger operatori $H_{\lambda\mu}(K)$, $K \in \mathbb{T}^2$, $L^{2,o}(\mathbb{T}^2)$ fazoda

$$H_{\lambda\mu}(K) := H_0(K) + V_{\lambda\mu},$$

ko’rinishda bo’ladi, bu yerda (qo’zg’almas) operator $H_0(K)-\varepsilon_K$ funksiyaga ko’paytirish operatori

$$(H_0(K)f)(p)=\varepsilon_K(p)f(p),\ \ f\in L^{2,o}(\mathbb{T}^2).$$

bunda

$$\varepsilon_K(p)=2\sum_{i=1}^{2}(1-\cos\frac{K_i}{2}\cos p_i)$$

va qo’zg’alish operatori $V_{\lambda\mu}$ quyidagi ko’rinishdagi

$$[V_{\lambda\mu}f](p)=\frac{\lambda}{(2\pi)^2}\sum_{i=1}^{2}\sin p_i\int\limits_{\mathbb{T}^2}\sin t_i f(t)\mathrm{d}t+\frac{\mu}{(2\pi)^2}\sum_{i=1}^{2}\sin 2p_i\int\limits_{\mathbb{T}^2}\sin 2t_i f(t)\mathrm{d}t.$$

integral operator bo’ladi.

Ravshanki, $V_{\lambda\mu}$ operatorning rangi ko’pi bilan to’rtga teng. Muhim spektr turg’unligi haqidagi Veyl teoremasiga ko’ra $H_{\lambda\mu}(K)$ operatorning muhim spektri $\sigma_{\rm ess}(H_{\lambda\mu}(K))$ berilgan $\lambda,\mu>0$ parametrga bog’liq emas va $H_0(K)$ operatorning spektri bilan ustma-ust tushadi. Shunday qilib

$$\sigma_{\rm ess}(H_{\lambda\mu}(K))=\sigma(H_0(K))=[\varepsilon_{\min}(K),\varepsilon_{\max}(K)],$$

tenglik o’rinli.

(λ,μ)-tekislikda

$$\mathrm{G}_2:=\Big\{(\lambda,\mu)\in\mathbb{R}^2:\ \lambda_0(\mu)>c_0+\frac{c_1}{\mu_1-\mu}\Big\},$$

sohani qaraymiz, bu yerda

$$c_0=\frac{30\pi-92}{3\pi}\frac{6\pi^2}{6\pi^2-8\pi-32}\approx 6.7741,$$

$$c_1=(1-\frac{30\pi-92}{3\pi}\frac{6\pi^2}{6\pi^2-8\pi-32}\frac{\pi-2}{2\pi})\frac{6\pi^2}{6\pi^2-8\pi-32}\approx -6.5551,$$

$$\mu_1=\frac{\pi-2}{2\pi}\frac{6\pi^2}{6\pi^2-8\pi-32}\approx 5.1606.$$

Teorema. Faraz qilaylik, $K=0$ va $\lambda,\mu>0$ bo’lsin, u holda:

(i) agar $(\lambda,\mu)\in\mathrm{G}_2$ bo’lsa, $H_{\lambda\mu}(0)$ operator $(-\infty,0)$ da ikkita $z_1(\lambda,\mu)$ va $z_2(\lambda,\mu)$ ikki karrali xos qiymatlarga ega, $(8,+\infty)$ da xos qiymatga ega emas;

(ii) agar $(\lambda,\mu)\in\partial\mathrm{G}_2$, bo’lsa, $H_{\lambda\mu}(0)$ operator $(\infty,0)$ da bitta $z_1(\lambda,\mu)$ ikki karrali xos qiymatga ega, $(8,+\infty)$ da xos qiymatga ega emas va $z=0$ soni $H_{\lambda\mu}(0)$ operator uchun ikki karrali virtual sath bo’ladi.

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BIR NOSTOXASTIK KVADRATIK OPERATOR QO‘ZG‘ALMAS NUQTALARI HAQIDA

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Nochiziqli dinamik sistemalar biologiya, fizika va boshqa sohalarning ko’pgina masalalarini hal qilishda qo’llaniladi [1-4]. Xususan, kvadratik dinamik sistemalar turli tabiatli population modellarni o’rganishda ishlatiladi, bunday modellar H Kesten, Yu.I Lyubich va U.A. Rozikovlarning ishlarida keltirilgan.

Aytaylik, $I = 1, 2, , m$ bo’lib, $x = (x_1, x_2, , x_m)$ - I to’plamning ehtimollik taqsimoti ya’ni

$$S^{m-1} = \{x = (x_1, x_2, ..., x_m) \in R^m; x_i \geq 0, \sum_{i=1}^{m} x_i = 1\} \tag{1}$$

S^{m-1} o’lchovli simpleks elementi bo’lsin.

Kvadratik stoxastik operator deb, S^{m-1} simpleksni o’zini-o’ziga akslantiruvchi quyidagi akslantirishga aytiladi:

$$V : x'_k = \sum_{i,j=1}^{m} P_{ij,k} x_i x_j, \;\; k = 1, \ldots, m. \tag{2}$$

bu yerda $P_(ij, k)$ koeffitsiyentlar

$$P_{ij,k} \geq 0, \;\; P_{ij,k} = P_{ji,k}, \;\; \sum_{k=1}^{m} P_{ij,k} = 1, \;\; i, j, k = 1, \ldots, m. \tag{3}$$

shartlarni qanoatlantiradi.

Har bir (3) shart bilan aniqlangan kvadratik stoxastik operator V biror $\mathbb{P} = (P_{ij,k})_{i,j,k=1}^{m}$ kubik matritsa yordamida bir qiymatli aniqlanadi.

(2) ko’rinishida aniqlangan kvadratik stoxastik operator ta’sirida $x^{(0)} \in S^{m-1}$ boshlang’ich nuqtaning trayektoriyasi (orbitasi) $x^{(n)}$ quyidagicha aniqlanadi: $x^{(n+1)} = V(x^{(n)}), n = 0, 1, 2, .$ Aytaylik, f biror bir X topologik fazoda aniqlangan uzluksiz akslantirish bo’lsin. Ushbu $f(x) = x$ tenglikni qanoatlantiruvchi $x \in X$ nuqtaga f akslantirishning qo’zg’almas nuqtasi deyiladi. $f^m(x) = x$tenglikni qanoatlantiruvchi $x \in X$ nuqtaga f akslantirish uchun m davriy nuqta deyiladi $f^m(x) = x$ tenglikni qanoatlantiruvchi eng kichik m soniga f akslantirishning tub davri deyiladi.

Ikki o’lchamli S^2 simpleksda aniqlangan nostoxastik kvadratik operatorga misol qaraymiz: Quyidagi ikki o’lchovli kvadratik stoxastik bo’lmagan operator V ni ko’rib chiqamiz.

$$V : (x, y, z) = S^2 \to (x', y', z') = V(x, y, z) \in S^2$$

$$V : \begin{cases} x' = 1/2x^2 + 2bxy + 1/2y^2 \\ y' = 1/2x^2 + 2(1-b)xy + 1/2y^2 \\ z' = z(2-z) \end{cases} \tag{4}$$

bu yerda, $b \in [-1/2, 3/2]$.

(4) ko'rinishdagi operatorning qo'zgalmas nuqtalarini topamiz. Buning uchun quyidagi

$$\begin{cases} x = 1/2x^2 + 2bxy + 1/2y^2 \\ y = 1/2x^2 + 2(1-b)xy + 1/2y^2 \\ z = z(2-z) \end{cases}$$

tenglamalar sistemasini yechamiz. $e_1 = (0,0,1)$, $e_2 = (\frac{-1-b-\sqrt{b^2-b+1/2}}{1-2b}, \frac{1-b+\sqrt{b^2-b+1/2}}{1-2b}, 0)$, $e_3 = (\frac{1}{2}, \frac{1}{2}, 0)$ qo'zg'almas nuqtalariga ega bo'ladi. (4) ko'rinishida aniqlangan ikki o'lchovli V kvadratik stoxastik bo'lmagan operator uchun

a) $e_1 = (0,0,1)$ nuqta tortuvchi qo'zg'almas nuqta bo'ladi. b) $b = \frac{1}{2}$ bo'lganda $e_3 = (\frac{1}{2}, \frac{1}{2}, 0)$ nuqta egar nuqta bo'ladi . c)Izoh: Analitik tahlil qilishni imkoni bo'lmaganligi tufayli, kompyuter dasturlash tillaridan foydalangan holda quyidagi natijalarni olamiz.

1) $b \in [-0,5, -04107)$ bo'lganda $|\lambda_{2,3}| > 1$ bo'lib, $e_2 = (\frac{-1-b-\sqrt{b^2-b+1/2}}{1-2b}, \frac{1-b+\sqrt{b^2-b+1/2}}{1-2b}, 0)$ nuqta itaruvchi qo'zgalmas nuqta bo'ladi.

2) $b = -0,4060$ bo'lganda $|\lambda_2| > 1$, $|\lambda_3| = 1$ bo'lib, $e_2 = (\frac{-1-b-\sqrt{b^2-b+1/2}}{1-2b}, \frac{1-b+\sqrt{b^2-b+1/2}}{1-2b}, 0)$ nuqta egar qo'zgalmas nuqta bo'ladi.

3) $b \in [-0,4013, -0,1475]$ bo'lganda $|\lambda_2| > 1$, $|\lambda_3| < 1$ bo'lib,

$$e_2 = (\frac{-1-b-\sqrt{b^2-b+1/2}}{1-2b}, \frac{1-b+\sqrt{b^2-b+1/2}}{1-2b}, 0)$$

nuqta egar qo'zgalmas nuqta bo'ladi. 4) $b \in [-0,1428, -0,03]$ bo'lganda $|\lambda_2| > 1$, $|\lambda_3| < 1$ bo'lib,

$$e_2 = (\frac{-1-b-\sqrt{b^2-b+1/2}}{1-2b}, \frac{1-b+\sqrt{b^2-b+1/2}}{1-2b}, 0)$$

nuqta egar qo'zgalmas nuqta bo'ladi.

5) $b \in [0,5, 0,609997]$ bo'lganda $|\lambda_{2,3}| > 1$, bo'lib,

$$e_2 = (\frac{-1-b-\sqrt{b^2-b+1/2}}{1-2b}, \frac{1-b+\sqrt{b^2-b+1/2}}{1-2b}, 0)$$

nuqta itaruvchi qo'zgalmas nuqta bo'ladi.

6) $b \in [0,6199988, 1,5]$ bo'lganda $|\lambda_2| < 1$, $|\lambda_3| > 1$ bo'lib,

$$e_2 = (\frac{-1-b-\sqrt{b^2-b+1/2}}{1-2b}, \frac{1-b+\sqrt{b^2-b+1/2}}{1-2b}, 0)$$

nuqta egar qo'zg'almas nuqta bo'ladi.

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TO‘RT O‘LCHAMLI FILIFORM LEYBNITS ALGEBRALARINING UZUNLIKLARI

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Nilpotent algebralarni o‘rganishda graduirlangan algebralar muhum ahamiyat kasb etadi. Graduirovkalarning ichida eng ko‘p qo‘llaniladigani bu Z-graduirovka hisoblanib, ushbu graduirovka algebraning xususiyatlarini batafsilroq o‘rganish imkonini beradi. Li algebralarining graduirovkalariga bag‘ishlangan bir qator ishlar chop qilingan bo‘lib, xususan [2], [3] ishlarda maksimal uzunlikdagi graduirovkaga ega bo‘lgan algebralar o‘rganilgan. B.A. Omirov esa [4] ishda maksimal uzunlikka ega bo‘lgan barcha filiform Leibniz algebralarining tasnifini olgan.

Ushbu ish ham nilpotent Leibniz algebralarining uzunliklarini o‘rganishga bag‘ishlangan bo‘lib, unda barcha to‘rt o‘lchamli filiform Leybnits algebralarining uzunliklari aniqlangan.

1-Ta’rif. F maydon ustida aniqlangan **L** algebra uchun quyiagi ayniyatlar bajarilsa, u holda u ***Li algebrasi*** deyiladi.

$$[x,x] = 0,$$

$$[[x,y],z] + [[y,z],x] + [[z,x],y] = 0$$

2-Ta’rif. F maydon ustida aniqlangan **L** algebraning ixtiyoriy x,y,z elementlari uchun quyidagi Leybnits ayniyati bajarilsaa, **L** algebra **Leybnits algebrasi** deyiladi.

$$[x,[y,z]] = [[x,y],z] - [[x,z],y].$$

Ushbu *quyi markaziy* qatorni aniqlaymiz

$$L^1 = L, \quad L^{k+1} = [L^k, L^1], \quad k \geq 1.$$

3-Ta’rif. Agar biror $k \in N$ soni uchun $L^k = 0$ shart bajarilsa, u holda L Leybnits algebrasi nilpotent deyiladi.

Agar L Leybnits algebrasi uchun $dim L^i = n - i$, $r \leq i \leq n$ shart o’rinli bo’lsa, u holda u filiform Leybnits algebrasi deyiladi.

Aytaylik, L chekli sondagi noldan farqli fazolardan tashkil topgan Z - graduirlangan Leybnits algebrasi bo’lsin, ya’ni $L = \oplus_{i \in Z} V_i$, bu yerda ixtiyoriy $i, j \in Z$ uchun $[V_i, V_j] \subseteq V_{i+j}$ bo’ladi. Agar $L = V_k \oplus V_{k+1} \oplus \cdots \oplus V_{k+m}$ da barcha i $(1 \leq i \leq m)$ uchun $V_{k+i} \neq 0$ bo’lsa, L nilpotent Leybnits algebrasi bog’liqli graduirovkaga ega deyiladi.

Bog’liqli graduirovka uchun $len(\oplus L) = m + 1$ soni graduirovkaning uzunligi deyiladi.

$$l(L) = max\{len(\oplus L) \mid L = V_k \oplus V_{k+1} \oplus \cdots \oplus V_{k+m}\}$$

soniga L Leybnits algebrasining **uzunligi** deb ataladi.

Quyidagi teoremada barcha to’rt o’lchamli filiform Leibnits algebralarining tasnifi berilgan

1-Teorema. [1]. Ixtiyoriy to’rt o’lchamli kompleks filiform Leybnits algebrasi quyidagi o’zaro izomorf bo’lmagan algebralarning biriga izomorf boladi.

$$
\begin{aligned}
R_2: &\quad [e_1,e_1]=e_3,\ [e_1,e_2]=e_4,\ [e_2,e_1]=e_3, [e_3,e_1]=e_4;\\
R_3: &\quad [e_1,e_1]=e_3;\ [e_2,e_1]=e_3;\ [e_3,e_1]=e_4;\\
R_4(\alpha): &\quad [e_1,e_1]=e_3;\ [e_1,e_2]=\alpha e_4;\ [e_2,e_1]=e_3, [e_2,e_2]=e_4, [e_3,e_1]=e_4;\\
R_5: &\quad [e_1,e_1]=e_3,\ [e_1,e_2]=e_4,\ [e_3,e_1]=e_4;\\
R_6: &\quad [e_1,e_1]=e_3,\ [e_2,e_2]=e_4,\ [e_3,e_1]=e_4;\\
R_7: &\quad [e_1,e_1]=e_4,\ [e_2,e_1]=e_3,\ [e_3,e_1]=e_4,\ [e_1,\ e_2]=-e_3, [e_1,e_3]=-e_4;\\
R_8: &\quad [e_1,e_1]=e_4,\ [e_2,e_1]=e_3,\ [e_3,e_1]=e_4,\ [e_1,e_2]=-e_3+e_4, [e_1,e_3]=-e_4;\\
R_9: &\quad [e_1,e_1]=e_4,\ [e_2,e_1]=e_3,\ [e_2,e_2]=e_4,\ [e_3,e_1]=e_4,\\
&\quad [e_1,\ e_2]=-e_3+2e_4,\ [e_1,e_3]=-e_4;\\
R_{10}: &\quad [e_1,e_1]=e_4,\ [e_2,e_1]=e_3,\ [e_2,e_2]=e_4,\ [e_3,e_1]=e_4,\\
&\quad [e_1,e_2]=-e_3,\ [e_1,e_3]=-e_4;
\end{aligned}
$$

Ushbu teoremada barcha to’rt o’lchamli filiform Leybnits algebralarining uzunliklari aniqlangan

2-Teorema. $R_2 - R_{10}$ algebralarning uzunliklari quyidagilarga teng:

$$
\begin{aligned}
&l(R_2)=1,\quad l(R_3)=3,\quad l(R_4(\alpha))=1,\quad l(R_5)=3;\\
&l(R_6)=1,\quad l(R_7)=3,\quad l(R_8)=1,\qquad l(R_9)=1,\quad l(R_{10})=1.
\end{aligned}
$$

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SPIN FAKTORLARI BO‘YICHA 2-LOKAL AVTOMORFIZMLAR TAVSIFI

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$A = e_1, e_2, ..., e_{mF}$ F maydondagi algebra bo‘lsin. a A algebraning elementi bo’lsin. U xolda $a = a_1e_1 + a_2e_2 + a_3e_3 + \cdots + a_me_m$ kabi yozishimiz mumkin, bu yerda $a_1, a_2, ..., a_m$ lar F ning elementlari. Ushbu maqolada $\bar{a} = (a_1, a_2, ..., a_m)^T$ sifatida qaraymiz. Aksincha, agar $\nu = (a_1, a_2, ..., a_m)^T$ F da $a_1, a_2, ..., a_m$ ko’rinishdagi ustun vektor bo’lsa, $\hat{\nu}$ni $a_1e_1 + a_2e_2 + a_3e_3 + \cdots + a_me_m$, ko’rinishda olamiz, ya’ni $\hat{\nu} = a_1e_1 + a_2e_2 + a_3e_3 + \cdots + a_me_m$. Aytaylik A algebra bo’lsin. Ma’lumki A algebrada $f(x) = Ax$ chiziqli akslantirish berilgan bo’lib $f(x, y) = f(x) \cdot f(y)$ tenglik bajarilsa, u holda bu akslantirish A algebralarda avtomorfizmdan ibrat bo’ladi.

Bizga cheksiz o’lchovli spin faktor $R1 \oplus H$ berilgan bo’lsin. Uning Gilbert fazosi sifatida to’la ortonormal sistemasi (bazisi) $B = (1, s_1, s_2, .., s_n, ..)$ bo’lsin. Ko’paytirish jadvali quyidagicha, ya’ni

1. $1 \cdot s_i = s_i \cdot 1$, $i = 1, 2, 3, ...$
2. $s_i \cdot s_j = 0$, $i \neq j$
3. $s_i^2 = 1$, $i = 1, 2, 3, ...$

Aytaylik, bu spin faktorning ixtiyoriy avtomorfizmi Δ berilgan bo’lib, uning matritsasi bo’lib, uning ko’rinishi quyidagicha bo’lsin

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} & ... \\ a_{21} & a_{22} & ... & a_{2n} & ... \\ ... & ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} & ... \\ ... & ... & ... & ... & ... \end{pmatrix}$$

Teorema. $R1 \oplus H$ spin faktorning har qanday avtomorfizm matritsasi kosasimmetrik matritsa bo’ladi.

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SINGULYAR KOEFFITSIENTGA EGA BO‘LGAN ARALASH TURDAGI TENGLAMA UCHUN BIR CHEGARAVIY MASALA HAQIDA

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$\Omega = \Omega_1 \cup \Omega_0$ sohada

$$0 = \begin{cases} (\partial/\partial x)\ {}_C D_{0y}^{\alpha} u(x,y) + \lambda u(x,y), \ (x,y) \in \Omega_1, \\ u_{xx}(x,y) + u_{yy}(x,y) + \dfrac{2\beta}{y} u_y, \ (x,y) \in \Omega_0, \end{cases} \tag{1}$$

tenglamani qaraylik, bu yerda $\Omega_1 = \{(x,y) : -1 < x < 1, \ 0 < y < q\}$, $\Omega_0 = \{(x,y) : x^2 + y^2 < 1, \ y < 0\}$, $u(x,y)$- noma’lum funksiya, α, β, λ - berilgan haqiqiy sonlar, $0 < \alpha < 1$, $0 < \beta < 1/2$, ${}_C D_{0y}^{\alpha}$ - y argument bo‘yicha ta’sir etuvchi Kaputo kasr tartibli integro-differensial operator [1]

$${}_C D_{0y}^{\alpha} u(x,y) = I_{0y}^{1-\alpha} u_y(x,y) = \frac{1}{\Gamma(1-\alpha)} \int_0^y (y-z)^{-\alpha} u_z(x,z)\, dz,$$

I_{ay}^{γ} - Riman - Liuvill ma’nosidagi kasr tartibli integral operator [1].

T **masala.** Quyidagi xossalarga ega bo‘lgan $u(x,y)$ funksiya topilsin:

1) $u(x,y) \in C(\overline{\Omega})$, $(\partial/\partial x)\ {}_C D_{0y}^{\alpha} u(x,y) \in C(\Omega_1)$, $u(x,y) \in C^2(\Omega_2)$;

2) $\Omega = \Omega_1 \cup \Omega_2$ sohada (1) tenglamani va

$$u(-1,y) = \varphi(y), \ 0 \le y \le q,$$

$$u(x,y)|_{\sigma_0} = \psi(x,y),$$

chegaraviy shartni va

$$\lim_{y \to -0} (-y)^{2\beta} u_y(x,y) = \lim_{y \to +0} {}_C D_{0y}^{\alpha}(x,y), \ 0 < x < p,$$

ulash shartini qanoatlantirsin, bu yerda $\varphi(y)$, $\psi(x,y)$ - berilgan funksiyalar, $\sigma_0 = \{(x,y) : x^2 + y^2 = 1, \ y \le 0\}$.

Teorema. T masala bittadan ortiq yechimga ega emas.

Bu teorema energiya integrallari usulidan foydalanib isbotlanadi.

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BA’ZI KICHIK O‘LCHAMLI LI ALGEBRALARINING TASVIRLARI

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1-ta’rif: $\mathbb{F}$ maydon ustidagi L vektor fazo berilgan bo‘lsin. $L \times L \to L$, $(x, y) \mapsto [x, y]$ ko‘rinishida aniqlangan akslantirish x va y ning qavsi yoki kommutatori deyiladi. Agar u quyidagi aksiomalarni qanoatlantirsa, u holda L vektor fazo $\mathbb{F}$ maydon ustidagi Li algebrasi deyiladi:

1. Qavs amali bichiziqli;
2. Ixtiyoriy $x \in L$ uchun $[x, x] = 0$ o‘rinli;
3. Ixtiyoriy $(x, y, z \in L)$ uchun $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ Yakobi ayniyati o’rinli.

2-ta’rif: Berilgan L Li algebrasining V vektor fazodagi **tasviri** deb $\phi : L \to gl(V)$ (bu yerda $gl(V) = End(V), dimV = n$) gomomorfizmga aytiladi. Boshqacha qilib aytganda, agar $\phi : L \to gl(V)$ chiziqli akslantirish ixtiyoriy $x, y \in L$ uchun $[\phi(x), \phi(y)] = \phi([x, y])$ shartni qanoatlantirsa, u holda ϕ akslantirishga L Li algebrasining V vektor fazodagi tasviri deyiladi.

L algebra sifatida quyidagi kichik o‘lchamli klassik Li algebralarini qaraylik.

1. $L = sp(2, \mathbb{R}) = \left\{ x \in gl(2, \mathbb{R}) \mid sx = -x^t s, s = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\}$ simplektiv Li algebrasi bo‘lsin va $\phi : sp(2, \mathbb{R}) \to gl(2, \mathbb{R})$ chiziqli akslantirishni ixtiyoriy $z \in gl(2, \mathbb{R})$ va barcha $x \in sp(2, \mathbb{R})$ lar uchun $\phi(x) = z \cdot x$ ni berilgan z matritsaga ko‘paytirish operatorini qaraylik. Quyida bu akslantirish qanday $z \in gl(2, \mathbb{R})$ lar uchun tasvir bo‘lishini tekshiramiz. Buning uchun quyidagi munosabatlarni qanoatlantiruvchi $sp(2, \mathbb{R})$ uchun quyidagi $\{e_1, e_2, e_3\}$ bazisdan foydalanamiz:

$$e_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \ e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},$$

$$[e_1, e_2] = 2e_2, \ [e_1, e_3] = -2e_3, \ [e_2, e_3] = e_1.$$

Ixtiyoriy $z = \begin{pmatrix} z_1 & z_2 \\ z_3 & z_4 \end{pmatrix} \in gl(2, \mathbb{R})$ elementni olaylik. Qaralgan akslantirishni yuqorida keltirilgan tasvir shartlariga qo‘yamiz. $sp(2, \mathbb{R})$ dagi barcha elementlar bazis elementlarning chiziqli kombinatsiyasidan tuzilganligi va akslantirish chiziqli ekanligidan faqat bazis elementlar uchun tekshirsak yetarli bo‘ladi.

Gomomorfizm shartini e_1 va e_2 bazislar uchun qo’yaylik:

$$\phi([e_1, e_2]) = [\phi(e_1), \phi(e_2)];$$
$$z \cdot [e_1, e_2] = [z \cdot e_1, z \cdot e_2];$$
$$z \cdot 2e_2 = z \cdot e_1 \cdot z \cdot e_2 - z \cdot e_2 \cdot z \cdot e_1.$$

Agar oxirgi tenglikni matritsa ko’rinishida yozsak, quyidagini hosil qilamiz:

$$\begin{pmatrix} 0 & 2z_1 \\ 0 & 2z_3 \end{pmatrix} = \begin{pmatrix} -z_3z_1 & z_1^2 + z_1z_4 - z_2z_3 \\ -z_3^2 & z_3z_1. \end{pmatrix}$$

Bu tenglikdan z matritsa uchun 2 ta yechimni hosil qilamiz: $\begin{pmatrix} 0 & z_2 \\ 0 & z_4 \end{pmatrix}, \begin{pmatrix} z_1 & z_2 \\ 0 & 2-z_1 \end{pmatrix}$.

Xuddi shu usulda e_1, e_3 va e_2, e_3 juftliklar uchun ham hisoblaymiz. Mos ravishda natijalar quyidagicha bo‘ladi:

$\begin{pmatrix} z_1 & 0 \\ z_3 & 0 \end{pmatrix}, \begin{pmatrix} z_1 & 0 \\ z_3 & 2-z_1 \end{pmatrix}; \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & z_2 \\ z_3 & 1 \end{pmatrix}$.

Olingan natijalardan ko‘rinib turibdiki, akslantirish tasvir bo‘lishi uchun z element aynan 0 yoki birlik matritsa bo‘lishi shart ekan. Bu esa trivial tasvir bo‘ladi.

2. Endi $L = o(3,\mathbb{R}) = \left\{ x \in gl(3,\mathbb{R}) \mid sx = -x^t s, s = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \right\}$ ortogonal Li algebrasi uchun ham yuqoridagi akslantirishni tekshiramiz. $\phi : o(3,\mathbb{R}) \to gl(3,\mathbb{R})$, $\phi(x) = z \cdot x$. Bu akslantirish qanday $z \in gl(3,\mathbb{R})$ lar uchun ham tasvir bo‘lishini tekshiramiz. Buning uchun quyidagi munosabatlarni qanoatlantiruvchi $o(3,\mathbb{R})$ uchun quyidagi $\{h_1, h_2, h_3\}$ bazisdan foydalanamiz:

$$h_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, h_2 = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; h_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix},$$

$$[h_1, h_2] = h_2, \; [h_1, h_3] = -h_3, \; [h_2, h_3] = -h_1.$$

Ixtiyoriy $z = \begin{pmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{pmatrix} \in gl(3,\mathbb{R})$ elementni olaylik. Yuqoridagi kabi hisob-kitoblarni amalga oshirib, quyidagi natijani olamiz:

Teorema: Berilgan $\phi : o(3,\mathbb{R}) \to gl(3,\mathbb{R}), \phi(x) = z \cdot x, x \in o(3,\mathbb{R}), z \in gl(3,\mathbb{R})$ akslantirish tasvir bo‘lishi uchun $z = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ yoki $z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ bo‘lishi zarur va yetarli.

Izoh: Yuqoridagi qaralgan ϕ akslantirish z matritsaga ko‘paytirish operatori bo‘ladi. Agar bu akslantirish ta’rifida matritsalarning oddiy ko‘paytirish amali o‘rniga Li qavsi amalini qarasak, yuqorida qaralayotgan akslantirish tasvirga aylanadi va bunday tasvir qo‘shma tasvir deb ataladi [1].

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INVOLYUTSIYALI BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMA UCHUN BITSADZE - SAMARISKIY MASALASI

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Oxirgi yillarda bir nechta xil argumentli noma’lum funksiya qatnashgan differensial tenglamalar bilan shug‘ullanishga bo‘lgan qiziqish ortib bormoqda. Bunga sabab ko‘plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo‘yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar kechikuvchi argumentli yoki invalutsiyalangan differensial tenglama deb yuritiladi. Shu sababdan biz ushbu ishda invalutsiyalangan birinchi tartibli differensial tenglama uchun Bitsadze - Samariskiy masalani tadqiq etamiz.

$(1, b)$ oraliqda ushbu

$$y'(x) = \lambda y\left(\frac{1}{x}\right) \tag{1}$$

invalutsiyalangan differensial tenglamani qaraylik, bu yerda $y(\cdot)$ - noma’lum funksiya, b, λ - berilgan haqiqiy sonlar bo‘lib, $|\lambda| < 1/2$, $b > 1$.

Ko‘rish qiyin emaski $\lambda = 0$ bo‘lgan holda (1) tenglama invalutsiyalangan bo‘lmaydi, shu sababdan biz ushbu ishda $\lambda \neq 0$ bo‘lgan holini qaraymiz.

Invalutsiyalangan tenglamlar haqida ma’lumotlar [1] - [3] ishlarda berilgan.

BS **masala.** (1) tenglamani va

$$y(1) = y(\xi) + B \tag{2}$$

boshlang‘ich shartni qanoatlantiruvchi shunday $y(\cdot) \in C[0,1] \cap C^1(0,1)$ funksiya topilsin, bu yerda ξ, B berilgan haqiqiy sonlar bo‘lib, $1 < \xi < b$.

(1) tenglamaning $|\lambda| < 1/2$ bo‘lgandagi umumiy yechimi

$$y(x) = \left(\frac{\lambda x^{k_1} + k_1 x^{k_2}}{k_1}\right) C_1 \tag{3}$$

ko‘rinishda bo‘ladi [4], bu yerda $k_1 = \left(1 + \sqrt{1 - 4\lambda^2}\right)/2$, $k_2 = \left(1 - \sqrt{1 - 4\lambda^2}\right)/2$, C_1 - ixtiyoriy o‘zgarmas son.

(3) ni (2) shartga bo‘ysundirib, C_1 ni

$$C_1 = \frac{k_1 B}{\left(1 - \xi^{k_1}\right)\lambda + \left(1 - \xi^{k_2}\right) k_1} \tag{4}$$

ko‘rinishda topamiz.

Bu yerda $\left(1 - \xi^{k_1}\right)\lambda + \left(1 - \xi^{k_2}\right) k_1 < 0$ bo‘lganligi uchun (4) ni (3) ga qo‘yib, BS masalaning yechimini

$$y(x) = B\frac{\lambda x^{k_1} + k_1 x^{k_2}}{\left(1 - \xi^{k_1}\right)\lambda + \left(1 - \xi^{k_2}\right) k_1} \tag{5}$$

ko‘rinishda topamiz.

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PANJARADAGI BIR ZARRACHALI SHRYODINGER OPERATORINING SPEKTRAL XOSSALARI

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Ushbu ishning asosiy natijasi bir o'lchamli $\mathbb{Z}$ panjarada eng kichik xos qiymati oddiy bo'lgan bir zarrachali diskret Shryodinger operatorini quramiz. Ushbu holat faqat panjaraga xos bo'lib, uzluksiz holda unga o'xshash holat yo'q.

Masalaning qo'yilishi. Biz $\mathbb{Z}$ bilan bir o'lchamli panjarani, $\ell_2(\mathbb{Z})$ bilan $\mathbb{Z}$ da aniqlangan va kvadrati bilan jamlanuvchi funksiyalarning Hilbert fazosini belgilaymiz. $\mathbb{Z}$ panjarada bir kvant zarrachasining $\hat{H}_0$ erkin hamiltoniani $\ell_2(\mathbb{Z})$ Hilbert fazosida quyidagicha aniqlanuvchi operator bo'ladi:

$$(\hat{H}_0\hat{\psi})(x) = \sum_{s\in\mathbb{Z}} \hat{\varepsilon}(s)\hat{\psi}(x+s),\ \ \hat{\psi} \in \ell_2(\mathbb{Z}).$$

Bu yerda

$$\hat{\varepsilon}(s) = \begin{cases} 1, & s = 0 \\ -\frac{1}{2}, & s = \pm 2 \\ 0, & s \in \mathbb{Z}\backslash\{0, \pm 2\}. \end{cases}$$

$\hat{H}_0$ erkin Hamiltonianning chegaralangan qo'zg'alishlarida $\hat{V}_{\mu\lambda}$ tashqi maydon ta'sirida d o'lchamli panjarada harakatlanuvchi bitta zarrachaning to'la hamiltoniani

$$\hat{H}_{\mu\lambda} = \hat{H}_0 - \hat{V}_{\mu\lambda}$$

formula orqali beriladi, $\hat{V}_{\mu\lambda}$ operator esa $\hat{v}_{\mu\lambda}$ funksiyaga ko'paytirish operatoridir, ya'ni

$$(\hat{V}_{\mu\lambda}\hat{\psi})(x) = \hat{v}_{\mu\lambda}(x)\hat{\psi}(x),\ \ \hat{\psi} \in \ell_2(\mathbb{Z}).$$

Bu yerda

$$\hat{v}_{\mu\lambda}(x) = \begin{cases} \mu, & x = 0 \\ \lambda, & x = 1 \\ 0, & x \in \mathbb{Z}\backslash\{0, 1\}. \end{cases}$$

bo'lib, μ va λ larni musbat deb faraz qilamiz, ya'ni $\mu, \lambda > 0$.

Shryodinger operatorining xos qiymatlari va xos funksiyalari

Fure almashtirishidan so'ng $\hat{H}_0$ va $\hat{V}_{\mu\lambda}$ operatorlar quyidagi mos unitar ekvivalent operatorlarga o'tadi:

$$(H_0 f)(p) = \varepsilon(p)f(p),\ \ \varepsilon(p) = 1 - \cos 2p$$

$$(V_{\mu\lambda f})(p) = \frac{1}{2\pi}\int_{-\pi}^{\pi} (\mu + \lambda e^{i(p-q)})f(q)dq = \mu c_1 + \lambda e^{ip}c_2,$$

bu yerda

$$c_1 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(q)dq,\ \ c_2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-iq}f(q)dq.$$

Bu unitary ekvivalentlik

$$F : L_2(\mathbb{T}) \to \ell_2(\mathbb{Z}),\ \ (Ff)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{-i(x,q)} f(q)dq,\ x \in \mathbb{Z} f \in L_2(\mathbb{T})$$

Furye akslantirishi orqali amalga oshiriladi. Bu yerda $\mathbb{T} = (-\pi; \pi]$ bir o'lchamli tor.

H_0 operator o'z-o'ziga qo'shma bo'lib, uning spektri absolyut uzluksiz va $\sigma(H_0) = [0; 2]$ ekanligini osongina ko'rsatish mumkin.

Eslatma. $\hat{\varepsilon}(\cdot)$ funksiyalarning $\varepsilon(p) = 1 - \cos 2p$ Furye akslantirishi $p = 0$ nuqtada yagona minimumga ega va $\varepsilon_{\min} = \min_{p\in\mathbb{T}} \varepsilon(p) = \varepsilon(0) = 0$.

Teorema. Agar $\mu \neq \lambda$ bo'lsa, u holda $H_{\mu\lambda}$ operator ikkita oddiy xos qiymatga ega va ularga mos xos funksiyalarning har biri musbat, lekin qa'tiy musbat emas;
Agar $\mu = \lambda$ bo'lsa, u holda $H_{\mu\lambda}$ operator bitta ikki karrali xos qiymatga ega va unga mos xos funksiyalardan birini qa'tiy musbat qilib tanlash mumkin.

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NAVBAT UZUNLIGI CHEGARALANGAN IMTIYOZLI XIZMAT KO‘RSATISH TARMOG‘INING BANDLIK DAVRI

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Bitta xizmat ko’rsatish qurilmasidan iborat bo’lgan xizmat ko’rsatish tarmog’iga λ_1 va λ_2 parametrli ikki xil Puasson talablar oqimi kelib tushayotgan bo’lsin. Xizmat ko’rsatish tartibi: oxirigacha xizmat ko’rsatiladigan mutlaq imtiyoz, ya’ni tarmoqda ikki turdagi talablar mavjud bo’lsa, qurilmaga avval birinchi turdagi talablar qabul qilinadi. Agar ikkinchi turdagi talabning xizmat vaqti davomida tarmoqqa birinchi turdagi talab tushsa, ikkinchi turdagi talabning xizmati shu joyda uzib qo’yiladi va kelgan talabga xizmat ko’rsatiladi. Tarmoq birinchi turdagi xamma talablardan bo’shagandan keyin xizmati uzib qo’yilgan talabga xizmat ko’rsatish davom ettiriladi. Talablarga xizmat ko’rsatish vaqti uzunliklari uzaro bog’liq bo’lmagan tasodifiy miqdorlar bo’lib, bunday holada B_1 (∞) va $B_2(\infty)$ umumiy taqsimot funksiyasiga va μ_1^{-1}, μ_2^{-2} o’rtacha qiymatga ega bo’lsin. SHuningdek, birinchi turdagitalablar uchun belgilangan kutish joylari soni chekli $N(N \geq 1)$ miqdorga teng deb faraz qilamiz. Qaralayotgan ishda yuqorida ta’riflangan xizmat ko’rsatish tarmog’i bandlik davri taqsimotining Laplas-Stiltes almashtirishi topilgan.

Quyidagi belgilarni kiritamiz:

$$\bar{\mathrm{b}}_{\mathrm{i}}(\mathrm{s}) = \int_0^{\infty} e^{-Sx} dB_{\mathrm{i}}(\mathrm{x}), \quad \mathrm{Res} \geq 0, \mathrm{i} = 1; 2$$

ξ - tarmoqning bandlik davri,

ξ_{i}-tarmoqning faqat birinchi turdagi talablarga xizmat ko’rsatilgan xolatdagi bandlik davri,

$$\bar{\mathrm{g}}(\mathrm{s}) = \int_0^{\infty} e^{-Sx} dp(\xi < x), \quad \mathrm{Res} \geq 0$$

$$\bar{\mathrm{g}}_{\mathrm{N}}(\mathrm{s}) = \int_0^{\infty} e^{-Sx} dp(\xi \mathrm{N} < x), \quad \mathrm{Res} \geq 0$$

Teorema. Quyidagi munosabat o‘rinli:

$$\bar{\mathrm{g}}(\mathrm{s}) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \quad \tilde{\mathrm{f}}(\mathrm{s}) + \frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \bar{\mathrm{b}}_1 \left[\mathrm{s} + \lambda_2 - \lambda_2 \mathrm{f}(\mathrm{s}) \right],$$

bu yerda $\mathrm{f}(\mathrm{s}) \quad \mathrm{x} = \bar{\mathrm{B}}_2 \left[\mathrm{s} + \lambda_1 - \lambda_1 \quad \bar{\mathrm{g}}_{\mathrm{N}}(\mathrm{s}) + \lambda_2 - \lambda_2 \mathrm{x} \right]$ tenglamalarning $|\tilde{\mathrm{f}}(\mathrm{g})| \leq 1$ Shartni qanoatlantiruvchi yagona yechimi,

$$\bar{\mathrm{g}}_{\mathrm{N}}(\mathrm{s}) = \frac{\Delta_{N-1}(s)}{\Delta_N(\ \mathrm{s})}$$

$$\sum_{K=0}^{\infty} v^{\mathrm{K}} \Delta_{\mathrm{K}}(\mathrm{s}) = \frac{v\bar{B}_1(s) - \bar{e}_1(s + \lambda_1 - \lambda_1 v)}{(1-v)[v - \bar{e}_1(s + \lambda_1 - \lambda_1 v)]}$$

xususiy holda, agar $\mathrm{B_2(x)} = 1 - e^{-\mu 2x}, \mathrm{x} > 0$ bo'lsa

$$\bar{\mathrm{g}}_{\mathrm{N}}(\mathrm{s}) = \frac{\mu_1}{\lambda_1} * \frac{\alpha_1^N(1-\lambda_1) + \alpha_2^N(\lambda_2 - 1)}{\alpha_1^{N+1}(1-\lambda_1) + \alpha_1^{N+1}(\lambda_2 - 1)}$$

$$\alpha_{1,2} = \frac{\lambda_1 + \mu_1 \pm \sqrt{(\lambda 1 + \mu_1 + s)^2 - 4\lambda_1\mu_1}}{2\lambda_1}$$

$$\tilde{\mathrm{f}}(\mathrm{s}) = \frac{\lambda_1 + \mu_1 + s + \lambda_1\left[1 - g_N(s)\right] - \sqrt{\lambda_2 + \mu_2 + s + \lambda_1\left[1 - g_N(s)\right]^2 - 4\lambda 2\mu_2}}{2\lambda_2}$$

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ABEL C*-ALGEBRALARNING TASNIFI HAQIDA GELFAND-NAYMARK TEOREMASI

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A kommutativ kompleks algebra bo‘lsin. A algebrada gomomorf chiziqli funksional algebraning **xarakteri** deyiladi. Algebraning barcha xarakterlari to‘plami Ω bilan belgilanadi, ya’ni

$$\Omega = \{\chi : A \to \mathbb{C},\ \ \chi - \text{chiziqli va } \chi(a \cdot b) = \chi(a) \cdot \chi(b),\ \ \forall a, b \in A\}$$

Muhim xossalar:

1) $\chi \equiv 0$ xarakterdir, unga trivial xarakter deyiladi;
2) Agar algebraning birlik $\mathbf{1}$ elementi bor bo‘lsa, u holda $\chi(\mathbf{1}) = 1$;
3) har qanday xarakter uzluksizdir;
4) $\|\chi\| \leq 1,\ \ \forall \chi \in \Omega$;
5) kommutativ C*-algebra uchun $\chi(a^*) = \overline{\chi(a)}, \forall a \in A, \forall \chi \in \Omega$;
6) Ω nuqtada yaqinlashish bo‘yicha Xausdorf fazosidir, ya’ni T_2-fazosidir.
7) Ω lokal kompakt fazodir.

Teorema 1. Ω metrikalanuvchi topologik fazo bo‘lishi uchun kommutativ C*-algebra A separabel bo‘lishi zarur va yetarlidir.

Endi biror lokal kompakt X fazosi uchun quyidagi fazoni qaraymiz:

$$C_o(X) = \{f : X \to \mathbb{C}, f - \text{uzluksiz va } f(\infty) = 0\}$$

bu yerda $f(\infty) = 0$ ifoda quyidagini bildiradi:
$\forall \varepsilon > 0$ uchun shunday $K \subset X$ kompakt to‘plam mavjudki: $|f(x)| < \varepsilon$, $\forall x \in X \setminus K$.

U holda $C_o(X)$ fazosi tabiiy amallarga ko‘ra kommutativ C*-algebrasi bo‘ladi.
Quyidagi asosiy teorema kommutativ (abel) C*-algebralarning tasnifi haqidadir.

Teorema 2. (Gelfand-Naymark teoremasi) A kommutativ C*-algebra bo‘lsin. U holda A algebra $C_o(\Omega)$ algebrasiga izometrik izomorfdir, ya’ni $A \cong C_o(\Omega)$.

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TEMPERLANGAN HILFER OPERATORI BUZILADIGAN ODDIY DIFFERENSIAL TENGLAMA UCHUN KOSHI MASALASI

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Ma’lumki oxirgi yillarda kasr tartibli differensial tenglamalarga qiziqish tobora ortib bormoqda. Bu bir tomondan, matematik umumlashma sifatida oldingi natijalarni o’z ichiga olsa, boshqa tomondan ko’pgina amaliy jarayonlarning matematik modelida turli kasr tartibli differensial tenglamalar ishlatilmoqda [1,2]. Bunday differensial tenglamalarni tadqiq etishda boshlang’ich shartli masalaning yaqqol ko’rinishdagi yechimlari muhim rol o’ynaydi [3]. Bunday masalalarni tadqiq etishda Laplas almashtirishlari, operator usullar yoki integral tenglamalarga keltirib ishlash usullaridan foydalaniladi [4].

Kasr tartibli operatorlarning asosiylari Riman-Liuvill va Kaputo ma’nosidagi operatorlar bo’lsa, ularning turli umumlashmalari ko’p ishlarda tadqiq etilmoqda [5].

Ushbu ishda temperlangan Hilfer operatori buziladigan oddiy differensial tenglama uchun Koshi masalasini tadqiq etamiz.

$(0,T)$ oraliqda

$$ {}^{TH}D_{0t}^{\alpha,\beta,\gamma}y(t)+\lambda y(t)=f(t) \tag{1}$$

tenglamani qaraylik, bu yerda ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}-$ temperlangan Hilfer kasr tartibli operator bo‘lib,

$$ {}^{TH}D_{0t}^{\alpha,\beta,\gamma}y(t)=e^{-\gamma t}I_{0t}^{\beta(1-\alpha)}\frac{d}{dt}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y(t)\right) \tag{2}$$

ko‘rinishda aniqlangan,

$$ I_{0t}^{\varphi}g(t)=\frac{1}{\Gamma(\varphi)}\int\limits_0^t(t-s)^{\varphi-1}g(s)ds \tag{3}$$

Riman-Liuvill ma’nosidagi kasr tartibli integral operator, $\alpha,\beta,\gamma,\lambda\in\mathbb{R}$, $\gamma\geq 0$, $0<\alpha<1$, $0\leq\beta\leq 1$, $f(t)$- berilgan funksiya.

Agar $\beta=0$ ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}$ dan ${}^{THL}D_{0t}^{\alpha,\gamma}$temperlangan Riman-Liuvill kasr tartibli hosila operator bo’ladi, agar $\beta=1$ ${}^{TH}D_{0t}^{\alpha,\beta,\gamma}$ dan ${}^{TC}D_{0t}^{\alpha,\gamma}$ temperlangan Kaputo kasr tartibli hosila operator bo’ladi.

Koshi masalasi. (1) tenglamani va

$$ \lim_{t\to 0}I_{0t}^{(1-\beta)(1-\alpha)}\left(e^{\gamma t}y(t)\right)=A \tag{4}$$

$t^{-(1-\beta)(1-\alpha)}y\left(t\right)\in C\left[0,T\right]$ shartlarni qanoatlantiruvchi funksiya topilsin, bu yerda A berilgan haqiqiy son.

Teorema. Agar $f(x)\in C^1\left[0,T\right]$ bo‘lsa, u holda

$$ y(t)=Ae^{-\gamma t}t^{-(1-\beta)(1-\alpha)}E_{\alpha,1-(1-\beta)(1-\alpha)}\left[-\lambda t^{\alpha}\right]+$$

$$ +e^{-\gamma t}\int\limits_0^t(t-s)^{\alpha-1}E_{\alpha,\alpha}\left[-\lambda(t-s)^{\alpha}\right]e^{\gamma s}f(s)ds$$

formula bilan aniqlangan $y(t)$ funksiya $\{(1),(4)\}$ masalaning yechimi bo‘ladi, bu yerda

$$E_{\alpha_1,\beta_1}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{\Gamma(\alpha_1 n+\beta_1)}$$

Mittag-Leffler funksiya.

Bevosita ko‘rsatish qiyin emaski (4) shartni

$$\lim_{t\to 0} t^{(1-\beta)(1-\alpha)}y(t)=B$$

shartga almashtirish mumkin.

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P-ADIK ALGEBRAIK TENGLAMA VA UNING TATBIQLARI

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Gibbs o‘lchovlari nazariyasining ba’zi masalalari, xususan k-tartibli Keli daraxtida berilgan Izing yoki Potts modeli uchun translyatsion-invariant Gibbs o‘lchovlari to‘plamini tavsiflash algebraik tenglamaning yechimlari soni bilan bevosita bog‘liq.

Quyidagi akslantirish

$$f(x)=\left(\frac{ax+1}{x+a}\right)^k, \tag{1}$$

ratsional akslantirishning Izing-Potts funksiyasi deb nomlanishining sababi ham Gibbs o‘lchovlari bilan bog‘liq. Izing-Potts funksiyasining qo‘zg‘almas nuqtalari to‘plamini topish

$$(x+a)^k x-(ax+1)^k=0 \tag{2}$$

algebraik tenglamaning yechimlarini topishga keltiriladi. Haqiqiy sonlar maydoni ustida Izing-Potts funksiyasining qo‘zg‘almas nuqtalari to‘plamining tavsifini [2] ishdan topish mumkin. Bu funksiya p-adik sonlar maydonida qaralsa, o‘ziga xos dinamika bo‘lishini [1] va undagi mualliflarning bir nechta ilmiy maqolalaridan topish mumkin.

p-adik sonlar nazariyasidan ma’lum bo‘lgan ba’zi to‘plamlarni keltiramiz.

$$\mathbb{Z}_p=\{x\in\mathbb{Q}_p : |x|_p\le 1\},\qquad \mathbb{Z}_p^*=\{x\in\mathbb{Q}_p : |x|_p=1\}.$$

Ta’kidlash joizki, $\mathbb{Z}_p$ va $\mathbb{Z}_p^*$ to‘plamlar mos ravishda *p-adik butun sonlar to‘plami* va *p-adik birlar to‘plami* deyiladi. Ravshanki, $\mathbb{Z}_p^*\subset\mathbb{Z}_p$ o‘rinli.

Ushbu ish (2) algebraik tenglamani $\mathbb{Q}_p$ maydonda yechishga bag‘ishlangan. Qulaylik uchun $a\in\mathbb{Z}_p^*$ deb olamiz.

[1] ishda $a\in\mathcal{E}_p\subset\mathbb{Z}_p^*$ hol to‘liq tahlil qilinganini hisobga olib, $a\in\mathbb{Z}_p^*\setminus\mathcal{E}_p$ holni qaraymiz. Bunda

$$\mathcal{E}_p:=\begin{cases}1+4\mathbb{Z}_2,, & \text{agar } p=2 \text{ bo‘lsa}\\ 1+p\mathbb{Z}_p, & \text{agar } p\neq 2 \text{ bo‘lsa.}\end{cases}$$

Ravshanki, $x_0=1$ soni (2) algebraik tenglamaning ildizi bo‘ladi. Qolgan ildizlarni topish murakkab ekanligini hisobga olib, $k=2$ holni qaraymiz. U holda (2) tenglama quyidagi ko‘rinishda bo‘ladi

$$(x-1)(x^2+(2-(a-1)^2)x+1)=0.$$

Yuqoridagi kubik tenglamaning p-adik sonlar maydonidagi barcha yechimlarini topish $y^2=(a-3)(a+1)$ tenglamning ildizlari mavjudligi masalasi bilan teng kuchli. Boshqacha aytganda $\mathbb{Q}_p$ maydonda $\sqrt{(a-3)(a+1)}$ mavjud bo‘lsa, kubik tenglama 3 ta p-adik ildizga, aks holda yagona p-adik ildizga ega bo‘ladi. [3] ishda kvadrat ildizning p-adik sonlar maydonida mavjud bo‘lishi uchun mezon berilgan. Shu mezondan foydalanamiz.

Teorema. $a\in\mathbb{Z}_p^*\setminus\mathcal{E}_p$ va $k=2$ bo‘lsin. U holda (2) algebraik tenglama quyidagi hollarda uchta turli ildizlarga ega:

1. $a \in \bigcup_{n=1}^{\infty}(1+2+2^{2n+2}+2^{2n+5}\mathbb{Z}_2)$;

2. $a \in \bigcup_{n=1}^{\infty}(-1+5^{2n}+5^{2n+1}\mathbb{Z}_5) \cup (-1+4\cdot 5^{2n}+5^{2n+1}\mathbb{Z}_5)$.

Shuningdek, (2) tenglama $p=3$ uchun yagona ildizga ega.

Ushbu teorema $p=2$ va $p=5$ hollarda (2) tenglama uchta ildizga ega bo‘ladigan hollarni to‘liq qamrab olmasa-da, a parametrning ba’zi qiymatlarida tenglama faqat bitta ildizga ega bo‘ladigan hollar mavjud. Masalan, $p=2$ bo‘lganda $a=1+4\mathbb{Z}_2$ uchun tenglama yagona ildizga ega. Shuningdek, $p=5$ uchun $a=2+5\mathbb{Z}_5$ bo‘lganda ham tenglama yagona ildizga ega.

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KICHIK O‘LCHAMLI SHRODINGER ALGEBRASINING ANTI-DIFFERENSIALLASHLARI

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Differensiallashlar va umumlashtirilgan differensiallashlar(jumladan, ikkilamchi differensiallash) halqalar va algebralarning tuzilishini o‘rganishda juda muhim hisoblanadi. Jumladan, δ-differensiallashlar Li algebralari nazariyasini o‘rganishda hamda ularning boshqa obyektlar bilan bog‘lashda ham muhim ahamiyatga ega [1, 2, 3]. Anti-differensiallashlar esa, δ-differensiallashlarning xususiy holi bo‘lib, $\delta = -1$ bo‘lganda biz anti-differensiallashga ega bo‘lamiz.

Biz bu ishda kichik o‘lchamli Shrodinger algebrasining anti-differensiallashlarini tasniflaymiz.

1-Ta’rif. Bazislari kompleks $\mathbb{C}$-maydondan olingan $\{e, f, h, z, x_i, y_i, s_{jk}(= -s_{kj}) \quad 1 \leq i \leq n, 1 \leq j < k \leq n\}$ bo‘lib, trivial bo‘lmagan ko‘paytmalari quyidagicha aniqlangan Li algebrasi S_n Shrodinger algebrasi deyiladi:

$$
\begin{array}{lll}
[e, f] = h, & [h, e] = 2e, & [f, h] = 2f, \\
[x_i, y_i] = z, & [h, x_i] = x_i, & [h, y_i] = -y_i, \\
[e, y_i] = x_i, & [f, x_i] = y_i, & \\
[s_{jk}, x_i] = \delta_{ki} x_j - \delta_{ji} x_k, & [s_{jk}, y_i] = \delta_{ki} y_j - \delta_{ji} y_k, & \\
[s_{jk}, s_{lm}] = \delta_{lk} s_{jm} + \delta_{jm} s_{kl} + \delta_{mk} s_{lj} + \delta_{lj} s_{mk}. &&
\end{array}
$$

bu yerda $\delta_{i,j}$-Kronecker soni.

2-Ta’rif. [1]. Berilgan $D : L \to L$ chiziqli akslantirish

$$D([x, y]) = \delta([D(x), y] + [x, D(y)])$$

shartni qanoatlantirsa L algebraning δ-differensiallashi deyiladi.

Biz o‘z ishimizda $\delta = -1$ holatni qaraymiz.

Tasdiq. S_2 Shrodinger algebrasining D anti-differensiallashi quyidagiga teng bo‘ladi:

$$D(f) = D(e) = D(h) = D(z) = D(x_i) = D(y_i) = 0, \quad D(s_{12}) = z, \quad i = 1, 2,$$

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YADROSI AJRALUVCHI GAMMERSHTEYN TIPIDAGI OPERATORLARNING QO‘ZG‘ALMAS NUQTALARI HAQIDA

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Gibbs o‘lchovlari nazariyasidagi asosiy masalalardan biri berilgan modellar uchun Gibbs o‘lchovlari to‘plamini tasniflash masalasi hisoblanadi. Spin qiymatlari continuum bo’lgan modellar uchun translatsion invariant Gibbs o’lchovlarini topish masalasi Gammershteyn tipidagi nochiziqli operatorning musbat qo’zg’almas nuqtalarini topish masalasiga keltiriladi ([1 − 3]). Ushbu tezisda ikkinchi tartibli Gammershtyen tipidagi operatorning qo’zg’almas nuqtalari soni yagona bo’lmaydigan operatorning yadrosini qurishga bag’ishlangan. Gammershteyn tipidagi operatorni quyidagicha aniqlaymiz:

$$(H_k f)(t) = \int_0^1 K(t,u) f^k(u) d(u), \;\; k \in N,$$

bu yerda

$$K(t,u) = \varphi_1(t)\psi_1(u) + \varphi_2(t)\psi_2(u) \in C^+[0,1] = \{f \in C[0,1] | f(x) > 0\}.$$

$\varphi_1(t),\;\; \varphi_2(t),\;\; \psi_1(u),\;\; \psi_2(u)$ funksiyalarni $[0,1]$ kesmada quyidagicha tanlab olaylik:

$$\varphi_1(t) = \begin{cases} f_1(t), & \text{agar } \; t \in [0, \frac{1}{4}] \\ f_2(t), & \text{agar } \; t \in [\frac{1}{4}, \frac{2}{4}], \\ f_3(t), & \text{agar } \; t \in [\frac{2}{4}, \frac{3}{4}] \\ f_4(t), & \text{agar } \; t \in [\frac{3}{4}, 1], \end{cases} \qquad \varphi_2(t) = \begin{cases} f_1(1-t), & \text{agar } \; t \in [0, \frac{1}{4}] \\ f_2(1-t), & \text{agar } \; t \in [\frac{1}{4}, \frac{2}{4}], \\ f_3(1-t), & \text{agar } \; t \in [\frac{2}{4}, \frac{3}{4}] \\ f_4(1-t), & \text{agar } \; t \in [\frac{3}{4}, 1], \end{cases}$$

$$\psi_1(u) = \begin{cases} g_1(u), & \text{agar } \; t \in [0, \frac{1}{4}] \\ g_2(u), & \text{agar } \; t \in [\frac{1}{4}, \frac{2}{4}], \\ g_3(u), & \text{agar } \; t \in [\frac{2}{4}, \frac{3}{4}] \\ g_4(u), & \text{agar } \; t \in [\frac{3}{4}, 1], \end{cases} \qquad \psi_2(u) = \begin{cases} g_1(1-u), & \text{agar } \; t \in [0, \frac{1}{4}] \\ g_2(1-u), & \text{agar } \; t \in [\frac{1}{4}, \frac{2}{4}], \\ g_3(1-u), & \text{agar } \; t \in [\frac{2}{4}, \frac{3}{4}] \\ g_4(1-u), & \text{agar } \; t \in [\frac{3}{4}, 1], \end{cases}$$

bu yerda $f_i, g_i \in C^+[0,1],\;\; i \in \{1,2,3,4\}$. Quyidagicha belgilashlarni kiritib olaylik:

$$\begin{cases} \alpha_1 = \int\limits_0^1 \psi_1(u)\varphi_1^2(u)d(u), \;\; \alpha_2 = \int\limits_0^1 \psi_1(u)\varphi_1(u)\varphi_2(u)d(u), \;\; \alpha_3 = \int\limits_0^1 \psi_1(u)\varphi_2^2(u)d(u), \\ \beta_1 = \int\limits_0^1 \psi_2(u)\varphi_1^2(u)d(u), \;\; \beta_2 = \int\limits_0^1 \psi_2(u)\varphi_1(u)\varphi_2(u)d(u), \;\; \beta_3 = \int\limits_0^1 \psi_2(u)\varphi_2^2(u)d(u), \end{cases}$$

Lemma. Quyidagi munosabatlar o’rinli: $\alpha_1 = \beta_3,\;\; \alpha_2 = \beta_2,\;\; \alpha_3 = \beta_1$.

Teorema. Berilgan $K(t,u)$ yadro uchun H_k operatorning musbat qo’zg’almas nuqtalari uchun quyidagi munosbatlar o’rinli:

i) Agar $2\alpha_2 - \alpha_1 + 3\alpha_3 > 0$ bo’lsa, u holda H_k operator ko’pi bilan ikkita qo’zg’almas nuqtalarga ega;

ii) Agar $2\alpha_2 - \alpha_1 + 3\alpha_3 = 0$ bo'lsa, u holda H_k operator ikkita qo'zg'almas nuqtalarga ega;

iii) Agar $2\alpha_2 - \alpha_1 + 3\alpha_3 < 0$ bo'lsa, u holda H_k operator kamida ikkita va ko'pida uchta qo'zg'almas nuqtalarga ega.

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О РАЗРЕШИМОСТИ КРАЕВОЙ ЗАДАЧИ ДЛЯ НЕОДНОРОДНОГО УРАВНЕНИЯ ЧЕТВЕРТОГО ПОРЯДКА С КРАТНЫМИ ХАРАКТЕРИСТИКАМИ

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В области $D = \{(x, y) : 0 < x < p,\ 0 < y < q\}$ рассмотрим уравнение

$$L[u] \equiv u_{xxxx} + a_1 u_{xx} + a_2 u - u_{yy} = f(x, y), \tag{1}$$

где $p, q, a_i \in R$, $i = 1, 2$ и для него исследуем следующую задачу.

Задача A_2. Найти функцию $u(x, y)$ из класса $C^{4,2}_{x,y}(D) \cap C^{3,1}_{x,y}\left(\bar{D}\right)$, удовлетворяющую в области D уравнению (1) и следующим краевым условиям:

$$u_y(x, 0) = 0, \quad u_y(x, q) = 0, \quad 0 \le x \le p, \tag{2}$$

$$\frac{\partial^m}{\partial x^m} u\Big|_{x=0} = \psi_m(y), \quad \frac{\partial^m}{\partial x^m} u\Big|_{x=p} = \psi_{m+1}(y), \quad m = 1, 3, \quad 0 \le y \le q, \tag{3}$$

где $\psi_i(y)$, $i = \overline{1, 4}$ – заданные достаточно гладкие функции.

Отметим, что для уравнения (1) в работах [1-2], исследованы некоторые корректные краевые задачи.

Теорема 1. Если задача A_2 имеет решение, то при выполнении условий $a_1 \le 0$, $a_2 \ge 0$, оно единственно.

Существование решения задачи доказано методом Фурье.

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ОБ ОДНОЙ КРАЕВОЙ ЗАДАЧЕ ДЛЯ НЕОДНОРОДНОГО УРАВНЕНИЯ ТРЕТЬЕГО ПОРЯДКА С КРАТНЫМИ ХАРАКТЕРИСТИКАМИ В ТРЕХМЕРНОМ ПРОСТРАНСТВЕ

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В области $D = \{(x, y, z) : 0 < x < p,\ 0 < y < q,\ 0 < z < r\}$ рассмотрим уравнение третьего порядка вида

$$L[u] \equiv u_{xxx} - u_{yy} - u_{zz} = f(x, y, z), \tag{1}$$

где $p, q, r \in R$ и для него исследуем следующую задачу.

Задача A. Найти решение уравнения (1) в области D из класса $C^{3,2,2}_{x,y,z}(D) \cap C^{2,1,1}_{x,y,z}\left(\overline{D}\right)$, удовлетворяющее краевым условиям:

$$u(x, 0, z) = u(x, q, z) = 0, \quad u(x, y, 0) = u(x, y, r) = 0, \tag{2}$$

$$\begin{cases} au(0, y, z) + bu_{xx}(0, y, z) = \psi_1(y, z), \\ cu(p, y, z) + du_{xx}(p, y, z) = \psi_2(y, z), \\ u_x(p, y, z) = \psi_3(y, z), \end{cases} \tag{3}$$

где $a,\ b,\ c,\ d \in R\backslash\{0\}$, а $\psi_i(y, z)$, $i = \overline{1, 3}$ – заданные достаточно гладкие функции.

Отметим, что для уравнения (1) в плоскости, т.е. $z = 0$ при $b = d = 0$ в работах [1-2], а при $a = c = 0$ в работе [3] исследованы краевые задачи.

Теорема 1. Если задача A имеет решение, то при выполнении условий $ab > 0,\ cd < 0$, оно единственно.

Существование решения задачи доказана с построением функции Грина.

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О ТРАНСЛЯЦИОННО-ИНВАРИАНТНЫХ P-АДИЧЕСКИХ КВАЗИ-МЕРАХ ГИББСА ДЛЯ МОДЕЛИ SOS

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Пусть $\mathbb{Q}-$поле рациональных чисел. Для фиксированного простого числа p каждое рациональное число $x \neq 0$ может быть представлено в виде

$$x = p^r \frac{n}{m}, \qquad r, n, m \in \mathbb{Z},$$

числа m и n взаимно просты с p. p-адическая норма числа $x \in \mathbb{Q}$ определяется по формуле

$$|x|_p = \begin{cases} p^{-r}, & \text{если } x \neq 0, \\ 0, & \text{если } x = 0. \end{cases}$$

Пополнение поля рациональных чисел $\mathbb{Q}$ по p-адической норме образует поле $\mathbb{Q}_p$ p-адических чисел (см.[1]).

Пусть $\Gamma^k = (V, L)$ дерево Кэли порядка k (см.[2]), и $\Phi := \{0, 1, 2, ..., m\}$, $m \geq 2$. Конфигурация σ на множестве A ($A \subset V$) определяется как функция, сопоставляющая $x \in A$ значение $\sigma(x) \in \Phi$; обозначим через Ω_A множество всех конфигураций на A.

Пусть $a \in \mathbb{Q}_p$ и $r > 0$, введем обозначения

$$B(a, r) = \{x \in \mathbb{Q}_p : |x - a|_p < r\}.$$

p-адический гамильтониан модели SOS на Ω_{V_n} определяется как (см.[3])

$$H_n(\sigma) = J \sum_{\langle x,y\rangle \in L_n} |\sigma(x) - \sigma(y)|_\infty, \qquad \sigma \in \Omega_{V_n}, \tag{1}$$

где $J \in B(0, p^{-1/(p-1)})$ параметр, L_n-множество ребер в V_n и $|\cdot|_\infty$ означает обычную абсолютную величину.

Пусть $z : x \mapsto z_x = (z_{0,x}, z_{1,x}, z_{2,x}, ..., z_{m,x}) \in \mathbb{Q}_p^{m+1}$ есть p-адическая векторнозначная функция на $x \in V \setminus \{x^0\}$. Для заданного $n \in \mathbb{N}$ рассмотрим p-адическое распределение вероятностей $\mu^{(n)}$ на Ω_{V_n}, заданное как

$$\mu_{\tilde{z}}^{(n)}(\sigma) = Z_{n,\tilde{z}}^{-1} \exp_p\{H_n(\sigma)\} \prod_{x \in W_n} \tilde{z}_{\sigma(x),x}. \tag{2}$$

Здесь $\sigma \in \Omega_{V_n}$, а $Z_{n,\tilde{z}}-$соответствующая статистическая сумма, т.е.

$$Z_{n,\tilde{z}} = \sum_{\omega \in \Omega_{V_n}} \exp_p\{H_n(\omega)\} \prod_{x \in W_n} \tilde{z}_{\omega(x),x}.$$

Пусть для p-адических распределений (2) выполняется условие согласованности:

$$\sum_{\omega_n \in \Omega_{W_n}} \mu^{(n)}(\sigma_{n-1} \vee \omega_n) = \mu^{(n-1)}(\sigma_{n-1}), \tag{3}$$

Тогда по неархимедовому аналогу теоремы Колмогорова (см. [4]) существует единственная p-адическая квази-мера Гиббса $\mu_{\tilde{z}}$ на Ω.

Теорема 1. p-адические распределения $\mu^{(n)}_{\tilde{\mathbf{z}}_x}(\sigma), n = 1, 2, ...$, заданные в (2), согласованы тогда и только тогда, когда для любого $x \in V \setminus \{x^0\}$ имеет место следующая система уравнений:

$$z_{i,x} = \prod_{y \in S(x)} \frac{\sum_{j=0}^{m-1} \theta^{|i-j|_\infty} z_{j,y} + \theta^{m-i}}{\sum_{j=0}^{m-1} \theta^{m-j} z_{j,y} + 1}, \qquad i = 0, 1, ..., m-1. \tag{4}$$

Здесь $\theta = \exp_p(J)$ *и* $z_{i,x} = \tilde{z}_{i,x}/\tilde{z}_{m,x}$ *для* $i = 0, 1, ..., m-1$ (см. [3]).

Если существуют две p-адические квази-меры Гиббса μ_1, μ_2 такие, что одна из них ограничена, а другая не ограничена, тогда говорят, что существует *фазовый переход*(см. [5]).

Пусть G_k- прямое произведение $k+1$ циклических групп второго порядка с соответствующими образующими $a_1, a_2, ..., a_{k+1}$. Известно [2], что существует взаимно однозначное соответствие между множеством V вершин дерева Кэли Γ^k и группой G_k.

Определение 1. Пусть G^*_k- нормальная подгруппа группы G_k. Совокупность z_x (соотв., конфигурация $\sigma(x)$), определенная для $x \in G_k$, называется G^*_k- *периодической*, если $z_{yx} = z_x$ (соотв., $\sigma(yx) = \sigma(x)$) для любых $x \in G_k$ и $y \in G^*_k$. G_k-периодическая совокупность называется *трансляционно-инвариантной*(ТИ).

Определение 2. μ_{z_x} квази-мера Гиббса, соответствующая ТИ совокупности z_x, называется ТИ p-адической квази-мерой Гиббса.

Теорема 2. Пусть $p > 3$ и $p \equiv 1(\mathrm{mod}\, 8)$ или $p \equiv 3(\mathrm{mod}\, 8)$. Тогда для p-адический модели SOS на дереве Кэли порядка два, существует не менее трех ТИ p-адических квази-мер Гиббса.

Теорема 3. Пусть $p > 3$ и $p \equiv 1(\mathrm{mod}\, 8)$ или $p \equiv 3(\mathrm{mod}\, 8)$. Тогда для p-адической модели SOS с тремя состояниями на дереве Кэли порядка два происходит фазовый переход.

Замечание 1. В работе [3] доказано, что $p \neq 3$ существует единственная p-адическая ТИ-мера Гиббса для p-адической модели SOS на дереве Кэли порядка k. Из Теоремы 2 следует, что для p-адической модели SOS на дереве Кэли порядка два существует не менее трех ТИ p-адических квази-мер Гиббса, которые содержат p-адическую ТИ-меру Гиббса, найденную в работе [3]. Остальные две квази-меры Гиббса, найденные в Теореме 2 – новые меры.

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АФФИННЫЕ ГОМЕОМОРФИЗМЫ ПРОСТРАНСТВА ПОЛУАДДИТИВНЫХ ФУНКЦИОНАЛОВ

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В данной заметке мы рассмотрим свойства аффинных гомеоморфизмов между пространствами $cc\left(P\left(X\right)\right)$ и $OS\left(X\right)$, где $P\left(X\right)$ – пространство всех вероятностных мер на компактном метрическом пространстве $(X,\,\rho)$, $OS\left(X\right)$ – пространство всех полуаддитивных функционалов, а X – компактное пространство [1].

Для выпуклого компакта K через $cc\left(K\right)$ обозначим пространство всех непустых выпуклых замкнутых подмножеств K, снабженное топологией Вьеториса [2].

Отметим, что каждое подмножество $A\in cc\left(P\left(X\right)\right)$ определяет функционал

$$\nu_A\left(\varphi\right)=\sup\left\{\mu\left(\varphi\right)\,:\quad \mu\in A\right\},\quad \varphi\in C\left(X\right),$$

более того, соответствие

$$A\in cc\left(P\left(X\right)\right)\mapsto\nu_A\in OS\left(X\right) \tag{1}$$

задает аффинный гомеоморфизм между пространствами $cc\left(P\left(X\right)\right)$ и $OS\left(X\right)$.

Для компакта X с метрикой $\rho=\rho_X$ определим функцию расстояния $\rho_{OS}=\rho_{OS(X)}$ на компакте $OS\left(X\right)$ следующим образом [3]:

$$\rho_{OS}\left(\nu_1,\ \nu_2\right)=\inf\left\{\nu\left(\rho\left(x_1,\ x_2\right)\right)\ :\ \nu\in\Lambda\left(\nu_1,\ \nu_2\right)\right\}. \tag{2}$$

Пусть ρ_P – метрика Канторовича на пространстве $P(X)$, которая задается аналогично (2). Рассмотрим метрику Хаусдорфа на пространстве $cc\left(P\left(X\right)\right)$, определяемую по правилу

$$\rho_H\left(A,\ B\right)=\max\left\{\max_{\mu\in A}\rho_P\left(\mu,\ B\right),\ \max_{\lambda\in B}\rho_P\left(\lambda,\ A\right)\right\},\ \ A,B\in cc(P(X)).$$

Основным результатом заметки является следующая

Теорема. Отображение, определенное по правилу (1) является аффинной изометрией между $(cc(P(X)),\rho_H)$ и $(OS(X),\ \rho_{OS})$.

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ТЕОРЕМА ГЕЛЬФАНДА-НАЙМАРКА

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Определение. Пусть A – алгебра. Линейное отображение $\chi : A \to \mathbb{C}$ со свойством $\chi(ab) = \chi(a)\chi(b)$ $(\forall a, b \in A)$ называется *характером* алгебры A. То есть всякий гомоморфизм из A в $\mathbb{C}$ называется характером.

Очевидно, что $\chi \equiv 0$ – есть характер, и если $\chi \not\equiv 0$ и алгебра A с единицей $\mathbf{e}$, то $\chi(\mathbf{e}) = 1$. Множество всех (ненулевых) характеров обозначим через Ω и Ω_+, соответственно: $\Omega_+ = \{\chi - \text{характер}\}$, $\Omega = \{\chi \in \Omega_+ : \chi \not\equiv 0\}$. Если $B(H)$ – алгебра всех линейно ограниченных операторов действующих в гильбертовом пространстве H, то $\Omega = \emptyset$. Непосредственно показывается, что если A – банахова алгебра, то всякий характер ограничен (а значит непрерывен), и более того, сжимающий функционал, т.е. $\|\chi\| \leq 1$.

Поскольку $\Omega_+ \subset A^*$, то на Ω_+ рассмотрим w*-топологию, т.е. топологию поточечной сходимости:

$$\chi_n \to \chi \;\Leftrightarrow\; \chi_n(a) \to \chi(a), \quad \forall a \in A$$

База окрестностей нуля задается множествами $U_{\varepsilon,F} := \{\chi : \; |\chi(x)| < \varepsilon, \; \forall x \in F\}$, где $F \subset A$ – конечное подмножество. Если $\chi_1, \chi_2 \in \Omega_+$ и $\chi_1 \neq \chi_2$, то $\exists x_0 \in A$: $\chi_1(x_0) \neq \chi_2(x_0)$. Тогда из непрерывности $\exists \varepsilon > 0$: $|\chi_1(x_0) - \chi_2(x_0)| > \varepsilon$. Положим

$$U_{\chi_1} = \big\{\chi : \; |\chi_1(x_0) - \chi(x_0)| < \frac{\varepsilon}{2}\big\}, \qquad V_{\chi_2} = \big\{\chi : \; |\chi_2(x_0) - \chi(x_0)| < \frac{\varepsilon}{2}\big\}$$

Очевидно, что множества U_{χ_1} и V_{χ_2} – есть открытая окрестность χ_1 и χ_2, соответственно, такие, что $U_{\chi_1} \cap V_{\chi_2} = \emptyset$. Таким образом, мы доказали, что Ω_+ – есть T_2-пространство, т.е. Хаусдорфово пространство. Аналогично показывается, что если алгебра с единицей, то пространства Ω и Ω_+ – замкнута. Отсюда следует, что пространство Ω_+ – компактен, и если алгебра с единицей, то пространство Ω – также компактен; а в общем случае пространство Ω – локально компактен.

Теперь, для каждого $a \in A$ рассмотрим отображению (функционал): $\hat{a}_+ : \Omega_+ \to \mathbb{C}$ с $\hat{a}(\chi) := \chi(a)$. Ясно, что $\hat{a}(0) = 0$. Сужению $\hat{a}_+$ на Ω обозначим как $\hat{a}$. Легко показать, что $\hat{a}_+$ – w*-непрерывен, и если $\Omega \neq \emptyset$, то $\hat{a}$ – также w*-непрерывен и $\hat{a}(\infty) = 0$, т.е. функционал, исчезающий в бесконечности. Напомним, что последнее условие означает, что для $\forall \varepsilon > 0$ существует компакт $K \subset \Omega$, такой, что $|\hat{a}(\chi)| < \varepsilon$, для всех $\chi \in \Omega \setminus K$. Совокупность всех таких $\hat{a}$ обозначим через $C_0(\Omega)$.

Рассмотрим отображение $\Gamma_+ : A \to C(\Omega_+)$, который сопоставляет каждому элементу $a \in A$ функционал $\hat{a}_+$. Непосредственно показывается, что Γ_+ является гомоморфизмом и сжимающим, а значит непрерывный. Если $\Omega \neq \emptyset$, то отображение $\Gamma : A \to C_0(\Omega)$ с $\Gamma(a) := \hat{a}$ является изоморфизмом. Таким образом, имеет место следующая теорема, которая есть теорема Гельфанда-Наймарка об описание коммутативных C*-алгебр.

Теорема. Если A – коммутативная (абелева) C*-алгебра, то она изометрически изоморфна алгебре всех непрерывных функций на Ω, исчезающих в бесконечности, т.е. $A \cong C_0(\Omega)$. Причем, алгебра A имеет единицу тогда и только тогда, когда Ω – компактен.

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ВЕЩЕСТВЕННЫЕ НЕИЗОМОРФНЫЕ W*-ПОДАЛГЕБРЫ

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Пусть $B(H)$ – алгебра всех ограниченных линейных операторов, действующих в комплексном гильбертовом пространстве H. Слабо замкнутая *-подалгебра M в $B(H)$, содержащая единичный оператор $\mathbb{1}$, называется W^*-*алгеброй*. Вещественная *-подалгебра $R \subset B(H)$ с $\mathbb{1}$ называется *вещественной* W*-*алгеброй*, если она слабо замкнута и $R \cap iR = \{0\}$. Комплексная или вещественная W^*-алгебра R называется *фактором*, если ее центр тривиален, т.е. состоит из скалярных кратных единичного оператора $\mathbb{1}$. Вещественная W^*-алгебра R имеет тип I_n, I_∞, II_1, II_∞ или III_λ, $(0 \le \lambda \le 1)$, если ее обвертывающая W^*-алгебра $R + iR$ имеет соответствующий тип в смысле обычной классификации W^*-алгебры.

Линейное отображение α алгебры M в себя с $\alpha(x^*) = \alpha(x)^*$ называется *-*автоморфизмом*, если $\alpha(xy) = \alpha(x)\alpha(y)$; *инволютивный *-антиавтоморфизмом*, если $\alpha(xy) = \alpha(y)\alpha(x)$ и $\alpha^2(x) = x$. Если α - инволютивный *-антиавтоморфизм W*-алгебры M, то через (M, α) мы обозначим вещественную W*-алгебру, порожденная антиавтоморфизмом α, т.е. $R = (M, \alpha) = \{x \in M : \alpha(x) = x^*\}$. Обратно, всякая вещественная W*-алгебра R имеет вид (M, α), где $M = R + iR$ - обвертывающая W*-алгебра и α - инволютивный *-антиавтоморфизм M, определенный как $\alpha(x + iy) = x^* + iy^*$ (см. [1-3]).

В дальнейщем, всякую вещественную W*-алгебру R мы отождествим с парой (M, α).

Пусть $< \cdot, \cdot >$ – скалярное произведение на H, и пусть $R \subset B(H)$ – вещественная *-подалгебра. Положим $< \xi, \eta >_r := Re(< \xi, \eta >)$, $\xi, \eta \in H$. Нетрудно показать, что $H_r := (H, < \cdot, \cdot >_r)$ есть вещественное гильбертово пространство такое, что

$$H_r + iH_r = H, \quad R \subset B(H_r) \subset B(H_r) + iB(H_r) = B(H).$$

Коммутант *-алгебры R определяется аналогично комплексному случаю: $R' = \{a \in B(H_r) : \ ab = ba, \ \forall b \in R\}$. Непосредственно проверяется, что $(R + iR)' = R' + iR'$.

Для каждого $x \in M$, определим отображение $\lambda(x)y = xy$, для всех $y \in M$. Так как $\|\lambda(x)y\|_2 \le \|x\|\|y\|_2$, то λ имеет единственное продолжение на $L^2(M)$, являющиеся ограниченным линейным оператором, обозначаемый снова через $\lambda(x)$. Тогда мы получим точное W*-представление $(\lambda, L^2(M))$ алгебры M. Аналогично, определяя отображение λ_r как $\lambda_r(x)y = xy$ (для всех $x, y \in R$), мы получим точное вещественное W*-представление (λ_r, L^2R) алгебры R. Можно показать [2], что $\lambda(M)$ есть комплексификация алгебры $\lambda_r R$, т.е. $\lambda_r R + i\lambda_r R = \lambda(M)$.

Пусть теперь $A \subseteq B$ – конечномерные вещественные C*-алгебры. Тогда $A \cong \bigoplus_{i+1}^{k} M_{n_i}(F)$ и $B \cong \bigoplus_{j+1}^{l} M_{m_j}(F)$, где $F = \mathbb{R}$ ($\mathbb{H}$ - тело кватернионов). Положим $\overrightarrow{n} = (n_1, ..., n_k)$ и $\overrightarrow{m} = (m_1, ..., m_l)$, и назовем их вектор-размерностями

A и B, соответственно. Определим элементы Λ_{ij} $k \times l$-матрицы Λ_A^B следующим образом: Λ_{ij} – число i-го слагаемого в представлении A в j-м слагаемом B. С помощью следов τ_A и τ_B построим вектора $\overrightarrow{s}$ и $\overrightarrow{t}$ так, что $s_i = \tau_A(f_i)$, $t_j = \tau_B(e_j)$, где f_i и e_j - минимальные проекторы i-го и j-го слагаемых A и B, соответственно. Непосредственно показывается, что $\overrightarrow{t} \cdot \overrightarrow{m} = 1$ и $\overrightarrow{s} \cdot \overrightarrow{n} = 1$ (см. [4-6]).

Предложение[5]. Пусть N и $\overline{N}$ - две вещественные матричные подалгебры вещественных матричных подалгебр M, заданные вместе с изоморфизмом $\varphi : N \to \overline{N}$. Если $\Lambda_N^M = \Lambda_{\overline{N}}^M$, то φ продолжается до автоморфизма θ в M.

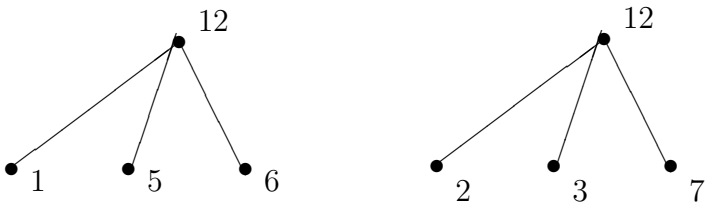
Замечание. Предложения не верна в вещественном случае. Это показывает следующий пример.

Пример. Рассмотрим две вещественные подалгебры: $Q = \mathbb{R} \oplus M_5(\mathbb{R}) \oplus M_6(\mathbb{R})$, $\overline{Q} = M_2(\mathbb{R}) \oplus M_3(\mathbb{R}) \oplus R_7(\mathbb{R})$ и вещественный фактор $R = M_{12}(\mathbb{R})$. Тогда вложение для подалгебр задается как:

$$Q \subset R \Rightarrow (x_1, y_1, z_1) \mapsto \begin{pmatrix} x_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & z_1 \end{pmatrix} : x_1 \in (\mathbb{R}), y_1 \in M_5(\mathbb{R}), z_1 \in M_6(\mathbb{R}).$$

$$\overline{Q} \subset R \Rightarrow (x_2, y_2, z_2) \mapsto \begin{pmatrix} x_2 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & z_2 \end{pmatrix} : x_2 \in M_2(\mathbb{R}), y_2 \in M_3(\mathbb{R}), z_2 \in M_7(\mathbb{R}).$$

Тогда $\Lambda_Q^R = \Lambda_{\overline{Q}}^R = (1, 1, 1)$



Очевидно, что графы и матрицы одинаковы. Однако, легко можно показать, что вещественные подалгебры Q и $\overline{Q}$ не изоморфны, соответственно [1].

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СОСТАВЛЕНИЕ УРАВНЕНИЙ НЕКОТОРЫХ КЛАССОВ ДИНАМИЧЕСКИХ СИСТЕМ С ЗАДАННЫМИ ИНВАРИАНТАМИ С ПОМОЩЬЮ СИМВОЛЬНЫХ ВЫЧИСЛЕНИЙ

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Моделирование перераспределения энергии между разномасштабными вихрями в турбулентных магнитогидродинамических (МГД) потоках удобно описывать с помощью каскадных моделей (shell models) турбулентности [1]. Эти модели представляют собой конечномерные динамические системы, комплексные фазовые переменные которых (коллективные переменные) интерпретируются по абсолютной величине как суммарные интенсивности турбулентных вихрей заданного диапазона пространственных масштабов $D_n = (q^{-n-1}; q^{-n}]$, где $n \in \mathbb{Z}$ и $q > 1$. Уравнения каскадной модели должны быть качественно подобны МГД-уравнениям в пространстве волновых векторов. Пусть $U_n(t)$ и $B_n(t)$ коллективные переменные для полей скорости и магнитной индукции. Максимальное сходство с МГД-уравнениями имеют модели типа GOY [1]:

$$\begin{aligned}
&\frac{dU_n}{dt} = \imath q^n \sum_{i,j} S_{ij} U^*_{n+i} U^*_{n+j} - \mathsf{Re}^{-1} q^{2n} U_n + \imath q^n \sum_{i,j} N_{ij} B^*_{n+i} B^*_{n+j} + f_n(t),\\
&\frac{dB_n}{dt} = \imath q^n \sum_{i,j} K_{ij} U^*_{n+i} B^*_{n+j} - \mathsf{Re}_m^{-1} q^{2n} B_n,\\
&n = -\infty, \ldots, +\infty, \quad S_{ij} = S_{ji}, \quad N_{ij} = N_{ji},
\end{aligned}$$

где Re и Re_m – гидродинамическое и магнитное числа Рейнольдса; S_{ij}, N_{ij}, K_{ij} – вещественные коэффициенты; $f_n(t)$ – моделируют внешний подвод энергии в масштабный диапазон D_n. Звездочка означает комплексное сопряжение. Суммирование по индексам i и j идет в промежутке от $-J$ до J, т.е. параметр J является мерой масштабной нелокальности модели. Можно считать, что суммирование идет по всем целочисленным значениям индексов, полагая нулевыми коэффициенты, у которых хотя бы один из индексов по модулю больше J.

В моделях типа (1) каждая переменная входит в нелинейные члены в сопряженном виде. Кроме моделей типа GOY исследуются и модели типа Sabra [2]. Их отличие от (1) в том, что в нелинейных членах множители-переменные либо без сопряжения, либо сопряжен только один из множителей-переменных члена.

В бездиссипативном пределе, когда $\mathsf{Re}^{-1} = \mathsf{Re}_m = 0$, и при $f_n(t) \equiv 0$ уравнения каскадной модели должны иметь несколько квадратичных инвариантов. Они являются аналогами сохраняющихся величин в идеальной МГД. Это инварианты:

1. энергии $E = \dfrac{1}{2} \sum_n \left(|U_n|^2 + |B_n|^2 \right)$;

2. перекрестной спиральности $H_C = \sum_n (U_n B^*_n + U^*_n B_n)$;

3. квадрата магнитного потенциала $A^2 = \sum_n q^{-n}|B_n|^2$ или магнитной спиральности $H_B = \sum_n (-1)^n q^{-1}|B_n|^2$.

В трехмерном МГД-потоке сохраняется магнитная спиральность, а в двумерном – квадрат магнитного потенциала. Выбор один из этих двух инвариантов косвенным образом вносит в модель информацию о размерности описываемого потока.

Получение уравнений конкретной каскадной модели сводится к нахождению таких коэффициентов квадратичных членов, при которых у системы будут нужные инварианты. Ясно, что требование постоянства выражений для E, H_C, A^2 и H_B требует тождественного равенства нулю некоторых кубичных форм от коллективных переменных. А коэффициенты этих форм являются линейными комбинациями коэффициентов каскадной модели. Поэтому вывод уравнений каскадной модели сводится к составлению системы алгебраических однородных линейных уравнений на коэффициенты и нахождению ее аналитического решения. Количество уравнений и неизвестных коэффициентов порядка сотен даже для малых значений J, поэтому вероятность ошибок при «ручных» выкладках очень велика.

В докладе представлена разработанная авторами технология автоматизированной генерации уравнений на коэффициенты модели с помощью символьных вычислений и нахождения параметрических выражений для коэффициентов. Это позволяет автоматизировать процесс построения новых каскадных моделей. Технология позволяет получать параметрические классы комбинированных моделей, обобщающих типы GOY и Sabra. Кроме существования инвариантов, можно обеспечивать также сохранение фазового объема и существование стационарных решений степенного типа. Такие требования также иногда предъявляют к каскадным моделям [1]. Представленная в докладе результаты обобщает разработанную ранее технологию для моделей с вещественными коллективными переменными [3].

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ТЕОРЕМА ТИПА КАРТАНА О ДИФФЕРЕНЦИАЛЬНЫХ СВОЙСТВАХ ЛОГАРИФМИЧЕСКОГО ПОТЕНЦИАЛА

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В работе рассматривается потенциал Рисса борелевской меры μ по рисскому ядру $K_\alpha(x) = \frac{1}{|x|^{n-\alpha}}$:

$$U_\alpha^\mu(x) = \int\limits_{R^n} \frac{d\mu(y)}{|x-y|^{n-\alpha}} \quad (0 < \alpha < n)\,,$$

и изучаются свойства данного оператора. В частности, установлены принадлежность классу непрерывных функций слабого решения $u(x)$, связанного с потенциалами Рисса эллиптического уравнения

$$P(D)u(x) = f(x),$$

где $P(D) = \sum\limits_{|\alpha|\le m} a_\alpha D^\alpha -$ эллиптический оператор порядка m с постоянными коэффициентами a_α, $\alpha = (\alpha_1, ..., \alpha_n), D^\alpha = D_1^{\alpha_1} ... D_n^{\alpha_n}$, действующий в пространстве обобщенных функций $D^/$, $f-$обобщчнная функция с компактным носителем $supp(f) \subset\subset B = \{x \in R^n : |x| < 1\}$ и порядка сингулярности $s : 0 \le s \le m$.

Напомним основные понятия теории потенциала, которыми мы будем пользоваться [1],[2],[3]. Одним из фундаментальных фактов в теории потенциала является следующая классическая теорема А. Картана о непрерывности по ёмкости - если $U_\alpha^\mu(x)-$ произвольный потенциал Рисса, то для любого $\varepsilon > 0$ существует открытое множество $U_\varepsilon \in \mathrm{R}^n,\ cap(U_\varepsilon) < \varepsilon$,и $U_\alpha^\mu(x)-$ непрерывен в $R^n \backslash U_\varepsilon$.

Будем говорить, что обобщённая функция f имеет порядок сингулярности не более, чем s, если она непрерывно продолжается в пространство $C^s(G)$, $G \subset R^n$, множество s раз непрерывно дифференцируемых функций. Если, кроме того, f не продолжается в $C^{s-1}(G)$, то говорят, что порядок обобщённой функции f равен точно s. Логарифмическим потенциалом меры μ называют функцию

$$U_0^\mu(x) = \int \ln \frac{1}{|x-y|}\, d\mu(y),\ x \in R^n. \tag{1}$$

Пусть $G \subset R^n-$ ограниченное борелевское множество. Положим

$$I(\mu) = \iint\limits_{G\times G} \ln \frac{1}{|x-y|}\, d\mu(x)\, d\mu(y).$$

Так как $\ln \frac{1}{|x-y|}$ ограничена снизу на G, то этот интеграл всегда существует. Положим также

$$V = \inf \{I(\mu) : \mu(G) = 1\}$$

и назовём $V-$ постоянной равновесия множества G. Пусть $d-$ диаметр множества G, так что $|x-y| \leq d$ в G. Тогда

$$\ln \frac{1}{|x-y|} \geq \log \frac{1}{d}.$$

Поскольку $\mu(G) = 1$, отсюда получаем, что $V \geq \log \frac{1}{d}$.

Определим теперь логарифмическую ёмкость множества G полагая $cap_0 = e^{-V_0}$. Основной результат работы приведён в следующей теореме.

Теорема. Пусть $f \in D^{/}(G)-$ обобщенная функция порядка сингулярности $k : 0 \leq n - m + k + \alpha < n$, $\alpha > 0$. Тогда для любого $\varepsilon > 0$ существует открытое множество $U_\varepsilon \subset G$ с ёмкостью $cap_{n-m+k+\alpha}(U_\varepsilon) < \varepsilon$ такое, что решение $u(x)$ уравнения $P(D)\ u = f$ принадлежит классу C^α на компактных подмножествах разности $G \backslash U_\varepsilon$.

Отметим, что в случае $n - m + k + \alpha = 0$, в качестве ёмкости $cap_{n-m+k+\alpha} = cap_0-$ понимается логарифмическая ёмкость, которая определяется с помощью логарифмического потенциала (1) [2],[4].

Согласно теореме Уитни о продолжении гладких функций из результата статьи вытекает, что потенциал Рисса $U^\mu_\alpha(x)$ и слабое решение $u(x)$ вне некоторого множества малой ёмкости является следом некоторой функции из класса $C^\alpha(R^n)$.

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О КОНСТРУКТИВНОМ ОПИСАНИИ МЕР ГИББСА ДЛЯ МОДЕЛИ ПОТТСА НА ДЕРЕВЕ КЭЛИ

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Пусть $\tau^k = (V, L), k \geq 1$ есть дерево Кэли порядка k, т.е. бесконечное дерево, из каждой вершины которого выходит ровно $k+1$ ребер, где $V-$ множество вершин, $L-$ множество ребер τ^k.

Известно, что τ^k можно представить как G_k- свободное произведение $k+1$ циклических групп второго порядка (см.[1]).

Для произвольной точки $x^0 \in V$ положим $W_n = \{x \in V | d(x^0, x) = n\}$, $V_n = \bigcup_{m=0}^{n} W_m$, где $d(x, y)-$ расстояние между x и y на дереве Кэли, т.е. число ребер пути, соединяющее x и y .

Мы рассмотрим модель, где спиновые переменные принимают значения из множества $\Phi = \{1, 2, \ldots, q\}$, $q \geq 2$ и расположены на вершинах дерева. Тогда *конфигурация* σ на V определяется как функция $x \in V \to \sigma(x) \in \Phi$; множество всех конфигураций совпадает с $\Omega = \Phi^V$. Пусть $\Omega_n = \Phi^{V_n}$ обозначает пространство конфигураций, определенных на V_n.

Гамильтониан модели Поттса определяется как

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)}, \tag{1}$$

где $J \in \mathbb{R}$, $\langle x, y\rangle-$ ближайшие соседи и $\delta_{ij}-$ символ Кронекера. Определим конечномерное распределение вероятностной меры μ в объеме V_n как

$$\mu_n(\sigma_n) = Z_n^{-1} \exp\left\{-\beta H_n(\sigma_n) + \sum_{x \in W_n} h_x \sigma(x)\right\}, \tag{2}$$

где $\sigma_n \in \Omega_n, \beta = 1/T$, $T > 0-$температура, $h_x \in \mathbb{R}^{q-1}$, $Z_n^{-1}-$ нормировочный множитель.

Теорема 1. [1] *Меры (2) удовлетворяют условию согласованию тогда и только тогда, когда для всех $x \in V \setminus \{x^0\}$ имеет место следующее:*

$$h_x = \sum_{y \in S(x)} F(h_y, \theta), \tag{3}$$

где функция $F : h = (h_1, \ldots, h_{q-1}) \in \mathbb{R}^{q-1} \to F(h, \theta) = (F_1, \ldots, F_{q-1}) \in \mathbb{R}^{q-1}$ определяется формулами

$$F_i = \ln\left(\frac{(\theta-1)e^{h_i} + \sum_{j=1}^{q-1} e^{h_j} + 1}{\theta + \sum_{j=1}^{q-1} e^{h_j}}\right), \qquad \theta = \exp(J\beta).$$

Каждому решению h_x функционального уравнения (3) соответствует одна мера Гиббса и наоборот.

В этом пункте мы находим новые решения функционального уравнения (3) при $q = 3$. Рассмотрим следующую матрицу

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

где a, b, c, d целые неотрицательные числа и

$$a + b = k, c + d = k. \tag{4}$$

Эта матрица определяет того, что сколько раз присутствуют векторные значения $\bar{h} = (h_1, h_2)$ и $\bar{l} = (l_1, l_2)$ в множестве $S(x)$ для каждого $h_x \in \{\bar{h}, \bar{l}\}$. Граничные условия (т.е совокупность векторов) $h = \{h_x, x \in V\}$, определяются следующими образом:

(i) если в вершине x имеем $h_x = \bar{h}$, то функция h_y принимающая векторное значение каждой вершине $y \in S(x)$ определим по следующему правилу: $\bar{h}$, на a вершинах $S(x)$ и $\bar{l}$, на b вершинах $S(x)$.

(ii) если в вершине x имеем $h_x = \bar{l}$, то функция имеет значения: $\bar{h}$, на c вершинах $S(x)$ и $\bar{l}$, на d вершинах $S(x)$.

Легко видеть, что граничные условия h_x (т.е. совокупность векторов) приведенное выше конструкции удовлетворяют функциональному уравнению (3), если векторы (h_1, h_2) и (l_1, l_2) удовлетворяют следующей системе уравнений:

$$\begin{cases} h_1 = a \ln \frac{\theta \exp(h_1)+\exp(h_2)+1}{\theta+\exp(h_1)+\exp(h_2)} + b \ln \frac{\theta \exp(l_1)+\exp(l_2)+1}{\theta+\exp(l_1)+\exp(l_2)}, \\ h_2 = a \ln \frac{\theta \exp(h_2)+\exp(h_1)+1}{\theta+\exp(h_1)+\exp(h_2)} + b \ln \frac{\theta \exp(l_2)+\exp(l_1)+1}{\theta+\exp(l_1)+\exp(l_2)}, \\ l_1 = c \ln \frac{\theta \exp(h_1)+\exp(h_2)+1}{\theta+\exp(h_1)+\exp(h_2)} + d \ln \frac{\theta \exp(l_1)+\exp(l_2)+1}{\theta+\exp(l_1)+\exp(l_2)}, \\ l_2 = c \ln \frac{\theta \exp(h_2)+\exp(h_1)+1}{\theta+\exp(h_1)+\exp(h_2)} + d \ln \frac{\theta \exp(l_2)+\exp(l_1)+1}{\theta+\exp(l_1)+\exp(l_2)}, \end{cases} \tag{5}$$

где a, b, c, d задаются в матрице M.

Теорема 2. Независимо от параметров система уравнений (5) имеет решение $(0, 0, 0, 0)$ и если

$$\mid (bc - ad)\Big(\frac{\theta - 1}{\theta + 2}\Big)^2 + (a + d)\Big(\frac{\theta - 1}{\theta + 2}\Big) \mid > 1,$$

тогда для система уравнений (5) существуют не менее семь различных решений: $(0, 0, 0, 0)$, $(h_1^{(1)}, 0, l_1^{(1)}, 0)$, $(h_1^{(2)}, 0, l_1^{(2)}, 0)$, $(0, h_2^{(1)}, 0, l_2^{(1)})$, $(0, h_2^{(2)}, 0, l_2^{(2)})$, $(h_1^{(1)}, h_1^{(1)}, l_1^{(1)}, l_1^{(1)})$, $(h_1^{(2)}, h_1^{(2)}, l_1^{(2)}, l_1^{(2)})$.

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ИНВАРИАНТНАЯ МЕРА И СИМВОЛИЧЕСКАЯ ДИНАМИКА ДЛЯ ОТОБРАЖЕНИЙ ОКРУЖНОСТИ С ИЗЛОМАМИ

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В настоящей работе мы изучаем связь между структурой орбит точек изломов, инвариантной мерой и символической динамикой. При изучении многих задач важную роль играет символическая динамика соответствующая рассматриваемую динамическую систему. Пусть f сохраняющий ориентацию гомеоморфизм окружности с иррациональным числом вращения $\rho := \rho_f$ (см. [**?**]). Пусть число вращения ρ имеет разложение в непрерывную дробь: $\rho := [a_1, a_2, \dots, a_n, \dots]$, где $a_i,\ i \in \mathbb{N}$. Отметим, что такое разложение единственно.Обозначим $\frac{p_n}{q_n},\ n \geq 1$. Числа $\frac{p_n}{q_n}$ называются подходящими дробями числа ρ. Используя арифметические свойства числа вращения ρ можно построить последовательность динамических разбиений.Возьмем произвольную $x_0 \in S^1$ и рассмотрим ее орбиту $x_i := f^i(x_0),\ i = 0, 1, 2, \dots$. Обозначим через Δ_0^n отрезок соединяющий точки x_0 и x_{q_n}. Положим $\Delta_s^n := f^s(\Delta_0^n),\ s \geq 1$. Хорошо известно, что для каждого $n \geq 1$ система отрезков $\{P_n(x_0),\ \ n \geq 1\}$ (см. [**?**],[**?**]):

$$P_n(x_0) = \{\Delta_0^{(n-1)}, \Delta_1^{(n-1)}, ..., \Delta_{q_n-1}^{(n-1)}\} \bigcup \{\Delta_0^{(n)}, \Delta_1^{(n)}, ..., \Delta_{q_n-1}^{(n)}\},$$

определяет разбиение окружности. При переходе от разбиения $P_n(x_0)$ к $P_{n+1}(x_0)$ все отрезки $\Delta_i^{(n)}(x_0), 0 \leq i \leq q_{n-1} - 1$, сохраняются, а каждый из отрезков $\Delta_j^{(n-1)}(x_0)$, разбивается на $(k_{n+1} + 1)$ отрезков. При помощи последовательности динамических разбиений $P_n(x_0)$ можно построить символическую динамику. Определим

$$\epsilon_0 := \epsilon_0(x) = 0, \ \text{ если}\, x \in \Delta_0^{(1)}(x_0),$$

$$\epsilon_0 := \epsilon_0(x) = a, \ \text{ если}\, x \in \Delta_0^{(0)}(x_0),$$

$$\epsilon_0 := \epsilon_0(x) = k_1 - s, \ \text{ если}\, x \in \Delta_s^{(0)}(x_0), 1 \leq s \leq k_1 - 1.$$

Пусть $x \in S^1 \backslash O_f(x_0)$. Положим $\epsilon_n = \epsilon_n(x) = a$, если $x \in \bigcup\limits_{i=0}^{q_{n-1}-1} \Delta_i^{(n)}(x_0)$. Пусть $x \in \Delta_j^{(n-1)}(x_0)$ при некотором $j, 0 \leq j < q_n$. Из структуры динамических разбиений следует, что $\Delta_j^{(n-1)}(x_0) = \Delta_j^{(n+1)}(x_0) \cup \bigcup\limits_{s=0}^{k_{n+1}-1} \Delta_{j+q_{n-1}+sq_n}^{(n)}(x_0)$. Положим $\epsilon_n := \epsilon_n(x) = 0$, если $x \in \Delta_j^{(n+1)}(x_0)$, $\epsilon_n := \epsilon_n(x) = k_{n+1} - s$, если $x \in \Delta_{i+q_{n-1}+sq_n}^{(n)}(x_0), 0 \leq s \leq k_{n+1} - 1$. Таким образом, мы получим взаимно-однозначное соответствие $\psi : S^1 \setminus O_f(x_0) \leftrightarrow \{\underline{\epsilon} = (\epsilon_0, \epsilon_1, \epsilon_2, ...\epsilon_n, ...), \epsilon_n \in \{a, 0, 1, ..., k_{n+1}\}$при этом, $\epsilon_{n+1} = a,$ тогда и только тогда, когда $\epsilon_n = 0, n \geq 1\} =: \Theta_+$. Отметим, что при этом каждому отрезку $\Delta^{(n)}$ динамического разбиения $P_n(x_0)$ соответствует единственное слово длины $n : (\epsilon_0, \epsilon_1, ..., \epsilon_{n-1})$. Пусть $\Delta^{(n)} = \Delta(\epsilon_0, \epsilon_1, ..., \epsilon_n)$. Мера Лебега на S^1 индуцирует вероятностную меру ℓ_0 на Θ_+ т.е. $\ell_0(\epsilon_0, \epsilon_1, ..., \epsilon_{n-1}) :=\mid \Delta(\epsilon_0, \epsilon_1, ..., \epsilon_{n-1}) \mid$. Отметим, что всем гомеоморфизмам окружности с одинаковым иррациональным числом вращения ρ соответствует одно и тоже символическое пространство $\Theta_+(\rho)$. Мы найдем

явное выражение для инвариантной меры отрезка $[b_1, b_2]$ соединяющего точки изломов b_1 и b_2 отображения f, при помощи символической динамики точки b_2. Хорошо известно, что каждое каждый гомеоморфизм окружности f с иррациональным числом вращения $\rho := \rho(f)$ обладает единственной вероятностной инвариантной мерой μ_f.

Теперь сформулируем основной результат нашей работы.

Теорема. Пусть $f \in C^1(S^1 \backslash \{b_1, b_2\})$ кусочно-гладкий гомеоморфизм окружности с иррациональным числом вращения $\rho := \rho(f)$ и точками изломов b_1 и b_2. Предположим, что $\epsilon(b_2) = (\epsilon_0, \epsilon_1, \epsilon_2, \epsilon_3, ...)$ символическое представление b_2 относительно последовательности динамических разбиений $\{P_f(b_1)\}$ точки излома b_1. Тогда

$$\mu_f([b_1, b_2]) = \sum_{n=0}^{\infty} (-1)^n \eta(\epsilon_n) \Delta_n,$$

где $\Delta_n = \mu_f([b_1, f^{q_n}(b_2)])$.

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КРАЕВАЯ ЗАДАЧА НА ПАРАЛЛЕЛЬНЫХ ХАРАКТЕРИСТИКАХ ДЛЯ НАГРУЖЕННОГО УРАВНЕНИЯ ПАРАБОЛО-ГИПЕРБОЛИЧЕСКОГО ТИПА

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Краевые задачи для вырождающегося нагруженного уравнения смешанного типа второго порядка исследовались сравнительно мало. Отметим работы А.М.Нахушева [1], В.М.Казиева [2], Б.Исломова и Ф.Джураева [3]. Это связано, прежде всего, с отсутствием представления общего решения для таких уравнений; с другой стороны, такие задачи сводятся к малоизученным интегральным уравнениям.

Пусть $\Omega-$ конечная односвязная область в плоскости переменных x, y , ограниченная кривыми:

$$S_j: \ |x|=1, \ 0<y<1, \ S_3: \ 0<x<1, \quad y=1, \ S_4: \ -1<x<0, \quad y=1,$$

$$\Gamma_j: \ |x|-\frac{2}{2-m}(-y)^{\frac{2-m}{2}}=0, \quad \Gamma_{j+2}: \ |x|+\frac{2}{2-m}(-y)^{\frac{2-m}{2}}=1, \ \ y\le 0,$$

здесь и далее $x\ge 0$, при $j=1$, $x\le 0$, при $j=2$, причем $m<0$.

Введем обозначения $\Omega_1^+=\Omega\cap\{(x,y): \ x>0, \ y>0\}$,

$$\Omega_2^+=\Omega\cap\{(x,y): \ x<0, \ y>0\}, \ \Omega_1^-=\Omega\cap\{(x,y): \ x>0, \ y<0\},$$

$$\Omega_2^-=\Omega\cap\{(x,y): \ x<0, \ y<0\}, \ \ I_1=\{(x,y): \ 0<x<1, \ y=0\},$$

$$I_2=\{(x,y): \ -1<x<0, \ y=0\}, \ I_3=\{(x,y): \ x=0, \ 0<y<1\},$$

$$\Omega_j=\Omega_j^+\cup\Omega_j^-\cup J_j, \ \Omega_3=\Omega_1^+\cup\Omega_2^-\cup J_3, \ \ A_j((-1)^{j+1},0)=\bar{I}_j\cap\bar{S}_j,$$

$$C_j\left[(-1)^{j+1}\frac{1}{2}; \ \ -\left((-1)^{j+1}\frac{2-m}{4}\right)^{2/(2-m)}\right]=\bar{\Gamma}_j\cap\bar{\Gamma}_{j+2}, \ (j=1,2),$$

$$O(0,0)=\bar{I}_1\cap\bar{I}_2, \ B_1(1,1)=\bar{S}_1\cap\bar{S}_3, \ B_2(-1,1)=\bar{S}_2\cap\bar{S}_4, \ B_0(0,1)=\bar{S}_3\cap\bar{S}_4.$$

В области Ω рассмотрим уравнение

$$0=\begin{cases} u_{xx}-|x|^p u_y-\rho_j u(x,0), & (x,y)\in\Omega_j^+, \\ u_{xx}-(-y)^m u_{yy}+\mu_j u(x,0), & (x,y)\in\Omega_j^-, \end{cases} \tag{1}$$

где $m, \quad p, \quad \rho_j, \quad \mu_j \ (j=1,2)-$любые действительные числа, причем

$$m<0, \ \ p>0, \ \ \rho_j>0, \ \ \mu_j>0, \ (j=1,2). \tag{2}$$

В области Ω для уравнения (1) исследуется следующая задача.

Задача 1. Найти функцию $u(x,y)$, обладающую следующими свойствами:

1) $u(x,y)\in C\left(\bar{\Omega}\right)\cap C^1(\Omega)\cap C^{2,1}_{x,y}\left(\Omega_1^+\cup\Omega_2^+\right)\cap C^2\left(\Omega_1^-\cup\Omega_2^-\right)$;

2) $u(x,y)$ является регулярным решением уравнения (1) в областях Ω_j^+ и Ω_j^- $(j=1,2)$;

3) $u(x,y)$ удовлетворяет краевым условиям

$$u|_{S_j} = \varphi_j(y), \quad 0 \le y \le 1, \tag{3}$$

$$u|_{\Gamma_1} = g_1(x), \quad 0 \le x \le \frac{1}{2}, \ u|_{\Gamma_4} = g_2(x), \quad -1 \le x \le -\frac{1}{2}, \tag{4}$$

4) на линии вырождения I_i $(i = \overline{1,3})$ выполняется условия склеивания

$$\lim_{y\to+0} u_y(x,y) = \lim_{y\to-0} u_y(x,y), \ (x,0) \in I_j, \ (j = 1,2), \tag{5}$$

$$\lim_{x\to+0} u_x(x,y) = \lim_{x\to-0} u_x(x,y), \ (0,y) \in I_3; \tag{6}$$

где $\varphi_1(y), \ \varphi_2(y), \ g_1(x), \ g_2(x), -$ заданные функции, причем $g_2(-1) = \varphi_2(0)$,

$$\varphi_j(y) \in C[0,1] \cap C^1(0,1), \ (j = 1,2), \tag{7}$$

$$g_1(x) \in C^1\left[0, \frac{1}{2}\right] \cap C^3\left(0, \frac{1}{2}\right), \ g_2(x) \in C^1\left[-1, -\frac{1}{2}\right] \cap C^3\left(-1, -\frac{1}{2}\right). \tag{8}$$

Лемма. Если $\varphi_1(y) \equiv \varphi_2(y) \equiv 0, \ \ \forall y \in [0,1]$, $g_1(x) \equiv 0, \ \ \forall x \in \left[0, \frac{1}{2}\right]$, и $g_2(x) \equiv 0, \ \ \forall x \in \left[-1, -\frac{1}{2}\right]$, то

$$\tau_j(x) \equiv 0, \quad \forall x \in \bar{I}_j \quad (j = 1,2), \tag{9}$$

здесь $\tau_j(x) \equiv \upsilon_j(x,0) = w_j(x,0)$.

Теорема. Если выполнены условия леммы и (7), (8),(9), то в области Ω решение задачи 1 для уравнения (1) существует и единственно.

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ЗАДАЧА ДИРИХЛЕ ДЛЯ УРАВНЕНИЯ ВЫСОКОГО ПОРЯДКА С ПРОИЗВОДНОЙ КАПУТО

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В области $\Omega = \Omega_x \times \Omega_y,\ \Omega_x = \{x : 0 < x < 1\},\ \Omega_y = \{y : 0 < y < 1\}$, рассмотрим уравнение

$$K(y)\, l(u(x,y)) + q_C D_{0x}^{\alpha} u = f(x,y), 0 \neq q \in R, \tag{1}$$

где

$$l(u) = (-1)^s \frac{\partial^{2s} u(x,y)}{\partial y^{2s}} + \frac{\partial^{s-1}}{\partial y^{s-1}} \left((-1)^{s-1} p_{s-1}(y) \frac{\partial^{s-1} u(x,y)}{\partial y^{s-1}} \right) + ...$$

$$+ \frac{\partial}{\partial y} \left(-p_1(y) \frac{\partial u(x,y)}{\partial y} \right) + p_0(y)\, u(x,y),$$

$$0 \leq p_j(y) \in C^j\left(\overline{\Omega}_y\right),\ j = 0,1,...,s-1,\ s \in N,$$

$$K(y) > 0, y \in (0,1), K(0) = K(1) = 0,$$

$$K^{(i)}(y) = O\left(y^{m-i}\right), y \to 0+0, 0 \leq m < s,$$

$$K^{(i)}(y) = O\left((1-y)^{n-i}\right), y \to 1-0, 0 \leq n < s, i = 0,1,...,$$

$$K(y) \in C^{2s}(0,1); 1 < \alpha < 2,$$

${}_C D_{0x}^{\alpha} u(x,y) = \frac{1}{\Gamma(2-\alpha)} \int\limits_0^x \frac{\frac{\partial^2 u(t,y)}{\partial t^2} dt}{(x-t)^{\alpha-1}}-$ производная Капуто, функция $f(x,y)$ является достаточно гладкой в $\overline{\Omega}$.

Для уравнения (1) исследуем следующую задачу.

Задача D. Найти решение уравнения (1) с условиями:

$u(x,y) \in AC^2[0,1]$, по переменной x;

$$u(x,y) \in C\left(\bar{\Omega}\right), \frac{\partial^{2s-1} u(x,y)}{\partial y^{2s-1}} \in C\left(\bar{\Omega}\right), \frac{\partial^{2s} u(x,y)}{\partial y^{2s}} \in C(\Omega),$$

удовлетворяющее краевым условиям

$$\frac{\partial^j u(x,0)}{\partial y^j} = \frac{\partial^j u(x,1)}{\partial y^j} = 0,\ 0 \leq x \leq 1,\ j = 0,1,...,s-1, \tag{2}$$

$$u(0,y) = \varphi_0(y), u(1,y) = \varphi_1(y), \tag{3}$$

где функции $\varphi_0(y), \varphi_1(x)$ достаточно гладкие.

В данной работе изучена краевая задача типа Дирихле для уравнения (1) на однозначную разрешимость. Рассмотрены отдельно случаи $q > 0$ и $q < 0$. Доказаны теоремы существования и единственности.

В силу условия (2) рассмотрена следующая спектральная задача:

$$\begin{cases} l\left(Y\left(y\right)\right) = \lambda \frac{Y(y)}{K(y)}, \\ Y^{(j)}(0) = Y^{(j)}\left(1\right) = 0, j = 0, 1, ..., s-1. \end{cases} \tag{4}$$

Показана , что задача (4) имеет счетное число собственных значений, причем все собственные значения положительны. Получены оценки рядов состоящих из собственных функций при помощи неравенства Бесселя. Найдены достаточные условия разложения данной функций в ряд Фурье по собственным функциям задачи (4).

Решение задачи D ищется в виде

$$u\left(x, y\right) = \sum_{k=1}^{\infty} X_k\left(x\right) Y_k\left(y\right), \tag{5}$$

где , с учетом условия (3), функции $X_k(x)$ есть решения задач

$$\begin{cases} \lambda_k X_k\left(x\right) + q_C D_{0x}^{\alpha} X_k\left(x\right) = f_k\left(x\right) = \int\limits_0^1 \frac{f(x,y) Y_k(y)}{K(y)} dy, \\ X_k\left(0\right) = \varphi_{0k} = \int\limits_0^1 \varphi_0\left(y\right) Y_k\left(y\right) dy, \\ X_k\left(1\right) = \varphi_{1k} = \int\limits_0^1 \varphi_1\left(y\right) Y_k\left(y\right) dy. \end{cases} \tag{6}$$

Решение задачи (6) имеет вид

$$X_k\left(x\right) = \int\limits_0^1 G\left(x, t\right) f_k\left(t\right) dt + \varphi_{0k} \frac{F_1\left(x\right) F_2\left(1\right) - F_2\left(x\right) F_1\left(1\right)}{F_2\left(1\right)} + \varphi_{1k} \frac{F_2\left(x\right)}{F_2\left(1\right)},$$

где

$$G\left(x, t\right) = -\frac{1}{qF_2\left(1\right)} \begin{cases} F_2\left(x\right) F_{\alpha}\left(1-t\right) - F_2\left(1\right) F_{\alpha}\left(x-t\right), 0 \le t < x \le 1, \\ F_2\left(x\right) F_{\alpha}\left(1-t\right), 0 \le x \le t \le 1 \end{cases}$$

- функция Грина;

$$F_{\beta}\left(x\right) = x^{\beta-1} E_{\alpha,\beta}\left(-\frac{\lambda_k}{q} x^{\alpha}\right),$$

$$E_{\alpha,\beta}\left(z\right) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma\left(\alpha n + \beta\right)}; \alpha, \beta, z \in C, \mathrm{Re}\alpha > 0$$

- функция Миттаг-Леффлера.

Найдены условия при которых ряд (5) будет решением задачи D.

О ДИНАМИЧЕСКОЙ СИСТЕМЕ, ЯВЛЯЮЩЕЙСЯ МОДЕЛЬЮ ДВУМОДОВОГО ГИДРОМАГНИТНОГО ДИНАМО С ПАМЯТЬЮ

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Механизм генерации космических магнитных полей является одной из фундаментальных проблем магнитной гидродинамики. Крупномасштабные магнитные поля планет, звезд и галактик формируются, по современным представлениям, механизмом гидромагнитного динамо [1]. Имеется ряд соображений в пользу того, что этот механизм обладает пространственной и временной нелокальностями [2]. Авторами доклада была предложена модель двумодового динамо, учитывающая временную нелокальность (память) [3-5]. Модель представляет следующую интегро-дифференциальную систему уравнений:

$$\begin{aligned} \frac{dx_1}{dt} &= \left(\eta^T - \frac{p}{s^2}x_3\right) x_2 - \eta^T x_1, \\ \frac{dy}{dt} &= (D - x_3)x_1 - x_2, \\ z(t) &= \int_0^t K(t-\tau)Q(x_1(\tau)x_2(\tau))\,d\tau. \end{aligned} \tag{1}$$

Здесь $x_1(t)$ и $x_2(t)$ представляют собой безразмерные амплитуды тороидальной и полоидальной компонент магнитного поля, положительные коэффициенты η^T, s, p, D являются управляющими параметрами модели.

Физический смысл интегрального члена – это память в обратной связи динамосистемы, когда подавление генерации магнитного поля обеспечивается взвешенной по прошлым состояниям функцией $Q(x, y)$ от компонент поля.

В докладе обсуждаются свойства системы (1), связанные с существованием и единственностью решения, а также возможностью сведения ее к дифференциальной системе при некоторых типах ядра $K(t)$ с экспоненциальной асимптотикой.

Теорема 1. Система уравнений (1) имеет одно и только одно решение в любой ограниченной области D переменных (x_1, x_2) для любых значениях начальных значений$x_1(0)$, $x_2(0) \in D$ и любом ограниченном ядре $K(t)$ на любом конечном интервале времени T.

Замечание. Теорема 1 не исключает возможность возникновения в некоторых случаях разрушения решения (blow-up) связанного с выходом решения, за конечное время, за пределы любой наперед заданной ограниченной области.

Теорема 2. Если ядро $K(t)$ - решение дифференциального уравнения

$$a_0K^{(n)}(t) + a_1K^{(n-1)}(t) + \cdots + a_{n-1}K'(t) + a_nK(t) = 0, \quad a_i - \text{const},$$

то интегральное представление

$$x_3(t) = \int_0^t K(t-\tau)Q(x_1(\tau), x_2(\tau))\,d\tau.$$

равносильно дифференциальному уравнению порядка n:

$$a_0\frac{d^n x_3}{dt^n} + a_1\frac{d^{n-1}x_3}{dt^{n-1}} + \dots a_n x_3 = \sum_{k=0}^{n} a_{n-k}\sum_{m=0}^{k-1} K^{(m)}(0)\frac{d^{k-m-1}}{dt^{k-m-1}}Q(x_1(t), x_2(t)).$$

для функции $x_3(t)$ с начальными условиями:

$$\left.\frac{d^l x_3(t)}{dt^l}\right|_{t=0} = \sum_{s=0}^{l-1} K^{(s)}(0)\left.\frac{d^{l-1-s}}{dt^{l-1-s}}Q(x_1(t), x_2(t))\right|_{t=0}, \quad l = 1, \dots, n-1.$$

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AW*-МОДУЛЬ НАД ВЕЩЕСТВЕННОЙ АБЕЛЕВОЙ AW*-АЛГЕБРОЙ

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По работе [1] дано понятие модуля над коммутативной C*-алгеброй который являются аналогом гильбертова пространства с применением вместо комплексных чисел, элементов коммутативной C*-алгебры как множества значений скалярного произведения заданного над модулем. Пусть Z - коммутативная C*-алгебра. C*-модуль H над Z понимается как правый Z-модуль таким образом, что единичный элемент Z действует как единичный оператор и скалярное произведение $(\cdot,\cdot) : H \times H \longrightarrow Z$ принимающее значения в коммутативной C*-алгебре Z, удовлетворяющее аксиомам скалярного произведения:
1. $(x,x) \geq 0$ и $(x,x) = 0 \iff x = 0\ \forall x \in H$
2. $(xa + yb, z) = (x,z)a + (y,z)b\ \forall x,y,z \in H\ \forall a,b \in Z$
3. $(x,y)^* = (y,x)$

Норму на модуле можно определить как модульный Z-фукционал $\|x\|_Z = \|(x,x)\|_Z^{\frac{1}{2}}$ где $|x| = (x,x)^{\frac{1}{2}}$ удовлетворяющий всем условиям C*-алгебры.

Если Z – коммутативная AW*-алгебра, то C*-модуль H называется AW*-модулем, если дополнительно выполняется следующие две условия:
(a) если $\{e_i\}$ – ортогональные проекторы в Z с точной верхней границей e, то равенство $e_i x = 0$ $(\forall i)$ влечет $ex = 0$, где $x \in H$;
(b) если $\{e_i\}$ – ортогональные проекторы в Z с точной верхней границей $\mathbf{1}$ и $\{x_i\}$ – ограниченное подмножество H, то существует $x \in H$ такой, что $e_i x = e_i x_i$, для всех i.

Для вещественных AW*-алгебр эти понятия определяются аналогично. В [2] приведены примеры вещественных абелевых AW*-алгебр комплексификация которых не является (комплексной) AW*-алгеброй. Следовательно комплексификация AW*-модуля над вещественной абелевой AW*-алгеброй, не всегда будет AW*-модулем.

Теорема. Пусть B – вещественная абелева AW*-алгебра такая, что $B + iB$ – AW*-алгебра. Тогда комлексификация $H + iH$ вещественного AW*-модуля H над вещественной AW*-алгеброй B, является (комплексным) AW*-модулем над $B + iB$.

Замечание. Если B – вещественная абелева AW*-алгебра, комплексификация $B + iB$ которой не AW*-алгебра (см.[2]), то комплексификация вещественного AW*-модуля над B является C*-модулем над $B + iB$, но не является AW*-модулем.

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ПОЧТИ ВНУТРЕННИЕ ДИФФЕРЕНЦИРОВАНИЯ НИЛЬПОТЕНТНЫХ n-ЛИЕВЫХ АЛГЕБР

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В этой заметке изучены почти внутренние дифференцирования нильпотентных n-Лиевых алгебр.

Понятие n-Лиевой алгебры было введено В. Т. Филипповым в 1985 году [1].

Определение 1. n-Лиевая алгебра - это векторное пространство A, наделенное кососимметричным умножением, удовлетворяющий n-арному тождеству Якоби:

$$[[x_1, ..., x_n], y_2, ..., y_n] = \sum_{i=1}^{n} [x_1, ..., [x_i, y_2, ..., y_n], ..., x_n].$$

В 1987 году С. М. Касымов [2] определил понятие нильпотентности n-Лиевой алгебры следующим образом:

Определение 2. n-Лиевая алгебра A называется нильпотентной, если $A^s = 0$ для некоторого целого неотрицательного s, в котором A^{i+1} определяется индуктивно как

$$A^1 = A \quad \text{и} \quad A^{i+1} = \left[A^i, A, ..., A\right].$$

n-Лиевая алгебра A нильпотентна класса c, если $A^{c+1} = 0$ и $A^c \neq 0$.

Определение 3. Дифференцированием n-Лиевой алгебры A называется линейное отображение A в себя, удовлетворяющее условию

$$d([x_1, ..., x_n]) = \sum_{i=1}^{n} [x_1, ..., d(x_i), ..., x_n]$$

для всех $x_1, ..., x_n \in A$. Обозначим множество всех дифференцирований n-Лиевой алгебры A через $Der(A)$.

Определение 4. Отображение $R(x_2, ..., x_n) : a \to [a, x_2, ..., x_n]$ n-Лиевой алгебры A называется правым умножением, определяемым элементами $x_2, ..., x_n$. Правые умножения и их линейные комбинации являются дифференцированиями, которые называются внутренними. Обозначим множество всех внутренних дифференцирований n-Лиевой алгебры A через $Inner(A)$.

Определение 5. Дифференцирование $D \in Der(A)$ n-Лиевой алгебры A называется почти внутренним дифференцированием, если $D(x) \in [x, A, ..., A]$ для всех $x \in A$.

То есть, существуют такие $a_2(x), ..., a_n(x) \in A$, что $D(x) = [x, a_2(x), ..., a_n(x)]$. Обозначим множество всех почти внутренних дифференцирований n-Лиевой алгебры A через $AID(A)$.

Теорема 1. [3] Любая $(n+3)$-мерная и $(n+4)$-мерная нильпотентная n-Лиевая алгебра изоморфна одной из попарно не изоморфных алгебр:

$A_{n+3,2}$: $[e_1, ..., e_n] = e_{n+3}$;

$A_{n+3,3}$: $[e_1, ..., e_n] = e_{n+1}$, $[e_2, ..., e_{n+1}] = e_{n+3}$, $[e_2, ..., e_n, e_{n+2}] = e_{n+3}$;
$A_{n+3,4}$: $[e_1, ..., e_n] = e_{n+1}$, $[e_2, ..., e_{n+1}] = e_{n+3}$;
$A_{n+3,5}$: $[e_1, ..., e_n] = e_{n+1}$, $[e_2, ..., e_n, e_{n+2}] = e_{n+3}$;
$A_{n+3,6}$: $[e_1, ..., e_n] = e_{n+1}$, $[e_2, ..., e_{n+1}] = e_{n+2}$, $[e_2, ..., e_n, e_{n+2}] = e_{n+3}$;
$A_{n+3,7}$: $[e_1, ..., e_n] = e_{n+1}$, $[e_2, ..., e_n, e_{n+2}] = [e_1, e_3, ..., e_{n+1}] = e_{n+3}$;
$[e_2, ..., e_{n+1}] = e_{n+2}$,
$A_{n+3,8}$: $[e_1, ..., e_n] = e_{n+1}$, $[e_2, ..., e_{n+1}] = e_{n+2}$, $[e_1, e_3, ..., e_{n+1}] = e_{n+3}$;
$A_{n+4,1}$: $[e_1, ..., e_n] = e_{n+3}$, $[e_2, ..., e_{n+1}] = e_{n+4}$;
$A_{n+4,2}$: $[e_1, ..., e_n] = e_{n+3}$, $[e_3, ..., e_{n+2}] = e_{n+4}$, $(n \geq 3)$;
$A_{n+4,3}$: $[e_1, ..., e_n] = e_{n+1}$, $[e_2, ..., e_n, e_{n+2}] = e_{n+3}$, $[e_1, e_3, ..., e_n, e_{n+2}] = e_{n+4}$;

Основными результатами являются следующие теоремы:

Теорема 2. Всякие почти внутренние дифференцирования $(n+3)$-мерных нильпотентных n-Лиевых алгебр являются внутренними.

Теорема 3. Всякие почти внутренние дифференцирования $(n+4)$-мерных нильпотентных n-Лиевых алгебр являются внутренним.

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ВЗАИМООТНОШЕНИЕ МЕЖДУ РАСПРЕДЕЛЕНИЯМИ СТАЦИОНАРНЫХ ДЛИН ОЧЕРЕДЕЙ СИСТЕМ M|G|1 И M|G|1|N

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Рассматриваются системы массового обслуживания F- M|G|1 и F_N - M|G|1|N, которое характеризуются следующими данными: в систему поступает поток Пуассона с параметром λ ,длительности времен обслуживания независимые, одинаково распределенные случайные величина со среднем μ^{-1} , число мест для ожидания в системе F_N ограничено величиной $N(N \geq 1)$.

Пусть ζ_k-период занятости система $F_k(k = \overline{1,N})$; ξ и ξ_N-стационарные длины очереди систем F и F_N, соответственно; $\rho = \lambda\mu^{-1}$- загрузка рассматриваемых систем.

В данной работе устанавливается связь между распределениями случайных величин ξ и ξ_N в случае $\rho < 1$.

Теорема. При $\rho < 1$ имеют место соотношения

$$P(\xi_N = k) = P(\xi = k)(1 - \rho P(\xi > N))^{-1}, 0 \leq k \leq N,$$

$$P(\xi_N = N + 1) = (1 - \rho)P(\xi > N)(1 - \rho P(\xi > N))^{-1}.$$

Доказательство. В работе [2] установлены следующие соотношения:

$$P(\xi_N = 0) = (1 + \lambda M\zeta_N)^{-1},$$

$$P(\xi_N = k) = \mu(M\zeta_k - M\zeta_{k-1})P(\xi_N = 0), 1 \leq k \leq N, \tag{1}$$

$$P(\xi_N = N + 1) = (1 - (\lambda - \mu)M\zeta_N)P(\xi_N = 0).$$

Известно, что при $N \to \infty(\rho < 1)$ [1,стр.62]

$$M\zeta_N \to \mu^{-1}(1 - \rho)^{-1}.$$

Учитывая это, из (1) при $\rho < 1$ получим

$$P(\xi = 0) = 1 - \rho,$$

$$P(\xi_N = k) = \mu(1 - \rho)(M\zeta_k - M\zeta_{k-1}), k \geq 1. \tag{2}$$

По соотношениям (1) и (2) находим формулу

$$P(\xi_N = k) = P(\xi = k)(1 + \lambda M\zeta_N)^{-1}(1 - \rho)^{-1}. \tag{3}$$

Пусть

$$f_N = P(\xi > N) = \sum_{k=N+1}^{\infty} P(\xi = k).$$

Тогда так как, по (3)

$$\sum_{k=1}^{N} P(\xi = k) = \mu(1-\rho)M\zeta_N$$

или

$$M\zeta_N = \frac{1}{\mu(1-\rho)} P(\xi \leq N),$$

то (3) принимает следующий вид:

$$P(\xi_N = k) = P(\xi = k)(1 - \rho f_N), k = \overline{1, N}.$$

Также, имеем

$$P(\xi_N = N+1) = 1 - \sum_{k=1}^{N} P(\xi_N = k) = \frac{(1-\rho)f_N}{1-\rho f_N}.$$

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НЕЕДИНСТВЕННОСТИ МЕРЫ ГИББСА ДЛЯ ОДНОЙ МОДЕЛИ НА ДЕРЕВЕ КЭЛИ

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Дереве Кэли Γ^k порядка $k \geq 1-$ бесконечное дерево, т.е. граф без циклов, из каждой вершины которого выходит ровно $k+1$ ребер.

Пусть $\Gamma^k = (V, L, i)$, $V-$ есть множество вершин Γ^k, $L-$ его множество ребер и функция инцидентности, сопоставляющая каждому ребер $l \in L$ его концевые точки $x, y \in V$. Если $i(l) = \{x, y\}$, то вершины x и y называются ближайшими соседями и обозначаются $< x, y >$.

Расстояние $d(x, y), x, y \in V$ на дереве Кэли определяется формулой

$$d(x, y) = min\{d | \exists x = x_0, x_1, ..., x_{d-1}, x_d = y \in V\}$$

где $< x_0, x_1 >, < x_1 = x_2 >, ..., < x_{d-1} = x_d >$ ближайших соседей.

Пусть $\Phi = \{1, 2, ..., q\}, q \geq 2$.Тогда конфигурация σ на V определяется как функция $x \in V \to \sigma(x) \in \Phi$. Множество всех конфигураций совпадает с $\Omega = \Phi^V$.

Пусть $A \subset V$. Обозначим через Ω_A пространство конфигураций, определенных на множестве A, принимающих значения из Φ.

В работа У.А.Розикова (см [1]) определен обобщенный символ Кронекера как функция:

$$U(\sigma_A) : \Omega_A \to \{|A| - 1, |A| - 2, ..., |A| - min\{|A|, |\Phi|\}\},$$

определенная следующим образом:

$$U(\sigma_A) = |A| - |\sigma_A \cap \Phi|. \tag{1}$$

где $|\sigma_A \cap \Phi|-$ число различных значений $\sigma_A(x), x \in A$.

Пусть $r \in N$ и $r' = [\frac{r+1}{2}]$.

Обозначим через M_r множество всех шаров $b_r(x) = \{y \in V : d(x, y) \leq r'\}$ с радиусом r', т.е. $\{b_r(x) : x \in V\}$.

Гамильтониан определяется следующим образом

$$H(\sigma) = -J \sum_{b \in M_r} U(\sigma_b), \tag{2}$$

где $J \in R$.

Основная проблема для данного гамильтониана-описание отвечающих ему мер Гиббса.

Определение меры Гиббса и других понятий связанные с теории гиббсовских мер можно найти например в [2].

Теорема. При $k = q = r = 2$ и достаточно низких температурах существуют три трансляционно-инвариантные Гиббсовские меры модели (2).

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ПЕРИОДИЧЕСКИЕ МЕРЫ ГИББСА ДЛЯ НС-МОДЕЛИ НА ДЕРЕВЕ КЭЛИ

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НС-модели вызывают интерес с точки зрения статической механики, комбинаторики и теории нейронных сетей [1],[2].

Дереве Кэли Γ^k порядка $k \geq 1-$ бесконечное дерево, т.е. граф без циклов, из каждой вершины которого выходит ровно $k+1$ ребер.

Пусть $\Gamma^k = (V, L, i)$, $V-$ есть множество вершин Γ^k, $L-$ его множество ребер и функция инцидентности, сопоставляющая каждому ребер $l \in L$ его концевые точки $x, y \in V$. Если $i(l) = \{x, y\}$, то вершины x и y называются ближайшими соседями и обозначаются $< x, y >$.

Расстояние $d(x, y), x, y \in V$ на дереве Кэли определяется формулой

$$d(x, y) = min\{d | \exists x = x_0, x_1, ..., x_{d-1}, x_d = y \in V\}$$

где $< x_0, x_1 >, < x_1 = x_2 >, ..., < x_{d-1} = x_d >$ ближайших соседей.

Пусть $x^0 \in V-$ фиксированная точка.

Обозначим:

$$W_n = \{x \in V \mid d(x^0, x) = n\}, \;\; V_n = \{x \in V \mid d(x^0, x) \leq n\}.$$

Мы рассматриваем модель, где спин принимает значения из множества $\Phi = \{0, 1, 2, 3\}$.

Конфигурация на дереве Кэли задается $\sigma = \{\sigma(x), x \in V\} \in \Phi^V$.

Рассмотрим некоторый граф $\mathcal{G}$ с ребрами $(i, j) \in \Phi \times \Phi$.

Пусть $A \subset V$. Мы назовем $\mathcal{G}$ допустимой конфигурацией если $(\sigma(x), \sigma(y)) \in \mathcal{G}$ для всех ближайших соседей x и y. Обозначим множество всех допустимых $\mathcal{G}$ конфигураций через Ω.

Рассмотрим матрицу

$$P = (P_{i,j}), i, j = \overline{0, 3}$$

где $P_{i,j} > 0$, если $(i, j) \in \mathcal{G}$; $P_{i,j} = 0$, если $(i, j) \notin \mathcal{G}$ и $\sum_{j \in \Phi} P_{i,j} = 1$

Для $\mathcal{G}$ и P Гамильтониан НС- модели определяется следующим образом

$$H(\sigma) = \begin{cases} \sum\limits_{\langle x,y \rangle} \ln P_{\sigma(x),\sigma(y)}, & \texttt{если} \;\; \sigma \in \Omega^{\mathcal{G}}, \\ \infty, & \texttt{если} \;\; \sigma \notin \Omega^{\mathcal{G}}. \end{cases} \tag{1}$$

Рассмотрим вероятностное распределение μ^n на Ω_{V_n} :

$$\mu^{(n)}(\sigma_n) = \frac{1}{Z_n} \prod_{\langle x,y \rangle \in V_n} P_{\sigma_n(x)\sigma_n(y)} \prod_{x \in W_n} t_{\sigma_n(x),x}, \tag{2}$$

где Z_n- нормирующий множитель.

Теорема 1.[3]. Вероятностное распределение $\mu^{(n)}(\sigma_n), n = 1, 2, ...$ в (2)-согласованное тогда и только тогда, когда для любого $x \in V$ имеют место следующие равенства:

$$z_{i,x} = \prod_{y \in S(x)} \frac{P_{i,0} + P_{i,1} z_{1,y} + P_{i,2} z_{2,y} + P_{i,3} z_{3,y}}{P_{0,0} + P_{0,1} z_{1,y} + P_{0,2} z_{2,y} + P_{0,3} z_{3,y}}, \tag{3}$$

где $z_{i,x} = t_{i,x}/t_{0,x}$, $i = 1, 2, 3$.

Известно, что Γ^k можно представить как группу G_k, которая является свободным произведением $k+1$ циклических групп $\{e, a_i\}$ второго порядка, т.е. $a_i^2 = e$ с образующими $a_1, ..., a_{k+1}$, соответственно.

Определение 1. Пусть $\widetilde{G}-$ подгруппа группы G_k. Совокупность векторов $z = \{z_x : x \in G_k\}$ называется $\widetilde{G}-$ периодической, если $z_{yx} = z_x$ для любых $x \in G_k$ и $y \in \widetilde{G}$.

Определение 2. Мера Гиббса, соответствующая $\widetilde{G}-$ периодической совокупности векторов называется $\widetilde{G}-$ периодической.

Рассмотрим случай:

$$P_{00} = 0,\ P_{01} = a,\ P_{02} = b,\ P_{03} = c,\ P_{10} = \alpha,\ P_{11} = 0,\ P_{12} = 1-\alpha,\ P_{13} = 0,$$

$$P_{20} = \beta,\ P_{21} = 1-\beta,\ P_{22} = 0,\ P_{23} = 0,\ P_{30} = 1,\ P_{31} = 0,\ P_{32} = 0,\ P_{33} = 0, \tag{4}$$

где $\alpha, \beta, a, b, c \in (0, 1); a + b + c = 1$.

Теорема 2. Если выполняется (4) и $k^2\alpha^2 > 1$, то существуют по меньшей мере три $G_k^{(2)}-$ периодические меры Гиббса.

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КАЧЕСТВЕННОЕ ИССЛЕДОВАНИЕ РЕЖИМОВ СТОХАСТИКИ В ОДНОЙ ДИНАМИЧЕСКОЙ СИСТЕМЕ

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Рассматривается многомерная автономная динамическая система [2,3] в общем случае и в приведенными там примерах с численными коэффициентами. Уравнения системы описываются суммами с n и ν членами, для чего вводится индексы i и j в этих суммах. В силу введенных в [2,3] ограничений мы рассмотрим случай $i = j \geq 2$. Система описывает динамику взаимоотношений в процессе возникновения потомства в случае, когда количество особей женского пола относится к типу j с вероятностью $0 \leq p \leq 1$.

Исследование произведем по этапам: в фазовой плоскости $\{x_1, y_1\}$, в её конечной части (т.е. внутри круга Пуанкаре P_∞) [4,5], а также в её бесконечной части (т.е. с включением поведения траекторий на P_∞ и вблизи P_∞). Во всех случаях уравнения в двумерном фазовом пространстве определяется одинаковыми старшими членами по $\{x, y\}$, что приводит к особому типу P_∞, при котором траектории системы уходят на P_∞ не при $t \to \pm\infty$, а при $t = const$. В случае, когда траектории остаются внутри P_∞ при $t \to \pm\infty$, тоже при наличии сепаратисных контуров [6,7] внутри P_∞, а не на P_∞, тоже могут возникать процессы стохастики [6,7,9].

Сепаратисные контуры различных типов возникают за счёт параметров бифуркации, которые в момент бифуркации полагаем нулями или желаемыми константами. Заметим, что в автоволновых процессах при встречных движениях уединённых волн (т.е. солитонов) происходит бифуркация их слияния (т.е. в этот момент они становятся бризерами). При слиянии сепаратриса ℓ идет либо из седла в тот же седло, либо через несколько сёдел и возвращаясь к исходному, соединяясь между собой сепаратрисами ℓ_1 и ℓ_2 или большим числом сепаратрис. Поскольку [6,7], малые изменения параметров, которые всегда имеют место системах, а также малые периодические возмущения (за счёт сезонности, периодов суток), да и силу самоорганизации систем [1] также возможны. Мы изучаем оба этих сценария бифуркаций, для этого используем процесс образования в расширенном фазовом пространстве $\{x, y, t\}$ при неавтономных внешних возмущениях, а также процесс самоорганизации грубых гомоклинических структур, образованных бесконечным пересечением двумерных сепаратрисных поверхностей $S^2_{\ell_{1,2}}$ [6,7]. Такие пересечения в расширенном фазовом пространстве возникают за счёт $\ell_{1,2}$, которые меняют взаимное расположение потоков траекторий при изменении взаиморасположения сепаратрис ℓ_1 и ℓ_2, что ведет к образованию стохастики или к хаосу.

Стохастика возникает и без внешних периодических возмущений, за счёт "прививки"одной системой двух уравнений, выделенных в виде подсистем из многомерной исходной системы [2,3]. Тогда одно и то же неизвестное входит в обе подсистемы, оказывая влияние на каждую из них, либо когда в одной из них существует несколько состояний равновесия, то есть особые точки (сёдел и антисёдел), образующих внутри петли сепаратрис периодические движения. Последнее возможно при, например,

бифуркациях седло-узла или вырожденного седла, и эти возможности определяются разбиением пространства параметров – бифуркационными циферблатами в окрестности состояния равновесия (такое разбиение, чаще всего, происходит в процессе экспериментов на ЭВМ, что также имеет место в [2,3]). Если при таком разбиении имеются параметрические области, в которых происходят периодические колебания, то они также могут служить "прививкой"от одной из подсистем к другой. Период колебания T может быть неизменным, и тогда мы пользуемся достаточными условиями существования обычной изохронности [8], а могут происходить при изменении периода T за счёт существования специальных видов изохронности. Варьируя бифуркационными параметрами, далее составляется цепочка подсистем за счёт введения общего неизвестного в обе соседние подсистемы, что приводит к различным случаям областей стохастики в таких подсистемах, а если их рассматривать в общей совокупности – то и в любом количестве подсистем, входящих в цепочку.

Рассматривается также практически почти невероятный случай, связанный с генетическим сбоем в процессе возникновения особей (или облучением и схожими влияниями). Тогда один из коэффициентов общей системы должен быть отрицательным (вероятность такая практически невозможна), но зато в одной из подсистем образуется область центра, изохронного или не изохронного [8].

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О СВОЙСТВЕ ЛОКАЛЬНО-ОТНОСИТЕЛЬНОЙ УПРАВЛЯЕМОСТИ ДИФФЕРЕНЦИАЛЬНОГО ВКЛЮЧЕНИЯ

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1.Постановка задачи. В теории динамических систем управления отдельный интерес представляют проблемы условной, относительной и локальной управляемости [1–3]. Для изучения этих задач можно использовать методы теории дифференциальных включений. Свойства типа локальной нуль-управляемости для дифференциальных включений рассмотрены в работах [4,5]. Некоторые свойства множества управляемости линейного дифференциального включения и свойство управляемости ансамбля траекторий дифференциальных включений изучены в [6].

Рассмотрим дифференциальное включение

$$\frac{dx}{dt} \in A(t)x + B(t), x(t_0) = x_0, t \geq \quad t_0. \tag{1}$$

Будем предполагать, что для правой части дифференциального включения (1) выполнены условия: 1) элементы матрицы $A(t)$ измеримы на произвольном отрезке $T = [t_0, t_1] \subset T_\infty = [t_0, +\infty)$ и $\|A(t)\| \leq a(t), t \in T = [t_0, t_1], a(\cdot) \in L_1(T)$; многозначное отображение $B : T_\infty \to \Omega(R^n)$ измеримо на любом $T \subset T_\infty$ и $\|B(t)\| \leq b(t), t \in T = [t_0, t_1], b(\cdot) \in L_1(T)$.

Пусть задано компактное терминальное множество $M \subset R^n$.

Определение. Будем говорить, что дифференциальное включение (1) локально-относительно M-управляемо,если существует $\varepsilon>0$ и некоторый отрезок времени $T = [t_0, t_1]$ такие, что любого $x_0 \in M + S_\varepsilon(0)$ существует абсолютно непрерывная траектория $x = x(t)$, удовлетворяющая условиям $x(t_0) = x_0$,$x(t_1) \in M$ т.е. имеет место соотношение $X(t_0, t_1, x_0, A, B) \cap M \neq \emptyset$, где $X(t_0, t_1, x_0, A, B)$ множество достижимости системы (1) в момент времени $t \geq t_0$.

При M=$\{0\}$ получим определение нуль-управляемости.

2.Необходимые условия локальной управляемости. Используя результаты теории многозначных отображений и выпуклого анализа, получим следующие необходимые условия локально-относительной M-управляемости дифференциального включения (1).

Теорема 1. Для локально-относительной M-управляемости дифференциального включения (1) необходимо, чтобы условие

$$\inf_{||\psi||=1} \sup_{\xi \in M} \int_{t_0}^{t_1} c(\Phi_A(t_1, \tau)[A(\tau)\xi + B(\tau)], \psi) d\tau \geq 0,$$

выполнялось при некотором моменте времени $t_1 > t_0$.

Следствие 1. Если дифференциальное включение (1) локально-относительно M-управляемо, то существует момент времени $t_1 > t_0$ такой, что имеет место включение

$$0 \in int \int_{t_0}^{t_1} \Phi_A(t_1, \tau)[A(\tau)M + B(\tau)]d\tau.$$

Следствие 2. Если дифференциальное включение (1) локально-относительно $\{m\}$- управляемо, то существует $t_1 > t_0$,такой,что

$$m \in intX(t_0, t_1, m, A, B).$$

Теорема 2. Если дифференциальное включение (1) локально-относительно M-управляемо, то существует $t_1 > t_0$, такой, что имеет место включение

$$0 \in int\overline{co} \bigcup_{t\in[t_0,t_1]} \Phi_A(t_1, t)[A(t)M + B(t)].$$

Следствие 3. Необходимым условием нуль-управляемости является справедливость включения

$$0 \in int\overline{co} \bigcup_{t\in[t_0,t_1]} \Phi_A(t_1, t)B(t).$$

при некотором $t_1 > t_0$.

Следствие 4. Если $A(t) \equiv 0$ и $B(t) \equiv 0$, то условие $0 \in int\overline{co}B$ является необходимым условием М-управляемости системы (1).

Теорема 3. Пусть $0 \in co[A(t)M + B(t)]$ для почти всех $t \in [t_0, t_1]$ и дифференциальное включение (1) локально-относительно М-управляема на отрезке времени $T = [t_0, t_1]$. Тогда для любого $\psi \in R^n, ||\psi|| = 1$ справедливо соотношение $\mu E > 0$, где μE – мера Лебега множества $E = \{t \in [t_0, t_1] : c(\Phi_A(t_1, t)[A(t)M + B(t)], \psi) \neq 0$.

Заключение. В работе для дифференциальных включений вида (1) получены необходимые условия локальной управляемости относительно компактного терминального множества $M \subset R^n$. Эти результаты представляют интерес для развития исследований по условиям управляемости динамических систем.

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МИНИМАКСНАЯ ЗАДАЧА УПРАВЛЕНИЯ ДЛЯ СИСТЕМЫ С ПАРАМЕТРАМИ В УСЛОВИЯХ НЕПОЛНОТЫ ИНФОРМАЦИИ

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1.Постановка задачи. Принцип оптимизации, основанное на достижения гарантированного значения критерия качества управления в динамических системах приводят к минимаксным задачам [1 – 2]. Развивая исследований [3], изучим условия оптимальности в одной минимаксной задаче для системы управления в условиях неопределенности вида

$$\dot{x} = A(t,y)x + b(t,u,w,y), t \in T = [t_0,t_1], x(t_0) \in D, \quad u \in U, w \in W, y \in Y \qquad (1)$$

где $x - n$ -вектор состояния, $u - m$ -вектор управления, $w - q$ - вектор возмущения, y - параметр модели, $A(t,y) - n \times n$ -матрица, $b(t,u,y) \in R^n$, D, U, W - выпуклые компактные из R^n, R^m, R^q соответственно; y принимает счетное множество значений $S = \{y_i, i = 1,2,...\}$ из компакта $Y \subset R^k$. Пусть: 1) элементы матрицы $A(t,y)$ суммируемы по $t \in T$ и непрерывно по $y \in Y$; 2) отображение $(t,u,w,y) \to b(t,u,w,y)$ измеримо по $t \in T$ и непрерывно по $(u,w,y) \in U \times W \times Y$ и $\|b(t,u,w,y)\| \leq \beta(t), \quad \beta(\cdot) \in L_1(T)$; для каждого $(t,u,y) \in T \times U \times Y$ множество $b(t,u,W,y) = \{b(t,u,w,y) | w \in W\}$ выпуклый компакт из R^n.

Допустимым управлением назовем измеримую m-вектор-функцию $u = u(t), \ t \in T$, принимающие значения из U. Возможной реализацией параметра возмущения будем считать измеримую q-вектор-функцию $w = w(t), \ t \in T$, принимающие значения из W. Обозначим: U_T - множество всех допустимых управлений; W_T - множество всех возможных возмущений.

Пусть задан функционал вида $J(u,w,x_0) = \inf\limits_{l \in L} \sum\limits_{y \in S} (P(y)x(t_1,x_0,u,w,y),l)\lambda(y)$, где $P(y)$, $y \in Y$ - непрерывно $s \times n$-матрица, $\lambda(y)$ - неотрицательная на Y функция, причем $\sum_{y \in S} \lambda(y)$ -конечная. Рассмотрим минимаксную задачу:

$$J(u) \equiv \max_{x_0 \in D, w \in W_T} \inf_{l \in L} \sum_{y \in Y} (P(y)x(t_1,x_0,u,w,y),l)\lambda(y) \to \min, u \in U_T \qquad (2)$$

2. Методы и результаты исследования. Пусть: $H_T(u,y)$ -множество всех абсолютно непрерывных траекторий системы (1), соответствующих начальному условию $x(t_0) = x_0 \in D$ и всем $w \in W_T$ при $u \in U_T$ и $y \in S$; $X_T(t_1,u,y) = \{\xi \in R^n | \xi = x(t_1, x(\cdot) \in H_T \in (u,y)\}$. В силу результатов работы [3] множество $X_T(u,y)$ является выпуклым компактным множеством пространства $C^n(T), aX_T(t_1,u,y)$ является выпуклым компактом из R^n. В силу теоремы о минимаксе из выпуклого анализа, задачу (2) эквивалента следующей задаче:

$$\inf_{l \in coL} \sum_{y \in S} C(P(y)X(t_1,u,y),l)\lambda(y) \to \min, u \in U_T, \qquad (3)$$

где $C(Q,l) = \sup\limits_{\xi \in Q}(\xi,l)$– опорная функция множества Q.

Пусть $\psi(t,y,l) = F_y'(t_1,t)P'(y)l, \quad F_y(t,\tau)$ – фундаментальная матрица решений уравнения $\dot{x} = A(t,y)x$. Введем функционалы:

$$\mu(u) = \inf_{l\in coL}\sum_{y\in Y}[C(D,\psi(t_0,y,l)) + \int_{t_0}^{t_1} C(b(t,u(t),W,y),\psi(t,y,l))dt]\lambda(y), u(\cdot)\in U_T, \quad (4)$$

$$\gamma(l) = \sum_{y\in S} C(D,\psi(t_0,y),l)\lambda(y) + \int_{t_0}^{t_1}\min_{u\in U}\sum_{y\in S} C(b(t,u,W,y),\psi(t,y,l))\lambda(y)dt, \quad l\in coL \quad (5)$$

Теорема. Для оптимальности управления $u^0(\cdot)$ в задаче (3) необходимо и достаточно существование точки глобального минимума $l^0 \in coL$ функции $\gamma(l), \quad l\in coL$, и выполнение для почти всех $t\in T$ условия

$$\min_{u\in U}\sum_{y\in S} C(b(t,u,W,y),\psi(t,y,l^0))\lambda(y) = \sum_{y\in S} C(b(t,u^0(t),W,y),\psi(t,y,l^0))\lambda(y) \quad (6)$$

Замечание. Используя приведенный метод, полученный результат можно обобщить для критерия качества с вогнутым терминальным функционалом.

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ИССЛЕДОВАНИЕ ХАОТИЧЕСКИХ И РЕГУЛЯРНЫХ РЕЖИМОВ ДИНАМИЧЕСКИХ СИСТЕМ С ПОМОЩЬЮ АЛГОРИТМА ТЕСТ 0-1

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Пусть у нас есть набор значений $x_k, k = 1, ..., N$, полученный, например, с помощью численного алгоритма, который представляет собой решение некоторой непрерывной динамической системы. Алгоритм Тест 0-1 реализует следующие шаги.

1. Генерируем случайное число $c \in (0, \pi)$ и переходим к новым переменным, используя формулы:

$$p(n) = \sum_{k=1}^{n} x_k \cos(ck), q(n) = \sum_{k=1}^{n} x_k \sin(ck), n = 1..N. \tag{1}$$

График, построенный по точкам (p, q) — фазовая траектория дискретного временного ряда, также может косвенно указывать на хаотическую или регулярную динамику. В частности, если фазовая траектория имеет вид траектории хаотического (броуновского) движения, то имеется хаос, а если траектория замкнута (ограничена), то существует регулярный режим.

2. Вычисляем стандартное среднеквадратическое отклонение $M(n)$

$$M(n) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} (p(k+n) - p(k))^2 + (q(k+n) - q(k))^2 \tag{2}$$

и модифицированное стандартное среднеквадратическое отклонение.

$$D(n) = M(n) - \left(\frac{1}{N}\sum_{k=1}^{N} x_k\right)^2 \frac{1-\cos(nc)}{1-\cos(c)}. \tag{3}$$

Обратим внимание, что предел в (3) обеспечивается вычислением $M(n)$ только для $n \le n_{\text{cut}}$, где $n_{\text{cut}} \ll N$. На практике мы можем выбрать $n_{\text{cut}} = N/10$. Если стандартное отклонение $M(n)$ ограничено, то мы имеем дело с регулярным режимом, а если оно возрастает, например, линейно, то с хаотическим режимом.

3. Затем вычисляем асимптотическую скорость роста стандартного среднеквадратического отклонения K_c. Для этого воспользуемся корреляционным методом. Пусть векторы $\xi = (1, 2, ..., n_{\text{cut}})$, $\Delta = (D(1), D(2), ..., D(n_{\text{cut}}))$. Тогда ковариация и вариация определяются по формулам:

$$\operatorname{cov}(x, y) = \frac{1}{q}\sum_{j=1}^{q} (x(j) - \bar{x})(y(j) - \bar{y}), \bar{x} = \frac{1}{q}\sum_{j=1}^{q} x(j),$$

$$\bar{y} = \frac{1}{q}\sum_{j=1}^{q} y(j), \operatorname{var}(x, x) = \operatorname{cov}(x, x).$$

Коэффициент корреляции между векторами ξ и Δ определяет асимптотическую скорость роста стандартного среднеквадратического отклонения:

$$K_c = \operatorname{corr}(\xi, \Delta) = \frac{\operatorname{cov}(\xi, \Delta)}{\sqrt{\operatorname{var}(\xi)\operatorname{var}(\Delta)}} \in [-1, 1]. \quad (4)$$

4. Шаги 1–3 выполняются для N_c значений c, выбранных случайно в интервале $(0, \pi)$. На практике достаточно взять $N_c = 100$. Затем мы вычисляем медиану этих N_c значений K_c по формуле (4). Для вычисления окончательного результата $K = \operatorname{median}(K_c)$.

Тест утверждает, что значение $K \approx 0$ указывает на регулярную динамику, а значение $K \approx 1$ — на хаотическую динамику. В таблице приведены сценарии динамических режимов по алгоритму Тест 0-1.

Таблица 1: Сценарии по алгоритму Тест 0-1

Режимы	Динамика (p, q)	Динамика $M(n)$	K
Регулярный	Ограниченная	Ограниченная	0
Хаотическая	Диффузионная	Линейный рост	1

Отметим, что существует проблема передискретизации в алгоритме Тест 0-1 для непрерывных динамических систем, вызванная слишком малым шагом выборки, что дает неверные результаты [1]. Обычно оптимальный шаг выборки выбирается визуально по графику. Однако можно применять и более глубокие подходы, например, метод первого локального минимума взаимной информации. В настоящей работе был применен другой подход, связанный с прореживанием исходных данных - удалением локальных экстремумов [2].

Далее в работе было проведено исследование различных нелинейных дробных динамических систем, обладающие хаотической динамикой с помощью алгоритма Тест 0-1 [3,4]. Показано, что Тест 0-1 можно эффективно применять для выявления хаотических и регулярных режимов.

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ОБ ОДНОЙ ДИНАМИЧЕСКОЙ СИСТЕМЕ ДВУПОЛОЙ ПОПУЛЯЦИИ

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Известно, что физические, химические, биологические, экономические и другие процессы представляются квадратично-стохастическими операторами (КСО) с дискретным и непрерывным временем, с помощью которых изучается эволюция рассматриваемой системы.

Проведено много исследований КСО с дискретным временем [1-3]. Так, в статье Бернштейна С.Н. [1] было введено понятие КСО дискретного времени и изучен вопрос эволюции биологических популяций. В работе Любича Ю.И. [2] исследована теория двуполой КСО. Далее, исследование вольтерровских квадратичных стохастических операторов двуполой популяции (ВКСОДП), принадлежащих к подклассу КСО [2] были развиты в [3] и представлены 16 частных случаев двуполых КСО, так называемых крайних операторов.

Отметим, что в настоящее время эта направление развивается очень интенсивно. Можно процитировать ряд монографий и статей. Например, в работах [4] изучались динамика двуполых биологических популяций и квадратичные стохастические операторы, а также теория случайных процессов, связанных с КСО с дискретным временем.

Наряду с этим, исследования КСО с непрерывным временем проводились в работах [5-6], посвященных изучению химических и биологических процессов.

В настоящей работе изучаем непрерывный аналог четвертого квадратичного стохастического оператора из [3], которое в нашем случае имеет вид

$$\begin{cases} \dot{x}_1(t) = x_2(t)y_1(t), \\ \dot{x}_2(t) = x_2(t)y_2(t) - x_2(t), \\ \dot{y}_1(t) = x_1(t)y_1(t) - y_1(t), \\ \dot{y}_2(t) = x_2(t)y_2(t) \end{cases} \tag{1}$$

или в векторном виде

$$\dot{X}(t) = F(X(t)),$$

где $X(t) = (x(t); y(t)) = (x_1(t), x_2(t); y_1(t), y_2(t))$, есть состояние некоторой системы в момент непрерывного времени при $t \geq 0, F(X(t)) = F(x_1(t), x_2(t); y_1(t), y_2(t)) = f_i(x_1(t), x_2(t); y_1(t), y_2(t)), i = \overline{1,4}$. Отметим, что в расматриваемом случае популяция аутосомна, т.е. мужские и женские типы равны ($n = v$).

Не трудно заметить, что система (1) схожа с уравнениями Лотки-Вольтерра, кроме этого имеются квадратичные стохастические процессы, связанные с системы (1) (см. [4, Глава 6]).

В настоящей статье найдены неподвижные точки исходной системы и установлены ее тип, найдены аналитические и численные решения системы и построены фазовая плоскость.

Следует отметить, что полученные аналитические и численные решения полностью соответствует теоретическим результатам работы [3]. Также, исследован качественные свойства данной системы в симплексе S^1, как частный случай системы (1).

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НОВЫЙ ПОДХОД К ИЗУЧЕНИЮ $m-$ВЫПУКЛЫХ ФУНКЦИЙ

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1. В последние годы Я и мои ученики из Ургенчского университета и Хорезмского филиала Института Математики разрабатываем новое научное направление об одном подходе исследования $m-$выпуклых $(m-c\nu)$ функций. Этот новый подход основан на связи $m-$выпуклых функций в евклидовом пространстве $\mathbb{R}^n$ $(1 \leq m \leq n)$, с $m-$субгармоническими (sh_m) функциями в комплексном пространстве $\mathbb{C}^n$.

Пусть $u(x) \in \mathbb{C}^2(D)$- дважды гладкая функция в области $D \subset \mathbb{R}^n$. Тогда матрица $\left(\frac{\partial^2 u}{\partial x_j \partial x_k}\right)$ является ортогональной, $\frac{\partial^2 u}{\partial x_j \partial x_k} = \frac{\partial^2 u}{\partial x_k \partial x_j}$ и после подходящего ортонормального преобразования она преобразуется в диагональную форму,

$$\left(\frac{\partial^2 u}{\partial x_j \partial x_k}\right) \to \begin{pmatrix} \lambda_1 & 0 & ... & 0 \\ 0 & \lambda_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \lambda_n \end{pmatrix},$$

где $\lambda_j = \lambda_j(x) \in \mathbb{R}$-собственные значения матрицы $\left(\frac{\partial^2 u}{\partial x_j \partial x_k}\right)$. Пусть $H^k(u) = H^k(\lambda) = \sum\limits_{1 \leq j_1 < ... < j_k \leq n} \lambda_{j_1} ... \lambda_{j_k}$-гессиан степени k вектора собственных значений $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$.

Определение 1. *Дважды гладкая функция $u(x) \in \mathbb{C}^2(D)$ называется $m-$выпуклой в $D \subset \mathbb{R}^n$, $u \in m - c\nu(D)$, если ее вектор собственных значений $\lambda = \lambda(x) = (\lambda_1(x), \lambda_2(x), ..., \lambda_n(x))$ удовлетворяет условиями*

$$m - c\nu \bigcap \mathbb{C}^2(D) = \left\{H^k(u) = H^k(\lambda(x)) \geq 0, \forall x \in D, k = 1, ..., n - m + 1\right\}.$$

При $m = n$ класс $n - c\nu$ совпадает с классом субгармонических функций $sh = \{\lambda_1 + \lambda_2 + ... + \lambda_n \geq 0\}$, а при $m = 1$ он совпадает с классом выпуклых функций $c\nu = \{\lambda_1 \geq 0, \lambda_2 \geq 0, ..., \lambda_n \geq 0\}$,: $c\nu = 1 - c\nu \subset 2 - c\nu \subset ... \subset n - c\nu = sh$. Теория субгармонических функций является развитой и важной частью Теория функций и Математической физики и Теория выпуклых функций хорошо изучена и отражена в работах А.Александрова, И.Бакельмана, А.Поздняк и др. (см. [2, 3, 7, 8, 15]). При $m > 1$ этот класс изучен в серии работ Н.Ивочкиной, Н.Трудингера, Х.Вонга, S.Y. Ли и др. (см.[9], [11], [18-20]). В работе Н.Трудингера, Х.Вонга [20] $m - c\nu$ функции введены в классе полунепрерывных сверху функций $u(x)$ в области $D \subset \mathbb{R}^n$, используя, так называемое, вязкое(viscosity) определение: что $H^k(q) \geq 0, k = 1, 2, ..., n - m + 1$, для любого квадратичного полинома $q(x)$: разность $u(x) - q(x)$ для которого имеет только лишь конечное число локальных максимумов в области D. Кроме того, в этой работе определен оператор $H^{n-m+1}(u)$, максимальной степени как борелевская мера и при помощи этого оператора введена емкость конденсатора $\mathbb{C}(E, D)$, доказаны ряд потенциальных свойств этой емкости.

Класс $m-$ субгармонических (sh_m) функций в комплексном пространстве $\mathbb{C}^n$, основываются на дифференциальных формах и потоках $(dd^c u)^k \wedge \beta^{n-k} \geq 0, k = 1, 2, ..., n-m+1$, где $\beta = dd^c \parallel z \parallel^2$-стандартная форма объема в $\mathbb{C}^n$. Теория sh_m- функций хорошо развита, в данное время является предметом изучения многих математиков (З. Блоцкий [6], С.Динев и С.Колодзей [10], С. Лі [12], Х.Ч.Лу [13],[14] и др). Достаточно полный обзор по этой теории имеется в статье А.Садуллаева и Б. Абдуллаева [1] в Трудах МИРАН.

2. В этой работе мы предлагаем совершенно иной подход к исследованию $m-c\nu$ функций, основанный на связях $m-c\nu$ функций с sh_m-функциями, с использованием богатых и хорошо изученных свойств sh_m- функций. Для этого вложим вещественное пространство $\mathbb{R}^n_x$ в комплексное пространство $\mathbb{C}^n_z$, $\mathbb{R}^n_x \subset \mathbb{C}^n_z = \mathbb{R}^n_x + i\mathbb{R}^n_y (z = x + iy)$, как вещественное $n-$мерное подпространство. Затем, функцию $u(x)$, заданную в области $D \subset \mathbb{R}^n_x$ поднимаем в область $\Omega = D \times i\mathbb{R}^n_y \subset \mathbb{C}^n_z$ полагая константой на параллельных плоскостях $\prod_{x^0} = \{z \in \mathbb{C}^n : x = x^0, y \in \mathbb{R}^n\}$, $u^c(z) = u^c(x + iy) = u(x)$.

Ключевым результатом работы является

Теорема 1. *Дважды гладкая функция $u(x) \in \mathbb{C}^2(D), D \subset \mathbb{R}^n_x$, является $m-c\nu$ в D тогда и только тогда когда функция $u^c(z) = u^c(x + iy) = u(x)$, которая не зависит от переменных $y \in \mathbb{R}^n_y$, является sh_m в области $D \times i\mathbb{R}^n_y$.*

Для исследования $m-$выпуклой функций $u(x)$, мы ее продолжим в комплексное пространство $\mathbb{C}^n$ как sh_m- функция $u^c(z)$, а затем применяем известные свойства $u^c(z) \in sh_m$ к $u(x)$, чтобы получить аналогичные свойства $m-$выпуклой функций. Все основные свойства $m-$выпуклой функций, полученные мною и моими учениками получены таким путем, с использованием связи $m-c\nu$ и sh_m- функций.

В результате мы существенно дополняем имеющихся прежде результатов по теории $m-c\nu$ функций и получаем ряд новых результатов. В частности мы даем полное построение Теории потенциал в классе $m-c\nu$ функций. Теорема 1 позволяет также определить нам $m-$выпуклой функции в классе полунепрерывных функций.

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НЕОБХОДИМОЕ УСЛОВИЕ СУЩЕСТВОВАНИЯ ПЕРИОДИЧЕСКИХ ВЕЩЕСТВЕННЫХ МЕР

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Напомним прежде всего определение действительной меры на полукольце.

Определение 1.([1],стр 304) Функция множества $m(A)$ называется мерой, если:
1) область определения S_m функции $m(A)$ есть полукольцо множеств,
2) значения функции $m(A)$ действительны и неотрицательны,
3) $m(A)$ аддитивна, т. е. для любого конечного разложения $A = A_1 \bigcup \ldots \bigcup A_n$ множества $A \in S_m$ на (попарно непересекающиеся) множества $A_k \in S_m$ выполнено равенство $m(A) = \sum\limits_{k=1}^{n} m(A_k)$ (т.е. аддитивно).

В множестве $E = [0; 1)$ рассмотрим систему множеств S, содержащие множества вида $[a; b)$. Нетрудно видеть что все такие множества образуют полукольцо. Если задать на этом полукольце какую-то меру m, то его можно продолжить единственном образом на минимальное кольцо, содержащее это полукольцо. Задать меру на полукольце можно несколькими способами.

На этой полукольце мы зададим понятие периодической меры.

Пусть $\xi = \{K^{(i)}, i \in I\}$ некоторое разбиение множества E т.е.

$$E = \bigcup_{i \in I} K^{(i)}, K^{(i)} \cap K^{(j)} = \emptyset \text{ при } i \neq j.$$

Определение 2. Меру m называем ξ-периодическим, если $m([a; b)) = \lambda_{ij}$ для любых $a \in K^{(i)}$ и $b \in K^{(j)}$. т.е. значение меры m на $[a; b)$ не зависит от a и b, но зависит от классов принадлежности a и b.

Заметим сразу что для любого разбиения ξ при всех $i \in I$ выполняется $\lambda_{ii} = 0$. Потому что если $a \in K^{(i)}$ можно написать $\lambda_{ii} = m([a; a)) = m(\emptyset)$=0. В силу аддитивности m, для любого $0 \leq a \leq c \leq b \leq 1$ имеем

$$m([a; b)) = m([a; c)) + m([c; b)) \tag{1}$$

то есть $\lambda_{ij} = \lambda_{ik} + \lambda_{kj}, a \in K^{(i)}, b \in K^{(j)}, c \in K^{(k)}$.

Можно привести пример таких величин из области физики. Рассмотрим потенциальную энергию материальной точки в зависимости от расположения в однородном потенциальном поле. Допустим при переходе с эквипотенциальной поверхности $K^{(i)}$ на поверхность $K^{(j)}$ изменение потенциальной энергии будет равняться λ_{ij}. При перемещении внутри любом эквипотенциальном слое $K^{(i)}$ выполняется $\lambda_{ii} = 0$. При переходе с $K^{(i)}$ на какую-то промежуточную эквипотенциальную поверхность $K^{(k)}$ и еще раз с $K^{(k)}$ на какую-то другую $K^{(j)}$ изменение потенциальной энергии будет соответственно равняться $\lambda_{ik}, \lambda_{kj}$. Для этих чисел имеет место равенство [см.6]

$$\lambda_{ij} = \lambda_{ik} + \lambda_{kj}, a \in K^{(i)}, b \in K^{(j)}, c \in K^{(k)}. \tag{2}$$

При фиксированном разбиении ξ можно найти периодическую меру на полукольце множеств S т.е.

$$S = \{[a; b) : 0 \leq a \leq b \leq 1\}.$$

В качестве ξ-разбиения множества $[0,1)$ возьмем

$$\xi_q = \left\{ \left[\frac{i-1}{q}; \frac{i}{q} \right), i = 1, \ldots, q \right\}, q \in N, q \geq 2 \text{ т.е. } E = \bigcup_{i=1}^{q} \left[\frac{i-1}{q}; \frac{i}{q} \right).$$

Таких периодических мер назовем ξ_q-периодическими.

Теорема 1. Любая ξ_q-периодическая мера имеет вид

$$m([a;b)) = \begin{cases} \sum\limits_{k=i}^{j} c_k, & \text{при } i < j, \\ 0, & \text{при } i = j. \end{cases}$$

где $a \in \left[\frac{i-1}{q}; \frac{i}{q} \right), b \in \left[\frac{j-1}{q}; \frac{j}{q} \right)$, $c_l \geq 0, l = 1, ..., q-1$ заданные числа.

Для некоторых ξ-разбиений может не существовать ξ-периодических мер. Следующая теорема выделяет класс таких ξ-разбиений.

Теорема 2. Если дано некоторое разбиение $\xi = \{K^{(i)}, i \in I\}$ множества $[0;1)$ и для некоторого $i_0 \in I$ выполняются следующие условия

1) Существуют $a_0, b_0 \in K^{(i_0)}$ такие, что $[a_0; b_0) \bigcap K^{(i)} \neq \emptyset$ для всех $i \in I$;

2) Для любого $i, j \in I$ существуют $a_i \in K^{(i)}, b_j \in K^{(j)}$ такие, что $[a_i; b_j) \bigcap K^{(i_0)} \neq \emptyset$.

Тогда не существует ненулевая ξ-периодическая мера.

В качестве применение теоремы 2 рассмотрим следующие два ξ-разбиения:

1) Пусть $[0;1) = K^{(1)} \bigcup K^{(2)}$, где $K^{(1)} = [0;1) \bigcap Q, K^{(2)} = [0;1) \setminus Q$. Так как на любом промежутке $[a;b)$ при $a < b$ существуют как иррациональные, так и рациональные числа, то выполняются оба условия теоремы 2. Отсюда получим не существование ξ-периодических мер для такого разбиения.

2) Пусть $[0;1) = K^{(1)} \bigcup K^{(2)}$ где,$K^{(1)} = \{x_1, \ldots, x_m\}$-конечное множество чисел из $[0;1), K^{(2)} = [0;1) \setminus K^{(1)}$. Не трудно видеть для таких разбиений тоже выполняется условие теоремы. Поэтому для таких разбиений тоже не существует ξ-периодических мер.

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О ДИСКРЕТНОМ СПЕКТРЕ ОДНОГО ЧАСТИЧНО ИНТЕГРАЛЬНОГО ОПЕРАТОРА С ВЫРОЖДЕННЫМ ЯДРОМ

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Пусть $\Omega_1 = [a,b]^{\nu_1}$ и $\Omega_2 = [c,d]^{\nu_2}$ $(\nu_1, \nu_2 \in \mathbb{N})$. В гильбертовом пространстве $L_2(\Omega_1 \times \Omega_2)$ рассмотрим следующий самосопряженный частично интегральный оператор (ЧИО)

$$H = H_0 - (T_1 + T_2). \tag{1}$$

Здесь действия операторов H_0, T_1 и T_2 определяются по формулам

$$H_0 f(x,y) = k_0(x,y) f(x,y), \;\; f \in L_2(\Omega_1 \times \Omega_2),$$

$$T_1 f(x,y) = \int_{\Omega_1} \gamma \left(\varphi_1(x)\overline{\varphi_1(s)} + \varphi_2(x)\overline{\varphi_2(s)} \right) f(s,y)ds, \;\; \gamma > 0$$

$$T_2 f(x,y) = \int_{\Omega_2} \mu \left(\psi_1(y)\overline{\psi_1(t)} + \psi_2(y)\overline{\psi_2(t)} \right) f(x,t)dt, \;\; \mu > 0$$

где $k_0(x,y)$ – неотрицательная непрерывная функция на $\Omega_1 \times \Omega_2$, $\varphi_1(\cdot)$ и $\varphi_2(\cdot)$, $\psi_1(\cdot)$ и $\psi_2(\cdot)$ – взаимно ортонормированные непрерывные функции на Ω_1, Ω_2, т.е.

$$(\varphi_i, \varphi_j) = \int_{\Omega_1} \varphi_i(\xi)\overline{\varphi_j(\xi)}d(\xi) = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases},$$

$$(\psi_i, \psi_j) = \int_{\Omega_2} \psi_i(\xi)\overline{\psi_j(\xi)}d(\xi) = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases}.$$

Здесь и в дальнейшем будем предполагать, что $ds = d\mu_1(s)$, $dt = d\mu_2(t)$ и $\mu_1(\Omega_1) = \mu_2(\Omega_2) = 1$, $\mu_j(\cdot)$ – мера Лебега на Ω_j, $j = 1,2$. Таким образом, оператор H (1) зависит от двух параметров γ и μ, т.е. $H = H(\gamma, \mu)$.

Через $\rho(\cdot)$, $\sigma(\cdot)$, $\sigma_{ess}(\cdot)$ и $\sigma_{disc}(\cdot)$ обозначим, соответственно, резольвентное множество, спектр, существенный спектр и дискретный спектр самосопряженных операторов [1].

Число

$$E_{\min}(V) = \inf\{\lambda : \lambda \in \sigma_{ess}(H)\}$$

будем называть *нижней гранью* (или *нижним краем*) существенного спектра оператора H.

В работе [2] изучены существенный спектр и количество собственных значений, лежащих ниже нижней грани существенного спектра в модели (1) при $\varphi_1(x) = \psi_1(x) = 0$, когда функция $k_0(x,y)$ имеет вид: $k_0(x,y) = u(x)u(y)$, где $u(x)$ – неотрицательная непрерывная функция на $\Omega = \Omega_1 = \Omega_2$ и $\int_\Omega \frac{dx}{u(x)} < \infty$. Дано точное описание существенного спектра частично интегрального оператора H и доказана конечность количества собственных значений, лежащих ниже нижнего края существенного спектра в модели (1). В [1] изучены спектральные свойства частично интегрального оператора H (1) в случае $k_0(x,y) = u(x)v(y)$. В [3] изучены существенный

спектр и собственные значения существенного спектра в модели (1), когда функция $k_0(x,y)$ имеет вид $k_0(x,y) = u(x)v(y)$. Вычислена нижняя грань существенного спектра для произвольных $\gamma > 0$ и $\mu > 0$.

Пусть $u(x)$, $v(y)$ – неотрицательные непрерывные функции на Ω_1 и Ω_2, соответственно, и $0 \in Ran(u) \cap Ran(v)$.

В этой работе мы исследуем количество отрицательных собственных значений, лежащих ниже нижнего края ЧИО H (1), когда функция $k_0(x,y)$ имеет вид $k_0(x,y) = u(x)v(y)$.

Теорема 1. Для спектра оператора $T = T_1 + T_2$ справедливы следующие соотношения: 1) $\sigma_{ess}(T) = \{0, \gamma, \mu\}$, 2) $\sigma_{disc}(T) = \{\gamma + \mu\}$.

Пусть $N(T)$ – количество собственных значений, лежащих в дискретном спектре самосопряженного оператора T с учетом кратности, $N_0(H)$ – количество собственных значений, лежащих ниже нижнего края существенного спектра $\sigma_{ess}(H)$, с учетом кратности.

Теорема 2. Для любого значения параметров $\gamma > 0$ и $\mu > 0$, количество собственных значений в модели (1), лежащих ниже нижней грани $E_{\min}(H)$ существенного спектра $\sigma_{ess}(H)$ не больше одного с учетом кратности, т.е. $N_0(H) \leq 1$.

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О ПЕРИОДИЧЕСКИХ МЕРАХ ГИББСА ДЛЯ НС-МОДЕЛИ С ЧЕТЫРЬМЯ СОСТОЯНИЯМИ В СЛУЧАЕ ГРАФА ТИПА "ПАЛКА"

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Пусть $\tau^k = (V, L)$ дерево Кэли порядка $k \geq 2$. Пусть $\Phi = \{0, 1, 2, 3\}$. Конфигурация $\sigma = \{\sigma(x), x \in V\}$ на дереве Кэли задается как функция из V в Φ. Множество всех конфигураций на V обозначается через Ω. Рассмотрим множество Φ как множество вершин некоторого графа G. Конфигурация σ называется G-допустимой конфигурацией, если $\{\sigma(x), \sigma(y)\}-$ребро графа G для любой ближайшей пары соседей x, y из V. Обозначим множество G-допустимых конфигураций через Ω^a.

Множество активности [3] для графа G есть функция $\lambda : G \to R_+$.

Для данных G и λ определим гамильтониан $G-$НС-модели следующим образом

$$H_G^\lambda(\sigma) = -\sum_{x \in V} \log \lambda_{\sigma(x)}, \text{ если } \sigma \in \Omega^a.$$

Определение меры Гиббса и понятия, связанные с теорией мер Гиббса, вводятся стандартным образом (см. например, [1]-[2]).

Определение 1. [3] Граф называется плодородным, если существует набор активности λ такой, что соответствующий гамильтониан имеет не менее двух трансляционно-инвариантных мер Гиббса (ТИМГ).

Мы рассмотрим случай $\lambda_0 = 1$, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ и изучим соответствующие этому случаю периодические меры Гиббса. Известно [3], что существуют три типа плодородных графов с четырьмя вершинами $0, 1, 2, 3$ (на множестве значений $\sigma(x)$): палка, ключ, обобщенный ключ.

Рассмотрим случай $G =$ *Палка*: $\{\{0, 1\}, \{1, 2\}, \{2, 3\}\}$.

Пусть $L(G)-$ множество ребер графа G, обозначим через $A \equiv A^G = \left(a_{ij}\right)_{i,j=0,1,2,3}$ матрицу смежности G, т.е.

$$a_{ij} \equiv a_{ij}^G = \begin{cases} 1, & \text{если } \{i, j\} \in L(G), \\ 0, & \text{если } \{i, j\} \notin L(G). \end{cases}$$

Известно [2], что каждой мере Гиббса для НС-модели на дереве Кэли можно сопоставлять совокупность величин $z = \{z_x, x \in G_k\}$, удовлетворяющих

$$\begin{aligned} z'_{0,x} &= \lambda \prod_{y \in S(x)} \frac{a_{00}z'_{0,y}+a_{01}z'_{1,y}+a_{02}z'_{2,y}+a_{03}}{a_{30}z'_{0,y}+a_{31}z'_{1,y}+a_{32}z'_{2,y}+a_{33}}, \\ z'_{1,x} &= \lambda \prod_{y \in S(x)} \frac{a_{10}z'_{0,y}+a_{11}z'_{1,y}+a_{12}z'_{2,y}+a_{13}}{a_{30}z'_{0,y}+a_{31}z'_{1,y}+a_{32}z'_{2,y}+a_{33}}, \\ z'_{2,x} &= \lambda \prod_{y \in S(x)} \frac{a_{20}z'_{0,y}+a_{21}z'_{1,y}+a_{22}z'_{2,y}+a_{23}}{a_{30}z'_{0,y}+a_{31}z'_{1,y}+a_{32}z'_{2,y}+a_{33}}, \end{aligned} \tag{1}$$

где $z'_{i,x} = \lambda z_{i,x}/z_{3,x}$, $i = 0, 1, 2$.

Известно, что τ^k можно представить как G_k- свободное произведение $k+1$ циклических групп второго порядка с образующими $a_1, .., a_{k+1}$, соответственно.

Пусть $\widehat{G}_k$ является подгруппой группы G_k.

Определение 2. Совокупность величин $z = \{z_x, x \in G_k\}$ называется $\widehat{G}_k$-периодической, если $z_{yx} = z_x$ для $\forall x \in G_k, y \in \widehat{G}_k$.

G_k-периодические совокупности называются трансляционно-инвариантными (ТИ).

Определение 3. Мера μ называется $\widehat{G}_k$-периодической, если она соответствует $\widehat{G}_k$-периодической совокупности величин z.

Справедлива следующая теорема.

Теорема 1. Пусть H_0-нормальная подгруппа конечного индекса в G_k. Тогда для плодородной НС-модели в случае $G = \textit{Палка}$ H_0-периодическая мера Гиббса является либо трансляционно-инвариантной, либо $G_k^{(2)}$-периодической.

Итак, по Теореме 1 для периодических мер Гиббса имеем только $G_k^{(2)}$-периодические меры Гиббса и для них из (1) можем получить следующую систему уравнений:

$$\begin{cases} z_0 = \lambda \left(\frac{s_1}{s_2}\right)^k, \ z_1 = \lambda \left(\frac{s_0+s_2}{s_2}\right)^k, \ z_2 = \lambda \left(\frac{s_1+1}{s_2}\right)^k, \\ s_0 = \lambda \left(\frac{z_1}{z_2}\right)^k, \ s_1 = \lambda \left(\frac{z_0+z_2}{z_2}\right)^k, \ s_2 = \lambda \left(\frac{z_1+1}{z_2}\right)^k. \end{cases} \tag{2}$$

Из работ [4] и [5] известно, что для НС-модели с четырьмя состояниями в случае плодородного графа $G = \textit{палка}$ при $k = 2, \ k = 3, \ k = 4$ и $\lambda > 0$ ТИМГ единственна, а при $k = 5$ найдены условия, при которых ТИМГ не единственна.

Анализируя систему (2), получена следующая теорема.

Теорема 2. Для НС-модели в случае $G = \textit{палка}$ справедливы следующие:

1. Пусть $k = 2, 3, 4$ и $\lambda > 0$. Тогда не существует $G_k^{(2)}$-периодической меры Гиббса, отличной от трансляционно-инвариантной.

2. Пусть $k \geq 5$. Тогда существуют $\lambda_{cr}^{(1)} = \varphi(x^{**}), \ \lambda_{cr}^{(2)} = \varphi(x^*)$ такие, что количество предельных мер Гиббса равно:

$$m(n) = \begin{cases} 0(1), & \textit{если} \ \ 0 < \lambda < \lambda_{cr}^{(1)}, \ \lambda > \lambda_{cr}^{(2)}, \\ 2(2), & \textit{если} \ \ \lambda = \lambda_{cr}^{(1)}, \\ 6(3), & \textit{если} \ \ \lambda_{cr}^{(1)} < \lambda < \lambda_{cr}^{(2)}, \\ 2(2), & \textit{если} \ \ \lambda = \lambda_{cr}^{(2)}, \end{cases}$$

где $n-$количество ТИМГ, $m-$количество $G_k^{(2)}$-периодических мер Гиббса, отличных от ТИ и $x^*, x^{**}-$точки максимума и минимума функции $\varphi(x)$, соответственно, где

$$\varphi(x) = \frac{x}{\left(\frac{x^k}{(x+1)^k} + 1\right)^k}.$$

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О МЕРАХ ГИББСА ДЛЯ ОДНОЙ ПЛОДОРОДНЫХ НС МОДЕЛЕЙ С ЖЕСТКИМИ ВЗАИМОДЕЙСТВИЯМИ

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Пусть $\Im^k = (V, L)$ дерево Кэли порядка $k \geq 2$. Пусть $\Phi = \{0, 1, 2\}$ и $\sigma \in \Omega = \Phi^V$ конфигурация. Обозначим через Ω множество всех конфигураций на V.

Рассмотрим множество Φ как множество вершин графа G. Конфигурация σ называется *G-допустимой конфигурацией* на дереве Кэли, если $\{\sigma(x), \sigma(y)\}-$ребро графа G для любой пары ближайших соседей x, y в V. Обозначим через Ω^a множество G допустимых конфигураций.

Набор активностей [1] для графа G представляет собой функцию $\lambda : G \to R_+$. Для заданных G и λ гамильтониан G-НС модели определяется как

$$H_G^\lambda(\sigma) = -\sum_{x \in V} \ln \lambda_{\sigma(x)}, \text{ если } \sigma \in \Omega^a.$$

Пусть $L(G)-$множество ребер графа G, обозначим через $A \equiv A^G = (a_{ij})_{i,j \in Z}$ матрицу смежности G. Рассмотрим граф $G =$ жезл: $\{0,1\}\{0,2\}\{1,1\}\{2,2\}$ (см. [3]).

Понятие меры Гиббса вводится стандартным образом (см.например, [1]-[3]). Известно [4], что каждой мере Гиббса для НС-модели на дереве Кэли можно сопоставлять совокупность величин $z = \{z_x, x \in G_k\}$, удовлетворяющих

$$z_{i,x} = \lambda \prod_{y \in S(x)} \frac{a_{i0} + a_{i1} z_{1,y} + a_{i2} z_{2,y}}{a_{00} + a_{01} z_{1,y} + a_{02} z_{2,y}}, \quad i = 1, 2. \tag{1}$$

В [3]-[7] изучены трансляционно-инвариантные и периодические меры Гиббса. Мы построим новые решения (1), т.е. возьмем m и r неотрицательные целые числа такие, что $0 \leq m \leq k$ и $0 \leq r \leq k$. Рассмотрим следующую матрицу

$$M = \begin{pmatrix} m & k-m \\ r & k-r \end{pmatrix}.$$

Эта матрица определяет, сколько раз значения z и t встречаются в множестве $S(x)$ для каждого $z_x \in \{z, t\}$. Более точно, граничное условие $z = \{z_x, x \in G_k\}$ со значениями $z = (z_1, z_2) \in R_+^2$ и $t = (t_1, t_2) \in R_+^2$ определяется следующими шагами:

• если на вершине x имеем $z_x = z$, то m вершин из множества $S(x)$ имеют значение z, а остальные $k - m$ вершины принимают значение t;

• если на вершине x имеем $z_x = t$, то r вершин из $S(x)$ имеют значение r, а остальные $k - r$ вершины принимают значение z.

Мера, соответствующая z_x, построенная указанным выше способом и не являющаяся трансляционно-инвариантной, называется альтернативной мерой Гиббса (АМГ) и обозначается как $\mu_{m,r}$.

Для АМГ из системы (1) получим следующую систему уравнений

$$\begin{cases} z_1 = \lambda \cdot \left(\frac{1+z_1}{z_1+z_2}\right)^m \cdot \left(\frac{1+t_1}{t_1+t_2}\right)^{k-m}, \quad t_1 = \lambda \cdot \left(\frac{1+t_1}{t_1+t_2}\right)^r \cdot \left(\frac{1+z_1}{z_1+z_2}\right)^{k-r}, \\ z_2 = \lambda \cdot \left(\frac{1+z_2}{z_1+z_2}\right)^m \cdot \left(\frac{1+t_2}{t_1+t_2}\right)^{k-m}, \quad t_2 = \lambda \cdot \left(\frac{1+t_2}{t_1+t_2}\right)^r \cdot \left(\frac{1+z_2}{z_1+z_2}\right)^{k-r}. \end{cases} \tag{2}$$

Рассмотрим инвариантное множество $I = \{(t_1, t_2, z_1, z_2) \in R^4 : z_1 = z_2, t_1 = t_2\}$.

Анализируя систему уравнений (2) на I, получим следующие утверждения.

Теорема 1. Пусть $k \geq 2$. Для HC-модели в случае $G =$ *жезл* на I для некоторых $\lambda > 0$ существует АМГ (не ТИ) тогда и только тогда, когда $m + r \leq k - 2$.

Теорема 2. Пусть $k = 4$. Тогда для НС модели в случае $G =$ *жезл* на инвариантном множестве I справедливы следующие утверждения:

1. Пусть $m = 1,\ r = 1$ и $\lambda_{cr}^{(1)} = 1$. Тогда при $\lambda \geq \lambda_{cr}^{(1)}$ АМГ совпадает с единственной ТИМГ μ_0 и при $0 < \lambda < \lambda_{cr}^{(1)}$ существуют ровно три меры Гиббса μ_0, μ_1 и μ_2, где μ_1, μ_2 являются АМГ (не ТИ).

2. Пусть $m = 2$, $r = 0$ или $m = 0$, $r = 2$ и $\lambda_{cr}^{(2)} = \frac{27}{16}$. Тогда при $\lambda > \lambda_{cr}^{(2)}$ АМГ совпадает с единственной ТИМГ μ_0', при $\lambda = \lambda_{cr}^{(2)}$ существуют две меры Гиббса μ_0', $\mu^{'}$, где $\mu^{'}-$АМГ(не ТИ), а при $\lambda < \lambda_{cr}^{(2)}$ существуют ровно три меры Гиббса μ_0', $\mu_1^{'}$, $\mu_2^{'}$, где $\mu_1^{'}$ и $\mu_2^{'}$ являются АМГ (не ТИ).

Замечание. Случаи $k = 3$, $m + r \leq 1$ и $k = 4$, $m = 1,\ r = 0$ изложены, соответственно, в работах [8] и [9].

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О СВЯЗАННЫХ СОСТОЯНИЯХ СИСТЕМЫ ТРЕХ ЧАСТИЦ НА РЕШЇТКЕ

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В работе [1] рассматривается система трех частиц (две из них - бозоны, а третья -произвольная), взаимодействующих с помощью парных контактных потенциалов притяжения. Описан существенный спектр ассоциированного этой системе трехчастичного оператора $H_{\mu,\lambda,\gamma}(\mathbf{K})$. Доказано существование эффекта Ефимова для $H_{\mu,\lambda,\gamma}(\mathbf{0})$ в случаях, когда либо две, либо три двухчастичные подсистемы трех частиц имеют вертуальные уровни на левом крае трехчастичного существенного спектра (т.е. когда $\mu_1 = \mu_1^0$ и $\mu_2 \in [0, \mu_2^0)$ или $\mu_\alpha = \mu_\alpha^0, \alpha = 1, 3$), а также показана конечность числа связанных состояний при малых значениях параметра $\mathbf{K} \neq \mathbf{0}$, для любого фиксированного $\gamma > 0$.

Одна из кдвухчастичных ветвейњ $[\tau_{\min,\gamma}(\mu, \boldsymbol{\pi}), \tau_{\max,\gamma}(\mu, \boldsymbol{\pi})]$ существенного спектра оператора $H_{\mu,\lambda,\gamma}(\boldsymbol{\pi})$ сдвигается к $-\infty$ с порядком μ при $\mu \to +\infty$, в результате которого собственные значения оператора "поглощаются"существенным спектром. Поэтому возникает естественный вопрос: существуют ли собственные значения оператора $H_{\mu,\lambda,\gamma}(\boldsymbol{\pi})$, лежащие левее существенного спектра при достаточно больших μ и фиксированном $\lambda > 0$, и если существуют, то сколько?

В данной работе доказывается, что существует критическое значение отношений масс $\gamma_1 \approx 4,7655$ такое, что при любом $\lambda > 0$ и $\gamma \in (0, \gamma_1)$ оператор $H_{\mu,\lambda,\gamma}(\boldsymbol{\pi})$ имеет по крайней мере одно собственное значение, а при $\gamma > \gamma_1$ имеет не менее четыре собственных значения, лежащих левее существенного спектра для достаточно больщих значений $\mu > 0$.

Пусть $\mathbb{T}^3$ – трехмерный тор, $L_2[(\mathbb{T}^3)^d]$, $d = 1, 2$ – гильбертово пространство квадратично-интегрируемых функций, определенных на $(\mathbb{T}^3)^d$ и $L_2^{sym}[(\mathbb{T}^3)^2] \subset L_2[(\mathbb{T}^3)^2]$ – подпространство симметричных функций относительно перестановки переменных.

Трехчастичный дискретный оператор Шредингера $H_{\mu,\lambda,\gamma}(\mathbf{K}), \mathbf{K} \in \mathbb{T}^3$, ассоциированный с системой трех частиц (две из них - бозоны с массой 1 и одна - произвольная с массой $m = 1/\gamma < 1$), взаимодействующих с помощью парных контактных потенциалов $\mu > 0$ и $\lambda > 0$ на трехмерной решетке $\mathbb{Z}^3$, действует в гильбертовом пространстве $L_2^{sym}[(\mathbb{T}^3)^2]$ по формуле (см. [1])

$$H_{\mu,\lambda,\gamma}(\mathbf{K}) = H_{0,\gamma}(\mathbf{K}) - \mu(V_1 + V_2) - \lambda V_3,$$

где

$$(H_{0,\gamma}(\mathbf{K})f)(\mathbf{p}, \mathbf{q}) = E_{\mathbf{K},\gamma}(\mathbf{p}, \mathbf{q})f(\mathbf{p}, \mathbf{q}),$$

$$E_{\mathbf{K},\gamma}(\mathbf{p}, \mathbf{q}) = \varepsilon(\mathbf{p}) + \varepsilon(\mathbf{q}) + \gamma\varepsilon(\mathbf{K} - \mathbf{p} - \mathbf{q}),$$

$$\varepsilon(\mathbf{p}) = 3 - \xi(\mathbf{p}), \qquad \xi(\mathbf{p}) = \sum_{i=1}^{3} \cos p_i, \qquad \mathbf{p} = (p_1, p_2, p_3) \in \mathbb{T}^3,$$

$$(V_1 f)(\mathbf{p}, \mathbf{q}) = \int\limits_{\mathbb{T}^3} f(\mathbf{p}, \mathbf{s}) d\mathbf{s}, \qquad (V_2 f)(\mathbf{p}, \mathbf{q}) = \int\limits_{\mathbb{T}^3} f(\mathbf{s}, \mathbf{q}) d\mathbf{s},$$

$$(V_3 f)(\mathbf{p}, \mathbf{q}) = \int\limits_{\mathbb{T}^3} f(\mathbf{p} + \mathbf{q} - \mathbf{s}, \mathbf{s}) d\mathbf{s}.$$

Пусть

$$\gamma_1 = \frac{1}{W_0} \quad \text{и} \quad W_0 = \int\limits_{\mathbb{T}^3} \frac{\sin^2 s_1 d\mathbf{s}}{\varepsilon(\mathbf{s})} \approx 0,2098.$$

Интеграл W_0 называется интегралом типа Ватсона (см.[3]).

Теорема. Пусть $\lambda > 0$. *i*) Если $\gamma \in (0, \gamma_1)$, то существует $\mu(\gamma, \lambda) > 0$ такое, что для любого $\mu > \mu(\gamma, \lambda)$ оператор $H_{\mu,\lambda,\gamma}(\boldsymbol{\pi})$ имеет по крайней мере одно собственное значение, лежащее левее существенного спектра;
ii) если $\gamma \in (\gamma_1, +\infty)$, то существует $\mu(\gamma, \lambda) > 0$ такое, что при всех $\mu > \mu(\gamma, \lambda)$ оператор $H_{\mu,\lambda,\gamma}(\boldsymbol{\pi})$ имеет не менее четире собственных значения, лежащих левее существенного спектра.

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О ЧИСЛЕ СОБСТВЕННЫХ ЗНАЧЕНИЙ ОПЕРАТОРА ШРЕДИНГЕРА, СООТВСТСТВУЮЩЕГО СИСТЕМЕ ТРЕХ ЧАСТИЦ НА РЕШЕТКЕ

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В данной работе рассматривается трехчастичный дискретный оператор Шредингера $H_{\mu,\gamma}(\mathbf{K})$, $\mathbf{K} \in \mathbb{T}^3$, ассоциированный с гамильтонианом системой трех частиц (две - фермионы с массой 1 и одна - произвольная с массой $m = 1/\gamma < 1$), взаимодействующих с помощью парных отталкивающих контактных потенциалов $\mu > 0$ на трехмерной решетке $\mathbb{Z}^3$.

Пусть $\mathbb{T}^3$ – трехмерный тор, $L_2^{as}[(\mathbb{T}^3)^2]$ – подпространство антисимметричных функций на $(\mathbb{T}^3)^2$.

Соответствующий трехчастичный дискретный оператор Шрѣдингера $H_{\mu,\gamma}(\mathbf{K})$ действует в $L_2^{as}[(\mathbb{T}^3)^2]$ по формуле

$$H_{\mu,\gamma}(\mathbf{K}) = H_{0,\gamma}(\mathbf{K}) + \mu(V_1 + V_2),$$

где

$$(H_{0,\gamma}(\mathbf{K})f)(\mathbf{p},\mathbf{q}) = E_{\mathbf{K},\gamma}(\mathbf{p},\mathbf{q})f(\mathbf{p},\mathbf{q}), \quad E_{\mathbf{K},\gamma}(\mathbf{p},\mathbf{q}) = \varepsilon(\mathbf{p}) + \varepsilon(\mathbf{q}) + \gamma\varepsilon(\mathbf{K}-\mathbf{p}-\mathbf{q}).$$

$$\varepsilon(\mathbf{p}) = 3 - \xi(\mathbf{p}), \qquad \xi(\mathbf{p}) = \sum_{i=1}^{3} \cos p_i, \qquad \mathbf{p} = (p_1, p_2, p_3) \in \mathbb{T}^3,$$

$$(V_1 f)(\mathbf{p},\mathbf{q}) = \int\limits_{\mathbb{T}^3} f(\mathbf{p},\mathbf{s})d\mathbf{s}, \qquad (V_2 f)(\mathbf{p},\mathbf{q}) = \int\limits_{\mathbb{T}^3} f(\mathbf{s},\mathbf{q})d\mathbf{s}.$$

Вводим следующие обозначения

$$W_{11} = \int\limits_{\mathbb{T}^3} \frac{\cos^2 s_1 d\mathbf{s}}{\varepsilon(\mathbf{s})}, \qquad W_{12} = \int\limits_{\mathbb{T}^3} \frac{\cos s_1 \cos s_2 d\mathbf{s}}{\varepsilon(\mathbf{s})}.$$

Интегралы W_{11} и W_{12} – интегралами типа Ватсона (см. [2]).

Пусть

$$\gamma_1 = \frac{1}{W_{11} + 2W_{12}} \approx 1,9336, \qquad \gamma_2 = \frac{1}{W_{11} - W_{12}} \approx 5,3985.$$

Теорема. а) Пусть $\gamma \in (0, \gamma_1)$. Тогда существует $\mu_1(\gamma) > 0$ такое, что для любого $\mu > \mu_1(\gamma)$ оператор $H_{\mu,\gamma}(\mathbf{0})$ не имеет собственных значений, лежащих правее существенного спектра.

в) Пусть $\gamma \in (\gamma_1, \gamma_2)$. Тогда существует $\mu_2(\gamma) > 0$ такое, что для любого $\mu > \mu_2(\gamma)$ оператор $H_{\mu,\gamma}(\mathbf{0})$ имеет единственное собственное значение, лежащее правее существенного спектра.

с) Пусть $\gamma \in (\gamma_2, +\infty)$. Тогда существует $\mu_3(\gamma) > 0$ такое,что для любого $\mu > \mu_3(\gamma)$ оператор $H_{\mu,\gamma}(\mathbf{0})$ имеет ровно три собственных значений, лежащих правее существенного спектра.

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НОВЫЕ НЕУПОРЯДОЧЕННЫЕ ТРАНСЛЯЦИОННО-ИНВАРИАНТНЫЕ МЕРЫ ГИББСА В МОЛЕКУЛАХ ДНК ДЛЯ МОДЕЛИ ПОТТСА

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В последние годы термодинамика изучалась для некоторых моделей ДНК в статистической физике [1]. Например, в работах [2]-[6] изучались соединения Холлидея между молекулами ДНК для моделей Изинга, Поттса и Блюма-Капеля на дереве Кэли. В частности, в работе [3] была представлена модель Поттса молекулы ДНК для изучения ее термодинамики. Исследована трансляционно-инвариантная мера Гиббса (ТИМГ) множества молекул ДНК на дереве Кэли. В данной работе для этой модели были найдены новые неупорядоченные ТИМГ.

Как известно, каждая молекула ДНК представляет собой двойную спираль, образованную двумя комплиментарными нитями нуклеотидов, которые удерживаются вместе водородными связями между парами оснований $C-G$ (цитозин-гуанин) и $A-T$ (аденин-тимин). Молекулы ДНК хранят генетическую информацию и содержат инструкции для всех белков, которые организм когда-либо синтезирует [7].

Мы рассматриваем множество молекул ДНК как конфигурацию модели Поттса на дереве Кэли. В настоящей работе рассматривается множество молекул ДНК, которые существуют на графе типа дерево.

Напомним некоторые определения. Дерево Кэли Γ^k порядка $k \geq 1$ есть бесконечное дерево, т.е. граф без циклов, из каждой вершины которого исходят ровно $k+1$ ребер. Пусть $\Gamma^k = (V, L, i)$, где V и $L-$ множества вершин и ребер дерева Кэли, а $i-$ функция инцидентности, сопоставляющая каждому ребру $l \in L$ его конечные точки $x, y \in V$. Если $i(l) = \{x, y\}$, то вершины x и y называются ближайшими соседями; это обозначается как $l = \langle x, y\rangle$.

Конфигурация $\sigma = \{\sigma(l), l \in L\}$ на ребрах дерева Кэли задается функцией из L в $\{-2, -1, 0, 1, 2\}$.

Мы рассматриваем следующую модель Поттса энергии конфигурации σ на множестве молекул ДНК:

$$H(\sigma) = J \sum_{\langle l,t\rangle \in L\times L;} \delta_{\sigma(l)\sigma(t)}, \tag{1}$$

где $J > 0-$константа связи, $\delta-$символ Кронекера, $\sigma(l) \in \{-2, -1, 0, 1, 2\}$ и $\langle l, t\rangle$ обозначает ребра, являющиеся ближайшими соседями, т.е. ребра, у которых имеется общая вершина.

Известно (см. [3]), что задача исследования ТИМГ в модели Поттса сводится к решению следующей системы нелинейных уравнений:

$$
\begin{aligned}
u_{-1} &= \left(\frac{1+\alpha v_{-1}+v_1+v_2}{\alpha+v_{-1}+v_1+v_2}\right)^2 \left(\frac{1+\alpha u_{-1}+u_0+u_1+u_2}{\alpha+u_{-1}+u_0+u_1+u_2}\right)^{k-2},\\
u_0 &= \left(\frac{1+v_{-1}+v_1+v_2}{\alpha+v_{-1}+v_1+v_2}\right)^2 \left(\frac{1+u_{-1}+\alpha u_0+u_1+u_2}{\alpha+u_{-1}+u_0+u_1+u_2}\right)^{k-2},\\
u_1 &= \left(\frac{1+v_{-1}+\alpha v_1+v_2}{\alpha+v_{-1}+v_1+v_2}\right)^2 \left(\frac{1+u_{-1}+u_0+\alpha u_1+u_2}{\alpha+u_{-1}+u_0+u_1+u_2}\right)^{k-2},\\
u_2 &= \left(\frac{1+v_{-1}+v_1+\alpha v_2}{\alpha+v_{-1}+v_1+v_2}\right)^2 \left(\frac{1+u_{-1}+u_0+u_1+\alpha u_2}{\alpha+u_{-1}+u_0+u_1+u_2}\right)^{k-2},\\
v_{-1} &= \frac{1+\alpha v_{-1}+v_1+v_2}{\alpha+v_{-1}+v_1+v_2} \left(\frac{1+\alpha u_{-1}+u_0+u_1+u_2}{\alpha+u_{-1}+u_0+u_1+u_2}\right)^{k-1},\\
v_1 &= \frac{1+v_{-1}+\alpha v_1+v_2}{\alpha+v_{-1}+v_1+v_2} \left(\frac{1+u_{-1}+u_0+\alpha u_1+u_2}{\alpha+u_{-1}+u_0+u_1+u_2}\right)^{k-1},\\
v_2 &= \frac{1+v_{-1}+v_1+\alpha v_2}{\alpha+v_{-1}+v_1+v_2} \left(\frac{1+u_{-1}+u_0+u_1+\alpha u_2}{\alpha+u_{-1}+u_0+u_1+u_2}\right)^{k-1}
\end{aligned}
\tag{2}
$$

где $u_i, v_j > 0$, $\alpha = exp(J/T)$, T-температура.

Теорема. Для модели (1) ДНК на дереве Кэли порядка $k \geq 2$ верны следующие утверждения:

A) Пусть $k = 2, 3, 4$. Тогда для любого $\alpha > 1$ существует единственная НТИМГ (неупорядоченная трансляционно-инвариантная мера Гиббса) μ_1;

B) Пусть $k > 4$.

1) если $\alpha \leq \alpha^*$, то существует единственная НТИМГ μ_1;

2) если $\alpha > \alpha^*$ и либо $(\frac{\alpha+3}{4})^2 = \rho_1(\alpha, k)$ или либо $(\frac{\alpha+3}{4})^2 = \rho_2(\alpha, k)$, то существуют две НТИМГ μ_1, μ_2;

3) если $\alpha > \alpha^*$ и $\rho_1(\alpha, k) < (\frac{\alpha+3}{4})^2 < \rho_2(\alpha, k)$, то существуют три НТИМГ μ_1, μ_2, μ_3, где

$$\alpha^* = \frac{\sqrt{9(k-3)^2 + 16(k-1)^2} - 3(k-3)}{2(k-3)}.$$

Замечание. Найденный НТИМГ является совершенно новым.

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СИСТЕМА ФУНКЦИОНАЛЬНЫХ УРАВНЕНИЙ РЕНАТУРАЦИИ ДНК-РНК ДЛЯ МОДЕЛИ БЛЮМА-КАПЕЛЯ С ВНЕШНИМ ПОЛЕМ

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Термодинамика ДНК для моделей статистической физики изучается в статьях [1]-[4]. В работах [5], [6] термодинамика ренатурации ДНК-РНК изучалась для некоторых моделей. В данной работе рассмотрена модель Блюма-Капеля и исследована термодинамика ренатурации ДНК-РНК для этой модели.

Известно, что в ДНК каждая пара $A+T$ соединена двумя водородными связями, а каждая пара $C+G$ - тремя водородными связями, где $A =$ аденин и $T =$ тимин, $C =$ цитозин, $G =$ гуанин. Поэтому мы моделируем их как $-1 = A + T, 1 = C + G$. Для разорванной водородной связи сопоставляется значение спина 0. Пары оснований $A + T$ (в ДНК) и $A + U$ (в РНК и $U =$ урацил) считаются идентичными в процессе ренатурации ДНК из РНК (вируса). Эти молекулы содержатся в клетках всех живых организмов, а также в некоторых вирусах (см.[5]).

Рассмотрим конфигурационное пространство Ω :

$$\Omega = \left\{\sigma = (d, r) \in \{0, -1, 1\}^{\mathbb{Z}} \times \{0, -1, 1\}^{\mathbb{Z}} : d_i r_i = 0, \forall i \in \mathbb{Z}\right\},$$

где

$$d = \{d_i \in \{0, -1, 1\} : i \in \mathbb{Z}\}, r = \{r_i \in \{0, -1, 1\} : i \in \mathbb{Z}\}$$

и $\mathbb{Z}-$ множество целых чисел.

Для каждой конфигурации $\sigma \in \Omega$ определим ее энергию (гамильтониан модели Блюма-Капеля) следующим образом:

$$H(\sigma) = H(d, r) = -J \sum_{i=-\infty}^{+\infty} ((d_i - d_{i+1})^2 + (r_i - r_{i+1})^2) - \alpha \sum_{i=-\infty}^{+\infty} (d_i + r_i), \qquad (1)$$

где $J \in R$ - константа связи между парами оснований, $\alpha \in R-$ внешнее поле.

Через σ_n обозначим ограничение конфигураций $\sigma \in \Omega$ на $\mathbb{Z}_n = \{-n, -n+1, ..., n-1, n\}$, а через Ω_n обозначим множество всех таких конфигураций. Пусть $A \subset \mathbb{Z}$. Обозначим через Ω_A пространство конфигураций, определенных на множестве A.

Рассмотрим вероятностное распределение μ_n на Ω_n :

$$\mu_n(\sigma_n) = Z_n^{-1} exp\left\{-\beta H_n(\sigma_n) + \sum_{m \in \{-n, n\}} h_{m, d_m, r_m}\right\}, \qquad (2)$$

где $\beta = 1/T$, $T > 0-$ температура, $Z_n^{-1}-$ нормирующий множитель, совокупность

$$h_{m,i,j} \in \mathbb{R}, i, j = -1, 0, 1, m = -n, n \qquad (3)$$

представляет собой набор действительных чисел и

$$H_n(\sigma_n) = -J \sum_{i=-n}^{+n} ((d_i - d_{i+1})^2 + (r_i - r_{i+1})^2) - \alpha \sum_{i=-n}^{+n} (d_i + r_i).$$

Говорят, что вероятностное распределение μ_n согласованно, если

$$\sum_{\omega_n \in \Omega_{\{-n,n\}}} \mu_n(\sigma_{n-1} \vee \omega_n) = \mu_{n-1}(\sigma_{n-1}) \tag{4}$$

для всех $n \geq 1$ и $\sigma_{n-1} \in \Omega_{n-1}$.

Здесь $\sigma_{n-1} \vee \omega_n$ есть объединение конфигураций. В этом случае существует единственная мера μ на Ω такая, что

$$\mu(\{\sigma \mid_{\mathbb{Z}_n} = \sigma_n\}) = \mu_n(\sigma_n)$$

для всех $n \geq 1$ и $\sigma_n \in \Omega_n$. Такая мера называется *мерой Гиббса*, соответствующей гамильтониану (1) и значениям (3).

Для ясности предположим, что

$$h_{-n,i,j} = h_{n,i,j}, i, j = 0, -1, 1. \tag{5}$$

Следующая теорема дает необходимые и достаточные условия на $h_{n,i,j}$, при которых выполняется (4).

Теорема. *Вероятностное распределение $\mu_n(\sigma_n), n = 1, 2, ...,$ в (2) согласованно тогда и только тогда, когда для любого $n \geq 1$ выполняются следующие функциональные уравнения:*

$$\begin{aligned}
x_{n-1} &= \tfrac{\theta+\eta^{-1}(x_n+\theta^2 y_n)+\theta^2\eta(\theta^2 u_n+v_n)}{1+\theta\eta^{-1}(x_n+y_n)+\theta\eta(u_n+v_n)} \\
y_{n-1} &= \tfrac{\theta+\eta^{-1}(\theta^2 x_n+y_n)+\theta^2\eta(u_n+\theta^2 v_n)}{1+\theta\eta^{-1}(x_n+y_n)+\theta\eta(u_n+v_n)} \\
u_{n-1} &= \tfrac{\theta+\theta^2\eta^{-1}(\theta^2 x_n+y_n)+\eta(u_n+\theta^2 v_n)}{1+\theta\eta^{-1}(x_n+y_n)+\theta\eta(u_n+v_n)} \\
v_{n-1} &= \tfrac{\theta+\theta^2\eta^{-1}(x_n+\theta^2 y_n)+\eta(\theta^2 u_n+v_n)}{1+\theta\eta^{-1}(x_n+y_n)+\theta\eta(u_n+v_n)}.
\end{aligned} \tag{6}$$

Где

$$\begin{aligned}
&\theta = exp(J\beta), \eta = exp(\alpha\beta), \\
&x_n = exp(h_{n,0,-1} - h_{n,0,0}), y_n = exp(h_{n,-1,0} - h_{n,0,0}), \\
&u_n = exp(h_{n,0,1} - h_{n,0,0}), v_n = exp(h_{n,1,0} - h_{n,0,0}).
\end{aligned} \tag{7}$$

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2-ПЕРИОДИЧЕСКИЕ ГРАДИЕНТНЫЕ МЕРЫ ГИББСА ДЛЯ МОДЕЛИ SOS С АЛЬТЕРНАТИВНЫМ МАГНЕТИЗМОМ

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Работа посвящена градиентные меры Гиббса (ГМГ) модели SOS с альтернативными значениями магнетизма и спина из множества $\mathbb{Z}$ всех целых чисел на деревьях Кэли.

Пусть $\Gamma^k = (V, L)$ - однородное дерево Кэли, каждая вершина которого имеет $k+1$ соседей, где V - множество вершин, а L - множество ребер (см.[2]).

Для $\sigma : x \in V \mapsto \sigma(x) \in \mathbb{Z}$ рассмотрим гамильтониан модели SOS с альтернативным магнетизмом, т.е.

$$H(\sigma) = -J \sum_{\langle x,y\rangle, x,y\in V;} \alpha\left(\mid \sigma(x) - \sigma(y) \mid\right) \mid \sigma(x) - \sigma(y) \mid, \tag{1}$$

где $J \in \mathbb{R}$ и

$$\alpha(\mid m \mid) = \begin{cases} 1, & \text{если } \ m \in 2\mathbb{Z}, \\ -1, & \text{если } \ m \in 2\mathbb{Z}+1. \end{cases}$$

Мы рассматриваем множество $\mathbb{Z}$ как множество вершин графа G. Мы используем граф G для определения G-допустимой конфигурации следующим образом. Конфигурация σ называется *G-допустимой конфигурацией* на дереве Кэли, если $\{\sigma(x), \sigma(y)\}$ является ребром графа G для любой пары ближайших соседей x, y в V. Через Ω^G обозначим множество G-допустимых конфигураций.

Пусть $L(G)$ - множество ребер графа G. Обозначим через $A \equiv A^G = \left(a_{ij}\right)_{i,j\in\mathbb{Z}}$ матрицу смежности графа G, т.е.

$$a_{ij} = a_{ij}^G = \begin{cases} 1, & \text{если } \ \{i,j\} \in L(G), \\ 0, & \text{если } \ \{i,j\} \notin L(G). \end{cases}$$

Пусть $\mathbf{B}$ есть σ-алгебра, порожденная цилиндрическими подмножествами Ω^G. Понятие меры Гиббса, градиентных мер Гиббса и q-периодических градиентных мер Гиббса для модели SOS с альтернативным магнетизмом определяется стандартным образом (см. [1],[3],[4],[5]). Известно, что (см. [1]) каждой (градиентной) мере Гиббса на $(\Omega^G, \mathbf{B})$ соответствует совокупность векторов $z_x = \{z_{i,x}, i \in \mathbb{Z}\} \in \mathbb{R}_+^\infty$ такая, что $\forall x \in V$ имеют место следующие равенства:

$$\begin{aligned} z_{2i,x} &= \prod_{y\in S(x)} \frac{a_{2i,0}\theta^{2|i|} + \sum\limits_{j\in\mathbb{Z}_0} a_{2i,2j}\theta^{2|i-j|} z_{2j,y} + \sum\limits_{j\in\mathbb{Z}} a_{2i,2j+1}\theta^{-|2(i-j)-1|} z_{2j+1,y}}{a_{0,0} + \sum\limits_{j\in\mathbb{Z}_0} a_{0,2j}\theta^{2|j|} z_{2j,y} + \sum\limits_{j\in\mathbb{Z}} a_{0,2j+1}\theta^{-|2j+1|} z_{2j+1,y}}, \\ z_{2i+1,x} &= \prod_{y\in S(x)} \frac{a_{2i+1,0}\theta^{-|2i+1|} + \sum\limits_{j\in\mathbb{Z}_0} a_{2i+1,2j}\theta^{-|2(i-j)+1|} z_{2j,y} + \sum\limits_{j\in\mathbb{Z}} a_{2i+1,2j+1}\theta^{2|i-j|} z_{2j+1,y}}{a_{0,0} + \sum\limits_{j\in\mathbb{Z}_0} a_{0,2j}\theta^{2|j|} z_{2j,y} + \sum\limits_{j\in\mathbb{Z}} a_{0,2j+1}\theta^{-|2j+1|} z_{2j+1,y}}, \end{aligned} \tag{2}$$

где $i \in \mathbb{Z}_0 = \mathbb{Z} \setminus \{0\}$, $\theta = \exp(J\beta)$.

В общем, решить систему (2) очень сложно. Сложность зависит от структуры графа G. В этой работе мы выбираем граф G, определяемый равенством

$$a_{ij} = \begin{cases} 1, & \text{если} \quad i = j \quad \text{или} \quad \mid i - j \mid = 1, \quad i, j \in \mathbb{Z}, \\ 0, & \text{в остальных случаях.} \end{cases}$$

Для этого конкретного графа из (2) мы получаем

$$z_{i,x} = \prod_{y \in S(x)} \frac{\theta z_{i,y} + z_{i-1,y} + z_{i+1,y}}{\theta + z_{-1,y} + z_{1,y}}, \quad i \in \mathbb{Z}_0. \tag{3}$$

Мы найдем некоторые постоянные решения (3), т. е. $z_x = z \in \mathbb{R}_+^\infty$ не зависит от вершин дерева Кэли. В этом случае из (3) получаем

$$z_i = \left(\frac{\theta z_i + z_{i-1} + z_{i+1}}{\theta + z_{-1} + z_1} \right)^k, \quad i \in \mathbb{Z}_0, \tag{4}$$

здесь $\theta > 0,\ z_i > 0,\ z_0 = 1$.

Из определения 2-периодических решений (см. [5]) для (4) полагаем:

$$z_j = \begin{cases} 1, & \text{если} \quad j \quad \text{четно}, \\ y, & \text{если} \quad j \quad \text{не четно}. \end{cases}$$

Тогда мы имеем

$$y = \left(\frac{\theta y + 2}{\theta + 2y} \right)^k. \tag{5}$$

Обозначая $a = \sqrt[k]{y}$ из (5), получаем

$$2a^{k+1} - \theta a^k + \theta a - 2 = 0. \tag{6}$$

Уравнение (6) имеет решение $a = 1$ независимо от параметров (θ, k). Разделив обе части (6) на $a - 1$, получим

$$2a^k + (2 - \theta)(a^{k-1} + a^{k-2} + \cdots + a) + 2 = 0. \tag{7}$$

Следующая теорема дает количество решений уравнения (7).

Теорема 1. Для каждого $k \geq 2$ существует точное критическое значение θ, т.е. $\theta_{\mathrm{c}} = \theta_{\mathrm{c}}(k) := \frac{2(k+1)}{k-1}$, такой что

1. если $\theta < \theta_{\mathrm{c}}$, то (7) не имеет положительного решения;
2. если $\theta = \theta_{\mathrm{c}}$, то уравнение имеет единственное решение $a = 1$;
3. если $\theta > \theta_{\mathrm{c}}$, то он имеет ровно два решения (оба отличны от 1), обозначаемые как a_1, a_2;

Таким образом, соответствующие решения (5) равны

$$\begin{array}{ll} 1) & 1 \text{ для } \ \theta \le \theta_c, \\ 2) & 1,\ a_1^k,\ a_2^k \ \text{ для } \ \theta > \theta_c. \end{array} \tag{8}$$

Таким образом, решения (8) позволяют сформулировать следующий результат.

Теорема 2. Пусть $k \ge 2$, $\theta_c = \frac{2(k+1)}{k-1}$. Тогда для гамильтониана (1) в пространстве G-допустимых конфигураций с графом G, число $\nu_2(k,\theta)$ 2-периодический по высоте ГМГ определяется по следующей формуле

$$\nu_2(k,\theta) = \begin{cases} 1, & \text{если } \ \theta \le \theta_c, \\ 3, & \text{если } \ \theta > \theta_c. \end{cases}$$

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О ТРАНСЛЯЦИОННО-ИНВАРИАНТНЫХ ОСНОВНЫХ СОСТОЯНИЯХ МОДЕЛИ SOS С КОНКУРИРУЮЩИМИ ВЗАИМОДЕЙСТВИЯМИ БЛЮМА-КАПЕЛЯ НА ДЕРЕВЕ КЭЛИ

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Пусть $\tau^k = (V, L), k \geq 1$ есть дерево Кэли порядка k, т.е. бесконечное дерево, из каждой вершины, которого выходит ровно $k+1$ ребер, где $V-$ множество вершин, $L-$ множество ребер τ^k (см. [1]).

Пусть G_k- свободное произведение $k+1$ циклических групп второго порядка с образующими $a_1, a_2, ...a_{k+1}$, соответственно, т.е. $a_i^2 = e$. Существует взаимно-однозначное соответствие между множеством вершин V дерева Кэли порядка k и группой G_k.

Мы рассматриваем модель, где спин принимает значения из множества $\Phi = \{-1, 0, 1\}$. Конфигурация σ на V определяется как функция $x \in V \to \sigma(x) \in \Phi$; множество всех конфигураций совпадает с $\Omega = \Phi^V$.

Пусть $G_k/G_k^* = \{H_1, ..., H_r\}-$ фактор группа, где G_k^*- нормальный делитель индекса $r \geq 1$.

Через $S_1(x)$ обозначим множество всех ближайших соседей точки $x \in G_k$, т.е. $S_1(x) = \{y \in G_k : \langle x, y\rangle\}$. Здесь $\langle x, y\rangle$ означает, что x и y ближайшие соседи.

Определение 1. Конфигурация σ называется G_k^*-периодической, если $\sigma(x) = \sigma_i$ при $x_i \in H_i, \forall x \in G_k$. G_k-периодическая конфигурация называется трансляционно-инвариантной.

Гамильтониан модели SOS с конкурирующими взаимодействиями Блюма-Капеля имеет вид

$$H(\sigma) = -J_1 \sum_{\langle x,y\rangle \in L} |\sigma(x) - \sigma(y)| - J_2 \sum_{d(x,y)=2} (\sigma(x) - \sigma(y))^2, \tag{1}$$

где $J = (J_1, J_2) \in \mathbb{R}^2$.

Пусть $M-$ множество единичных шаров с вершинами на V. Мы назовем сужение конфигурации σ на шаре $b \in M$ ограниченной конфигурацией σ_b. Определим энергию конфигурации σ_b на b следующим образом:

$$U(\sigma_b) = -\frac{1}{2} J_1 \sum_{\langle x,y\rangle : x,y \in b} |\sigma(x) - \sigma(y)| - J_2 \sum_{d(x,y)=2} (\sigma(x) - \sigma(y))^2, \tag{2}$$

где $J = (J_1, J_2) \in \mathbb{R}^2$ – произвольный фиксированный параметр.

Рассмотрим случай $k = 2$. Через c_b обозначим центр единичного шара b.

Пусть

$$B_- = \{x \in S_1(c_b) : \varphi_b(x) = -1\},$$
$$B_0 = \{x \in S_1(c_b) : \varphi_b(x) = 0\},$$
$$B_+ = \{x \in S_1(c_b) : \varphi_b(x) = 1\}.$$

Лемма. Для каждой конфигурации σ_b верно следующее:

$$U(\sigma_b) \in \{U_1, U_2, U_3, \ldots, U_{12}\},$$

где

$$U_1 = 0, \quad U_2 = -\frac{1}{2}J_1 - 2J_2, \quad U_3 = -J_1 - 8J_2, \quad U_4 = -\frac{3}{2}J_1 - 6J_2,$$
$$U_5 = -2J_1 - 8J_2, \quad U_6 = -J_1 - 2J_2, \quad U_7 = -\frac{3}{2}J_1, \quad U_8 = -\frac{3}{2}J_1 - 8J_2,$$
$$U_9 = -J_1 - 6J_2, \quad U_{10} = -3J_1, \quad U_{11} = -\frac{5}{2}J_1 - 2J_2, \quad U_{12} = -2J_1 - 2J_2.$$

Определение 2. Конфигурация φ называется основным состоянием гамильтониана H, если $U(\varphi_b) = \min\{U_1, U_2, U_3, \ldots, U_{12}\}$ для любого $b \in M$.

Обозначим

$$A_i = \left\{J \in \mathbb{R}^2 : U_i = \min\{U_1, U_2, U_3, \ldots, U_{12}\}\right\}, \tag{3}$$

где $i = 1, 2, \ldots, 12$.

Нетрудный, но достаточно громоздкий анализ показывает, что A_i имеет вид:

$$A_1 = \{(J_1, J_2) \in \mathbb{R}^2 | J_1 \le 0; J_1 + 8J_2 \le 0\},$$
$$A_2 = A_4 = A_6 = A_9 = A_{11} = A_{12} = \{(J_1, J_2) \in \mathbb{R}^2 | J_1 = 0; J_2 = 0\},$$
$$A_3 = \{(J_1, J_2) \in \mathbb{R}^2 | J_1 \le 0; J_1 + 8J_2 \ge 0\},$$
$$A_5 = \{(J_1, J_2) \in \mathbb{R}^2 | J_1 \ge 0; 0 \le J_1 \le 8J_2\},$$
$$A_7 = \{(J_1, J_2) \in \mathbb{R}^2 | J_1 = 0; J_2 \le 0\},$$
$$A_8 = \{(J_1, J_2) \in \mathbb{R}^2 | J_1 = 0; J_2 \ge 0\},$$
$$A_{10} = \{(J_1, J_2) \in \mathbb{R}^2 | J_1 \ge\ 0; 8J_2 \le J_1\}.$$

Теорема. Пусть $k = 2$. Для модели SOS с конкурирующими взаимодействиями Блюма-Капеля верны следующие утверждения:

1) на множестве A_1 существуют три трансляционно-инвариантные основные состояния и они имеют вид $\varphi(x) = m,\ m \in \Phi, \forall x \in G_k$;

2) если $(J_1, J_2) \in \mathbb{R}^2 \setminus A_1$, тогда не существует трансляционно-инвариантного основное состояния.

Замечание. Отметим, что модель (1) совпадает с моделью SOS, если $J_2 = 0$. В этом случае из теореми получим, что сушествуют три трансляцион-инвариантные основные состояния при $J_1 \le 0$. Это повторяет результат для SOS модели который получен в работе [3]. Также отметим, если в работе [2] предпологать, что $\alpha = 0$, а в теореме $J_2 = 0$, то полученные результаты совпадают.

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